LIFE AND WORK OF

PROVOST GEORGE SALMON FRS

1819-1904

Lecture by

Roderick Gow

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The year 2005 is significant for the history of mathematics in Ireland, since it marks the two hundredth anniversary of the birth of William Rowan Hamilton. Lectures have already been given, and others are planned, to celebrate his achievements, which are highly valued nowadays. To a lesser degree, the year 2004 was also noteworthy in the history of mathematics in Ireland, since it saw the hundredth anniversary of the death of George Salmon, mathematician, theologian and Provost of Trinity College from 1888. We are not aware of any commemoration of Salmon's life and work that occurred in 2004, and it is our intention, somewhat belatedly, to acknowledge today Salmon's contributions to mathematics, theology and education.

It is not true to say that the centenary of Salmon's death passed unnoticed, for 1904 was also the year in which women were first admitted to Trinity College to study for degrees. As Provost, Salmon was an implacable enemy of the admission of women to the college and he is reported to have said that *women would only enter TCD over his dead body*. Salmon and a few like-minded colleagues had certainly resisted a full discussion of the merits of women's admission to the college but he withdrew his veto in July 1903, thus enabling the necessary changes in the statutes to be made. Salmon died on 22 January 1904, eight days after Letters Patent granting the admission of women had been issued. The first three women students began attending lectures at TCD in Hilary Term, 1904.

We have rather reversed the normal procedure for a biographical lecture by starting with events that occurred at the end of a long and productive life. Let us therefore return to the early years of his life. He was born on 25 September, 1819. This much seems to be agreed. In his 1897 entry for *Who's who*, Salmon wrote that he was born in Dublin. Other articles written before his death confirm this place of birth. With one exception that we have observed, contemporary obituaries claim that he was born in Dublin or else make no pronouncement on the subject. The exceptional obituary, a brief note written for the Royal Society of London by its president, the astronomer William Huggins, states that Salmon was born in Cork. The definitive article about Salmon, written by his former pupil John Henry Bernard, later Archbishop of Dublin and Provost of TCD, was that which was published in the supplement to the *Dictionary of National Biography* in 1913. Here, Bernard states, without explanation, that Salmon was born in Cork, and subsequent writers have largely adhered to this opinion.

It seems that we may never know the truth for certain. Pigott's *Commercial Directory* for 1820 lists Michael Salmon, father of George, as a linen merchant at the Linen Hall, Dublin (located in the north of Dublin, near the far end of Capel Street). In Pigott's *Commercial Directory* for 1824, Michael Salmon is listed as a linen merchant with premises

at 80 Grand Parade, Cork. This is interesting, as one Richard Dowden is also listed as a linen merchant at 96 Grand Parade. Now Salmon was related to the Dowden family, for he was the cousin of Alicia Bennett Dowden, herself the wife of John Wheeler Dowden (1799–1891). John Wheeler Dowden must have inherited the family linen business, which he ran successfully into his old age. His two sons John (1840-1910) and Edward (1843-1913) both achieved eminence. John became bishop of Edinburgh in 1886, and at his consecration, Salmon preached on the historic claims of episcopacy. Edward Dowden was the first professor of English Literature at TCD, holding the chair from 1867 until his death. He was a prolific author of books on poets and playwrights, such as Shakespeare, Shelley and Browning. He maintained a vigorous correspondence with family memebers, literary critics and academics, much of which is preserved in the archives of TCD. There are, for example, eighteen letters from Salmon to him, some of which make interesting reading. The same is not true of Salmon, whose own correspondence does not seem to have survived. Anyone who attempts to discover new facts about Salmon will probably have to rely on archive sources containing his letters to J. J. Sylvester, J. C. Adams, Dowden, J. H. Bernard, W. E. H. Lecky, T. A. Hirst, among others. Selections of Edward Dowden's letters were published soon after his death and one or two letters provide us with some insight into Salmon's character.

In a letter of 1873, Dowden wrote:

> Dr Salmon is, I don't know whether you know, a cousin of mine, my mother's first cousin. I had a month with him in Switzerland in 1865, I think. His mind is like an extraordinary natural force, operating in a sure instinctive way, and he seems able to go on with two or three sets of operations at the same time. He has usually about a dozen novels on hand, and the only things he can't read are poetry and metaphysics.

In a letter to Bernard (1911), he also wrote:

> I am glad you find a sense of beauty in his [Salmon's] geometrical work–of which I know nothing. I remember Boole wrote or spoke to me of the great possibilities of mathematical beauty and Norton of Montreal my class fellow, then a mathematical scholar, spoke to me of the great beauty of Boole's work and the lack of this in Salmon's, the latter pushing for results in any possible way.

> I remember his [Salmon's] reply to a letter from the country in which I said I was reading a volume of his Sermons. He said it was as Arctic voyagers eat old boots when they run short of other provisions.

Salmon was educated at the Cork school of Hamblin and Porter, where he received a good foundation in the study of classics. He matriculated at Trinity College, Dublin in 1833, and graduated as first Senior Moderator in Mathematics and Physics in 1838, when just nineteen years old. Salmon sat the Fellowship Examination in 1840 and was elected a Fellow of the College in 1841. He was ordained deacon in 1844 and priest in the Church of Ireland in 1845. Salmon's social milieu was very much that of a cleric. His mother was the daughter of a clergyman, his two sisters married English clergymen, his own wife Frances Anne Salvador whom he married in 1844, was the daughter of a Church of England clergyman, and we noted that his relative John Dowden became a bishop. Salmon himself did not attain the highest offices in the Church of Ireland. A person of his ability and organizational skills might have become a bishop, but Salmon's commitment to TCD perhaps disinclined him from seeking preferment elsewhere. Equally his robust opinions and writing style, combined with a certain tendency to biting or sardonic humour, may have acted against him in some ecclesiastical circles.

Salmon lived with his family for forty years in Wellington Road, Dublin. There were six children of the marriage, four boys and two girls, but only his son Edward, who lived in Dundrum, and daughter Frances Mary, who kept house for him while he was Provost, survived Salmon. A son Francis James died in 1860, aged two, and a son George died in New South Wales in 1874.

Nowadays, we would say that Salmon worked mainly in the field of algebraic geometry. Aspects of his work are still described in modern textbooks on the subject, for example, those of Abhyankar and Hartshorne. The most long lasting and original of Salmon's contribution to mathematical research arose from his correspondence with the English mathematician Arthur Cayley. Cayley had shown in 1849 that a non-singular cubic surface in the three-dimensional complex projective space contains at most a finite number of lines, and furthermore, that a such a surface always contains at least one line. Salmon was able to make Cayley's findings more precise, as he showed that there are exactly twenty seven lines on a non-singular cubic surface. To this day, the incidence structure known as the twenty seven lines on a cubic surface, with its associated symmetry group of order 51,840, remains one of the deepest and most intriguing subjects in algebraic geometry.

To illustrate the theory, consider the example of the affine cubic surface

$$x^3 + y^3 + z^3 = 1.$$

Exactly 27 complex lines lie on this surface. They are given by the following sets of

parametric equations:

$$x = \omega_1, \quad y = t, \quad z = -\omega_2 t$$

$$x = t, \quad y = \omega_1, \quad z = -\omega_2 t$$

$$x = t, \quad y = -\omega_2 t, \quad z = \omega_1,$$

where ω_1 and ω_2 satisfy

$$\omega_1^3 = \omega_2^3 = 1.$$

Note that there are exactly three real lines on the surface, namely

$$x = 1, \quad y = t, \quad z = -t$$

$$x = t, \quad y = 1, \quad z = -t$$

$$x = t, \quad y = -t, \quad z = 1.$$

While this is a simple example, to show that no more than 27 lines lie on the surface is not totally straightforward by elementary elimination techniques.

It is interesting to note that, during the summer of 1857, Salmon began working with William Rowan Hamilton on subjects related to quaternions and octonions. Hamilton wrote thus of Salmon:

> He is a splendid pupil, of whom I am justly proud: and very glad shall I be, when he comes to excel his teacher in this as in other departments... there is a genuine *commerce* of intellect between us; for *he* has been teaching *me* a variety of things, which I did not know before, about invariants, covariants, cogredients, contragredients, *etc*

Salmon's real achievement in mathematics lies in the influence exerted by four textbooks he wrote. These were:

A Treatise on Conic Sections (1848)

A Treatise on the Higher Plane Curves (1852)

Lessons Introductory to the Modern Higher Algebra (1859)

A Treatise on the Analytic Geometry of Three Dimensions (1862).

Each of these textbooks was translated into both French and German, and these translations often went through several editions. Interestingly enough, a Russian translation of the *Conic Sections* was said to have been made in 1860 by Mikhail Vashchenko-Zakharchenko, who worked in Kiev, but we have not been able to find independent confirmation of this fact. The German translations (in effect, reworkings), each by the German

mathematician Wilhelm Fiedler, were especially highly regarded. Fiedler's *Analytische Geometrie der Kegelschnitte*, freely adapted from Salmon's *Conic Sections*, appeared in nine editions between 1860 and 1918, and the seventh edition of 1907 contains a substantial commemoration of Salmon, written from Fiedler's personal recollections.

The most famous and enduring textbook, *A Treatise on Conic Sections*, was written for undergraduates at Trinity College. It passed through five further editions, and became one of the most popular and frequently quoted geometric textbooks of the second half of the 19th century. The book contains eight introductory chapters, dealing with the elementary coordinate geometry of lines and circles. This part of the book still makes useful reading for beginners in coordinate geometry and it is well supplied with examples for study. The next five chapters deal with the three types of conic section, that is, the ellipse, hyperbola, and parabola. The final chapter, on geometric methods, is the most advanced and it is an introduction to the modern geometry developed by Poncelet and Michel Chasles. The mixture of elementary and more sophisticated material seems to have appealed to students, teachers and researchers for several decades.

In 1866, Salmon was elected regius professor of divinity at TCD. From that time, he devoted himself mainly to New Testament scholarship, and his mathematical activity gradually diminished. Writing in his book *The Church of Ireland 1869-1969* (1975), R. B. McDowell described Salmon thus:

> As a theologian, he belonged to the common-sense school, and a devoted pupil [Newport White] who praised him for never shirking difficulties implies that Salmon's forte lay in critical analysis rather than in constructive synthesis...

> Fundamentally he was a strong protestant, averse to sentimentality, quick to scrutinize the claims of ecclesiastical authority and contemptuous of ritualism.

Salmon's writing style was robust and combative, and often hinted at the scathing humour others have observed in him. In a letter to a fellow clergyman, he wrote:

> I have been in several fights and have sometimes given hard knocks, but I have always been glad to make peace because I have found the giving usually pains myself as much as the adversary.

We can see something of Salmon's polemical style in his book *Historical Introduction to the Study of the Books of the New Testament* (1885). Here, he sought to challenge and refute some of the sceptical revisionist tendencies of the German school of theologians centred

on Ferdinand Baur (1792-1860), professor of historical theology at Tübingen University. Salmon wrote:

> On the one hand it is a waste of energy to try to kill what, if left alone, will be sure to die of itself: on the other hand, there is the danger that you might afterwards find notions, which I had passed by as too contemptible for refutation, circulating among half–learned people as the 'latest results' which 'eminent critics' had arrived at in Germany. But in the present case, I think I am safe in deciding that it is practically unnecessary for me to trouble myself about the opinions of those who carry their scepticism to a further point than Baur....

> But Baur is far from marking the lowest point of negative criticism. He found disciples who bettered his instruction, until it became as hard for a young Professor, anxious to gain a reputation for ingenuity, to make a new assault on a New Testament book, as it is now for an Alpine clubman to find in Switzerland a virgin peak to climb.

We may wonder if Salmon ever regretted his migration from mathematics to theology and, later, to administration. A letter of his to James Sylvester, written in 1877 gives us some of his opinions:

> But alas I am becoming very rusty; learning nothing new and forgetting half the old. I suppose you will be as little pleased with me for giving my time to the study of Gnostic heresies as I was at your giving yours to translating Horace....

> I think some Oxford men talk a great deal of nonsense about the endowment of Research. What security have you that the men you endow will research? But if you give your researcher a class of intelligent young men, you make sure of getting at least some good out of him in the way of teaching; the better the man he is the more he will stimulate his class: and if you don't overburden him with teaching, the class will stimulate.

In some ways, Salmon's provostship marked the highwater mark of TCD's prestige, seeing as it did the three hundredth anniversary of the foundation of the college. The Great War and independence of Ireland were to begin a new era in the fortunes of Trinity.

ADDITIONAL READING

- T. D. Spearman, *George Salmon 1819-1904*, Hermathena **151** (1991), 25–38.

- R. Gow, *George Salmon 1819-1904: his mathematical work and influence*, Irish Math. Soc. Bulletin **39** (1997), 26-76.

  Available at `http://maths.ucd.ie/ rodgow`

- Article: *George Salmon*, Oxford Dictionary of National Biography, Oxford University Press, 2004.