

First Science Summer Tutorial Homework 2005

1. Find the critical points of the function $f(x) = x^2 - 2x + 5$.
2. Find the critical points of the function $f(x) = x^3 - 6x^2 - 15x + 12$.
3. Find the local maximum and minimum values of the function $f(x) = -x^3 - 2x^2 + x + 2$ and sketch the curve $y = f(x)$.
4. Find the vertical asymptotes of the function

$$f(x) = \frac{x^4 + 3x^2 + 5}{x^2 - 4}.$$

5. Find two positive numbers whose sum is 20 and whose product is as large as possible. Do not simply guess an answer!
6. The radius of a sphere is measured to be 25.5cm with an error of at most ± 0.01 cm. Use differentials to estimate the error in the resulting calculation of the volume V of the sphere ($V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere).
7. The population P of a city is growing exponentially and is given by the formula

$$P = (150,000)(1.04)^t,$$

where t is the number of years since 1995. What was the population in 1995? What was the population in 2000? How long will it take for the population to double the 1995 total? These questions require a pocket calculator.

8. Find all functions whose derivative is $7x^5$.
9. Evaluate the definite integrals

$$\int_0^2 x^2 + x^3 dx, \quad \int_1^3 2x^2 - 2x dx$$

10. Sketch the region in the plane enclosed by the curves $y = 3x^2 - 2$ and $y = x^2 + 6$. Express the area of this region as a definite integral and then evaluate the integral.