

**First Arts Modular Degree**  
**Mathematical Studies 2004–2005**

**Combinatorics and Number Theory Problem Sheet 2**

1. Evaluate  $\binom{10}{7}$  and  $\binom{14}{7}$ .
2. Let  $m$  and  $n$  be whole numbers with  $0 \leq m \leq n$ . Prove that

$$m \binom{n}{m} = n \binom{n-1}{m-1}.$$

3. In a set of size  $n$ , there are 7 times as many subsets of size 3 as there are subsets of size 2. Find the value of  $n$ .
4. Prove that  $\binom{2n}{n} = 2 \binom{2n-1}{n}$  and deduce that  $\binom{2n}{n}$  is an even integer.
5. Find the number of rearrangements of the letters of the word PERMUTATION. In how many rearrangements are the two T's next to each other? In how many rearrangements is A the first vowel to appear, then E, then I, then O, then U (counting from the left, as in PAMRETTINO)?
6. Ten different books are arranged on a shelf. In how many ways can this be done if one particular book  $A$  must always be to the left (but not necessarily immediately to the left) of a particular book  $B$ ? In how many of these arrangements (with  $A$  to the left of  $B$ ) are  $A$  and  $B$  next to each other?
7. How many integers between 1 and 999 inclusive contain exactly one 7 and at least one 8 among their digits?
8. A committee of four people is to be chosen from six men and six women. In how many ways can this be done? In how many of the committees so formed are there more men members than women?
9. How many different 10-digit numbers may be made from the number 1233212441? How many of these numbers begin and end with a 1?
10. At a certain examination, eight different papers are set, two of which are mathematical. In how many ways can the papers be arranged so that the two mathematical papers are not consecutive.