1. a) (3, 3, 4, 4, 4, 6, 7, 7, 8, 8) satisfies Chvátal’s criterion, whence any graph with this degree sequence is Hamiltonian.
   b) (3, 3, 3, 3, 3, 3, 3, 3) doesn’t satisfy Chvátal’s criterion; no conclusion can be drawn.
   c) (1, 2, 3, 4, 4, 5, 5) doesn’t satisfy Chvátal’s criterion. We see that a graph with this degree sequence cannot have a Hamiltonian cycle because the first degree in the sequence is 1.

2  a, c) Can one visit each square of a $4 \times 4$ chessboard by a sequence of knight moves ending at the same square where one started?

Since $\kappa(G') = 6 > |S| = 4$, graph $G$ is not Hamiltonian. A knight tour is impossible on a $4 \times 4$ chessboard.

2 b) The degree sequence of $G$ is (2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4). It doesn’t satisfy either Ore’s or Chvátal’s condition.
3 a) To see that the Petersen graph has no Hamiltonian cycle \( C \), consider the edges in the cut disconnecting the inner 5-cycle from the outer one. If there is a Hamiltonian cycle, an even number of these edges must be chosen. If only two of them are chosen, their end-vertices must be adjacent in the two 5-cycles, which is not possible. Hence 4 of them are chosen. Assume that the top edge of the cut is not chosen (all the other cases are the same by symmetry). Of the 5 edges in the outer cycle, the two top edges must be chosen, the two side edges must not be chosen, and hence the bottom edge must be chosen. The top two edges in the inner cycle must be chosen, but this completes a non-spanning cycle, which cannot be part of a Hamiltonian cycle.

![Diagram of the Petersen graph](image)

3 b) The degree sequence of the Petersen graph is \((3, 3, \ldots, 3)\). To obtain a Hamiltonian graph with this degree sequence one can for example take the cyclic graph \( C_{10} \) and join every vertex to its “opposite”, that is to the vertex which is fifth in the sequence after the vertex under consideration.