UNIVERSITY COLLEGE DUBLIN

AUTUMN EXAMINATIONS, 2004

SCBDF0001, SCBDF0015

FIRST SCIENCE EXAMINATION

B.Sc. COMPUTER SCIENCE, Year 1

MATH 1200: PASS MATHEMATICS Paper 1

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Instructions for Candidates

Candidates should attempt all 40 questions.
Each question should be answered A, B, C, D, or E on the Answer Sheet provided.
There is no negative marking.

Time allowed: two hours and thirty minutes

Notes for Invigilators

This is a *Multiple Choice Test*.

The candidates should record their answers on the EDPAC Answer Sheets provided.

The *SUBJECT* is *MATH1200*.

The candidates may use answer books for roughwork calculation.

Non-programmable calculators may be used during this test.

Programmable calculators, mathematical tables and graph paper may *not* be used.

ALGEBRA

1. What is the rank of the matrix $\begin{pmatrix} 1 & 2 & -1 \\ 3 & 1 & -2 \\ 1 & -1 & 1 \end{pmatrix}$?

   (A) 2 (B) 3×3 (C) 3 (D) 1 (E) 9

2. If $x,\ y$ and z are real numbers satisfying

$$\begin{array}{rcrcrcl} x & + & y & - & 2z & = & 5 \\ 2x & - & y & - & z & = & 4 \\ x & + & 2y & + & 2z & = & 2 \end{array}$$

   what is $x + y + z$?

   (A) 2 (B) 0 (C) 5 (D) -2 (E) 4

3. Which of the following matrices is not in row-echelon form?

   (A) $\begin{pmatrix} 1 & -4 \\ 0 & 1 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & 15 & 2 \\ 0 & 0 & 0 \end{pmatrix}$ (C) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

   (D) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$ (E) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

4. Which of the following is the reduced row-echelon form which can be obtained by elementary row operations from the matrix

$$\begin{pmatrix} 1 & -2 & -5 & 8 \\ 2 & 1 & -5 & 6 \\ 2 & 3 & -3 & 2 \end{pmatrix}?$$

   (A) $\begin{pmatrix} 1 & -2 & -5 & 8 \\ 0 & 1 & 1 & -2 \\ 0 & 1 & 1 & -2 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & -2 & -5 & 8 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & -2 \end{pmatrix}$ (C) $\begin{pmatrix} 1 & 0 & -3 & 4 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

   (D) $\begin{pmatrix} 1 & -2 & -5 & 8 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ (E) $\begin{pmatrix} 1 & -2 & -5 & 8 \\ 0 & 1 & 1 & 6 \\ 0 & 0 & 1 & 2 \end{pmatrix}$

5. Which of the following statements is false?

   (A) A system of 3 linear equations in 4 variables cannot have a unique solution.

   (B) A system of 3 linear equations in 4 variables can be inconsistent.

   (C) A system of 4 linear equations in 3 variables is always consistent.

   (D) A system of 4 linear equations in 3 variables can have infinitely many solutions.

   (E) A system of 4 linear equations in 3 variables can have a unique solution.

6. The following is a reduced row-echelon form obtained by elementary row operations from the augmented matrix of a system of linear equations.

$$\begin{pmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

   Which of the following statements about the system is false?

   (A) The system is consistent.

   (B) The system has four variables.

   (C) The system has three leading variables.

   (D) The system has infinitely many solutions.

   (E) The system has two free variables.

7. What is the general solution of the following system of equations?

$$\begin{array}{rcrcrcr} x & - & 2y & - & z & = & -3 \\ 2x & - & 3y & - & 4z & = & -4 \\ 2x & + & y & - & 12z & = & 4 \end{array}$$

(A) $\begin{array}{rcl} x &=& 1+5t \\ y &=& 2+2t \\ z &=& t, \quad t \in \mathbb{R} \end{array}$

(B) $\begin{array}{rcl} x &=& 1 \\ y &=& 2 \\ z &=& 0 \end{array}$

(C) $\begin{array}{rcl} x &=& -5+t \\ y &=& 2-2t \\ z &=& t, \quad t \in \mathbb{R} \end{array}$

(D) $\begin{array}{rcl} x &=& 6 \\ y &=& 4 \\ z &=& 1 \end{array}$

(E) $\begin{array}{rcl} x &=& 1-5t \\ y &=& 2-2t \\ z &=& t, \quad t \in \mathbb{R} \end{array}$

8. For what value of k is the following system *inconsistent*?

$$\begin{array}{rcrcrcr} x & - & 2y & + & 3z & = & -4 \\ x & - & y & + & 2z & = & 2 \\ 3x & - & 4y & + & kz & = & -4 \end{array}$$

(A) 7 (B) 0 (C) -1 (D) Consistent for all k (E) -7

9. If $A = \begin{pmatrix} 3 & -1 \\ -3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 4 \\ -5 & 2 \end{pmatrix}$, what is $(A - B^{tr})^{tr}$? (The superscript tr denotes transpose).

(A) $\begin{pmatrix} 2 & 2 \\ -5 & 0 \end{pmatrix}$ (B) $\begin{pmatrix} 2 & 4 \\ -7 & 0 \end{pmatrix}$ (C) $\begin{pmatrix} 2 & -5 \\ 2 & 0 \end{pmatrix}$

(D) $\begin{pmatrix} 2 & -7 \\ 4 & 0 \end{pmatrix}$ (E) $\begin{pmatrix} 4 & 1 \\ -6 & 4 \end{pmatrix}$

10. If $A = \begin{pmatrix} 1 & 5 \\ 2 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} -3 & 1 \\ 2 & -1 \end{pmatrix}$, what is AB?

(A) $\begin{pmatrix} -13 & 6 \\ -2 & 0 \end{pmatrix}$ (B) $\begin{pmatrix} 7 & -4 \\ -10 & 4 \end{pmatrix}$ (C) $\begin{pmatrix} -1 & -17 \\ 0 & 12 \end{pmatrix}$

(D) $\begin{pmatrix} -3 & 5 \\ 4 & 2 \end{pmatrix}$ (E) $\begin{pmatrix} -2 & -1 \\ -3 & 5 \end{pmatrix}$

11. Calculate the matrix product

$$\begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} 3 & -2 & 0 \\ 1 & 4 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}.$$

(A) (11) (B) (-28) (C) (7 0 4) (D) (28) (E) (0)

12. If $A = \begin{pmatrix} -2 & 3 \\ 5 & -8 \end{pmatrix}$, what is $\text{adj}(A)$?

(A) $\begin{pmatrix} 8 & 3 \\ 5 & 2 \end{pmatrix}$ (B) $\begin{pmatrix} 2 & -3 \\ -5 & 8 \end{pmatrix}$ (C) 1

(D) $\begin{pmatrix} -8 & 3 \\ 5 & -2 \end{pmatrix}$ (E) $\begin{pmatrix} -8 & -3 \\ -5 & -2 \end{pmatrix}$

13. What is the inverse of the matrix $\begin{pmatrix} -5 & -2 \\ 9 & 4 \end{pmatrix}$?

(A) $\frac{1}{2}\begin{pmatrix} -4 & -2 \\ 9 & 5 \end{pmatrix}$ (B) $\begin{pmatrix} 4 & 2 \\ -9 & -5 \end{pmatrix}$ (C) $-\frac{1}{2}\begin{pmatrix} -5 & -2 \\ 9 & 4 \end{pmatrix}$

(D) -2 (E) $-2\begin{pmatrix} 4 & 2 \\ -9 & -5 \end{pmatrix}$

14. If A and B are invertible 2×2 matrices, and X is a 2×2 matrix for which

$$AXB = A + B + AB,$$

which of the following is true?

(A) $X = B^{-1} + A^{-1}$.
(B) $X = 3I_2$ (where I_2 denotes the 2×2 identity matrix).
(C) $X = AB$.
(D) $X = B^{-1} + A^{-1} + B^{-1}ABA^{-1}$.
(E) $X = B^{-1} + A^{-1} + I_2$.

15. If $A = \begin{pmatrix} 4 & 2 \\ 5 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -2 \\ -2 & 2 \end{pmatrix}$, find a 2×2 matrix X for which $AX = B$.

(A) $\frac{1}{2}\begin{pmatrix} 18 & 10 \\ 23 & 13 \end{pmatrix}$ (B) $\frac{1}{2}\begin{pmatrix} 12 & 14 \\ 16 & 19 \end{pmatrix}$ (C) $\frac{1}{2}\begin{pmatrix} 13 & -10 \\ -23 & 18 \end{pmatrix}$

(D) $\frac{1}{2}\begin{pmatrix} 19 & -14 \\ -16 & 12 \end{pmatrix}$ (E) $\frac{1}{2}\begin{pmatrix} 9 & 4 \\ 10 & 8 \end{pmatrix}$

16. If $A = \begin{pmatrix} 2 & -2 & 3 \\ 4 & 5 & -2 \\ 2 & -1 & 0 \end{pmatrix}$, what is the cofactor of $(A)_{12}$ (the entry in the 1st row and 2nd column of A)?

(A) -4 (B) 8 (C) -2 (D) 4 (E) 2

17. If A is the matrix of Q. 16 above, what is $\det(A)$?

(A) 10 (B) 0 (C) -38 (D) -6 (E) -30

18. If A is the matrix of Q. 16 above, what is $\text{adj}(A)$?

(A) $\begin{pmatrix} -2 & 3 & -11 \\ 4 & -6 & -16 \\ -14 & 2 & 18 \end{pmatrix}$ (B) $\begin{pmatrix} -2 & 4 & -14 \\ 3 & -6 & 2 \\ -11 & -16 & 18 \end{pmatrix}$ (C) $\begin{pmatrix} -2 & -4 & -14 \\ -3 & -6 & -2 \\ -11 & 16 & 18 \end{pmatrix}$

(D) $\begin{pmatrix} 2 & -4 & 2 \\ 2 & 5 & 1 \\ 3 & 2 & 0 \end{pmatrix}$ (E) $\begin{pmatrix} -2 & -3 & -11 \\ -4 & -6 & 16 \\ -14 & -2 & 18 \end{pmatrix}$

19. Find (all) the values of k for which the system

$$\begin{aligned} x - 2y + z &= 12 \\ 3x - 6y + kz &= 500 \\ 2x + ky + 5z &= -10000 \end{aligned}$$

does *not* have a unique solution.

(A) 3 (B) $-4, 3$ (C) $-3, -4, 0$ (D) $-3, 4$
(E) $3, -3, 4, -4$

20. Which of the following statements is false?

(A) If A is an invertible square matrix, then every system of linear equations having A as coefficient matrix is consistent.

(B) If A and B are invertible square matrices of the same size, then the product AB is also invertible.

(C) If A and B are invertible square matrices of the same size, then their sum $A + B$ is also invertible.

(D) If the determinant of the square matrix A is a positive number, then A is invertible.

(E) If A is an invertible square matrix, then the transpose of A is also an invertible square matrix.

CALCULUS

21. One and only one of the following statements in mathematical notation is true. Which one?

    (A) $-4 \leq -5$.

    (B) $\mathbb{R} \subset \mathbb{Q}$.

    (C) $\pi \in \mathbb{R}$.

    (D) $\sqrt{-\pi} \in \mathbb{Z}$.

    (E) $\mathbb{Z} \subset \mathbb{N}$.

22. The domain of the function

$$f(x) = \sqrt{2 - x}$$

    is

    (A) $(-2, 2)$ (B) $(-\infty, 2)$
    (C) $(-\infty, 2]$ (D) $(2, \infty)$ (E) $[2, \infty)$

23. The domain of the function

$$f(x) = \frac{1}{x^2 - 1}$$

    is

    (A) Every real number except -1 and 1.

    (B) All positive real numbers.

    (C) Every real number except 0.

    (D) All real numbers less than -1.

    (E) The natural numbers.

24. The exact value of $49^{-3/2}$ is

    (A) 1/7 (B) 343 (C) 1/343 (D) 0.002915 (E) 0.000004249

25. Let f be the function $f(x) = \lfloor 11x \rfloor$ (where $\lfloor a \rfloor$ denotes the greatest integer which is smaller than or equal to a). Then $f(2.32) =$

    (A) 2 (B) 3 (C) 25 (D) 26 (E) 0.52

26.

$$\lim_{x \to -2} \frac{1}{x^2 + 1} =$$

    (A) -2 (B) 5 (C) 1.25 (D) $-1/5$ (E) 1/5

27.
$$\lim_{x\to\infty} \frac{2x^3+1}{5-3x^4} =$$

(A) 2/3 (B) $-2/3$ (C) 0 (D) 1/5 (E) It does not exist.

28.
$$\lim_{x\to 1} \frac{x^2-2x+1}{x^2-1} =$$

(A) 1 (B) 0 (C) -1 (D) -2
(E) does not exist.

29. If $f(x) = 5x^4$, then $f'(x) =$

(A) x^5 (B) $x^5 + C$ (C) $20x^3$ (D) $20x^3 + C$
(E) $-20/x^5$.

30. The derivative of the function $f(x) = x^2 - 5x + 2$ at 2 is -1. The point $(2, -4)$ lies on the graph of this function. The equation of the tangent line to the graph at this point is:

(A) $y = -x - 6$ (B) $y = -x + 6$ (C) $y = x - 6$

(D) $y = -x - 2$ (E) $y = -x + 2$

31. If $f(x) = 2/x^7$, then $f'(x) =$

(A) $14/x^8$ (B) $-14/x^8$ (C) $14/x^6$ (D) $1/(14x^6)$
(E) $-14/x^6$.

32. If $f(x) = \sqrt{x^3}$, then $f'(x) =$

(A) $\frac{3\sqrt{x}}{2}$ (B) $\frac{1}{2\sqrt{x^3}}$ (C) $\frac{1}{3x^{2/3}}$
(D) $\frac{3}{2\sqrt{x}}$ (E) $3x$

33.
$$\frac{d}{dx}\left(\frac{4x}{x^2+1}\right) =$$

(A) $\dfrac{4}{2x}$ (B) $\dfrac{4}{(x^2+1)^2}$ (C) $\dfrac{12x^2+4}{(x^2+1)^2}$
(D) $\dfrac{4x^2-4}{(x^2+1)^2}$ (E) $\dfrac{4-4x^2}{(x^2+1)^2}$

34. Let $f(x) = \sqrt{x}$ and $g(x) = x^2 + 3x^3$. Then $(g \circ f)(x) =$

(A) $\sqrt{x^2+3x^3}$ (B) $x + \sqrt{3x^3}$
(C) $x + 3x\sqrt{x}$ (D) $2x + 9x^2$ (E) $1 + \frac{9}{2}\sqrt{x}$

35. If $f(x) = \sqrt{x^6 + 5}$, then $f'(x) =$

(A) $\sqrt{6x^5}$ (B) $\dfrac{x^6 + 5}{2\sqrt{x}} + 6x^5\sqrt{x}$ (C) $\dfrac{1}{2\sqrt{6x^5}}$

(D) $\dfrac{1}{2\sqrt{x^6 + 5}}$ (E) $\dfrac{6x^5}{2\sqrt{x^6 + 5}}$

36. If $f(x) = (x^5 + 2)^2$, then $f'(1) =$

(A) 50. (B) 10.
(C) 20. (D) 18.
(E) 30.

37. If $f(x) = x^2\sqrt{x^2 + 1}$, then $f'(x) =$

(A) $2x$ (B) $\dfrac{3x^3 + 2x}{\sqrt{x^2 + 1}}$ (C) $\dfrac{2x^2}{\sqrt{x^2 + 1}}$ (D) $\dfrac{4x^3 + x^2 + 4x}{2\sqrt{x^2 + 1}}$

(E) $\dfrac{x^4 + x^3 + x^2}{\sqrt{x^2 + 1}}$

38. If $y = \sqrt{x}$, then $\dfrac{d^2y}{dx^2} =$

(A) $\dfrac{1}{4\sqrt{x}}$ (B) $\dfrac{1}{2}x^{-1/2}$ (C) $-\dfrac{1}{4\sqrt{x^3}}$

(D) $-\dfrac{1}{4}x^{3/2}$ (E) $\dfrac{1}{6}x^{1/4}$

39. If $y = 3/x^5$, then $\dfrac{d^2y}{dx^2} =$

(A) $\dfrac{60}{x^3}$ (B) $-\dfrac{15}{x^6}$ (C) $\dfrac{90}{x^7}$ (D) $\dfrac{3}{20x^3}$

(E) $-\dfrac{90}{x^7}$

40. A heavy rock falls over the edge of a 1500 metre cliff. After t seconds it has fallen $s = 4.9t^2$ metres. At approximately what speed is it moving when it hits the ground?

(A) 17 metres per second. (B) 86 metres per second.
(C) $9.8t$ metres per second. (D) 170 metres per second.
(E) 300 metres per second.