First Science Mathematics Tutorial Homework Problems 2005

1. For the vectors

$$\overrightarrow{u} = [3, 1, 5], \quad \overrightarrow{v} = [1, 1, 2],$$

find $\text{proj}_{\overrightarrow{u}} \overrightarrow{v}$ and find the cross product $\overrightarrow{u} \times \overrightarrow{v}$.

2. Find the distance of the point $A(2, -1, 0)$ from the plane $P : x - 2y - 5z = 0$.

3. Find the equation of the plane containing the points $A(1, 1, 6)$, $B(-2, 2, 4)$, $C(0, 1, 4)$.

4. Find the vector equation for the line that contains the point $(2, -1, 2)$ and is parallel to both the planes $P_1 : x - y + 2z = 4$ and $P_2 : 2x - 3y + z = 4$ (note that a line is parallel to a plane if the direction vector of the line is orthogonal to the normal vector of the plane).

5. Decide whether or not the following lines in $\mathbb{R}^3$ intersect, and if they intersect, find the coordinates of the intersection point.

$$L_1 : \overrightarrow{r}(t) = [-3, 1, 5] + t[1, 2, -4]$$
$$L_2 : \overrightarrow{m}(s) = [8, -1, 0] + s[3, -2, 1]$$

6. Show that

$$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \quad \text{is an eigenvector of the matrix} \quad \begin{pmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{pmatrix}$$

and find the corresponding eigenvalue.

7. Find the eigenvalues of the matrix

$$\begin{pmatrix} 0 & -3 & 5 \\ -4 & 4 & 10 \\ 0 & 0 & 4 \end{pmatrix}$$

and find a non-zero eigenvector for each eigenvalue.