# Correction to Hermitian Morita Theory: a Matrix Approach

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Our description of the adjoint involution $\mathrm{ad}_{h_0}$ in §1 should be corrected as follows:

$$\mathrm{ad}_{h_0}(X) = S^{-1}\overline{X}^t S, \quad \forall X \in M_n(D).$$

Consequently, $S$ should be replaced by $S^{-1}$ and vice versa everywhere in §2.

The proof of [1, Prop. 3.1] contains an error: the matrices $e_{ii}$ are not defined for all values of $i$ when $k > n$. We are very grateful to Bhanumati Dasgupta for pointing this out to us and for contributing to a correct proof which is presented below.

**Proposition 3.1.** *There exists an $\varepsilon$-hermitian $k \times k$-matrix $B \in M_k(D)$ such that*

$$h(x, y) = \overline{x}^t By, \ \forall x, y \in D^{k\times n}. \tag{1}$$

*Proof.* Let $B = (b_{ij})$. We will determine the entries $b_{ij}$. Let $e_{ij} \in D^{k\times n}$, $e'_{ij} \in D^{n\times k}$ and $E_{ij} \in M_n(D)$ respectively denote the $k \times n$-matrix, the $n \times k$-matrix and the $n \times n$-matrix with 1 in the $(i, j)$-th position and zeroes everywhere else. One can easily verify that

$$e_{if} E_{f\ell} = e_{i\ell}, \tag{2}$$

for all $1 \leq i \leq k$ and all $1 \leq f, \ell \leq n$. Also note that if $C \in M_n(D)$, then computing the product $E_{ij}C$ picks the $j$-th row of $C$ and puts it in row $i$ while making all other entries zero. Similarly, computing the product $CE_{ij}$ picks the $i$-th column of $C$ and puts it in column $j$ while making all other entries zero. The matrices $e_{ij}$ and $e'_{ij}$ behave in a similar fashion.

The matrices $\{e_{ij} \mid 1 \leq i \leq k, \ 1 \leq j \leq n\}$ generate $D^{k\times n}$ as a right $M_n(D)$-module. Thus it suffices to compute $h(e_{if}, e_{jg})$ for all

$1 \le i, j \le k$ and all $1 \le f, g \le n$. Using (2) we have for arbitrary $1 \le \ell, r \le n$ that

$$\begin{aligned} h(e_{if}, e_{jg}) &= h(e_{i\ell}E_{\ell f}, e_{jr}E_{rg}) \\ &= E_{f\ell}h(e_{i\ell}, e_{jr})E_{rg} \\ &= \big(h(e_{i\ell}, e_{jr})\big)_{\ell r}E_{fg}, \end{aligned}$$

where $\big(h(e_{i\ell}, e_{jr})\big)_{\ell r}$ denotes the $(\ell, r)$-th entry of the $n \times n$ matrix $h(e_{i\ell}, e_{jr})$. It follows that $\big(h(e_{i\ell}, e_{jr})\big)_{\ell r}$ is independent of the choice of $\ell$ and $r$. For all $1 \le i, j \le k$ we define

$$b_{ij} := \big(h(e_{i\ell}, e_{jr})\big)_{\ell r}.$$

Thus $h(e_{if}, e_{jg}) = b_{ij}E_{fg}$ for all $1 \le i, j \le k$ and all $1 \le f, g \le n$. We also have

$$\overline{e_{if}}^t Be_{jg} = e'_{fi}Be_{jg} = b_{ij}E_{fg}$$

for all $1 \le i, j \le k$ and all $1 \le f, g \le n$. Therefore,

$$h(e_{if}, e_{jg}) = \overline{e_{if}}^t Be_{jg}$$

for all $1 \le i, j \le k$ and all $1 \le f, g \le n$, which establishes (1).

Finally,

$$b_{ji}E_{gf} = h(e_{jg}, e_{if}) = \varepsilon\overline{h(e_{if}, e_{jg})}^t = \varepsilon\overline{b_{ij}}\,\overline{E_{fg}}^t = \varepsilon\overline{b_{ij}}E_{gf},$$

for all $1 \le i, j \le k$ and all $1 \le f, g \le n$, which implies $b_{ji} = \varepsilon\overline{b_{ij}}$, for all $1 \le i, j \le k$. In other words, $\overline{b_{ji}} = \varepsilon b_{ij}$, for $1 \le i, j \le k$, so that $\overline{B}^t = \varepsilon B$, which finishes the proof. ■

## REFERENCES

[1] Lewis, D.W. and Unger, T., Hermitian Morita Theory: a Matrix Approach. *Irish Math. Soc. Bulletin* **62** (2008), 37–41.

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