

A Miscellany of Mathematical Marvels

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Outline

Bang! Enormous Numbers

The Art of Googology

The Enigma of Infinity

Foucault Pendulum in Dublin

Fermi Resonance and Climate Change



Introduction

Given the diverse nature of the audience,
I will present a **mathematical miscellany**.

There are elements of pure maths
and a diversity of applications.

I will conclude with some ideas on a project
that may be of interest to you here in TUS.

I hope there is something for everyone.



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A Simple Demonstration

I will begin with a simple demonstration of something that has never been done before.

Take a pack of fifty-two playing cards, and mix them up by shuffling them well.

Claim: the resulting arrangement of cards has never occurred before and never will again, during the entire life of our Universe!



How Many Possible Arrangements?

The first card may be chosen in any of **52 ways**.

The next one can be any of the remaining **51 cards**.

For the third, there are **50 choices**, and so on, until just one card remains.



How Many Possible Arrangements?

The first card may be chosen in any of **52 ways**.

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For the third, there are **50 choices**, and so on, until just one card remains.

Therefore, the total number of possibilities is

$$52! \equiv 52 \times 51 \times 50 \times \cdots \times 3 \times 2 \times 1.$$

This number is called **factorial 52**.



Factorial 52 is Enormous

To call 52! “large” is a gross understatement.

52! = 8065817517094387857166063685640376 ...
... 6975289505440883277824000000000000

To a single figure of accuracy, this is 10^{68} .
That is, 1 followed by 68 zeros.

Note: We use **superscripts** to indicate large numbers:

$1000 = 10^3$ $1,000,000 = 10^6$ $1,000,000,000 = 10^9 \dots$



Uniqueness of the Card Deck

The Universe is 4×10^{17} seconds old.

Choose a random arrangement of cards every second. Only a tiny fraction of all orderings would be selected during the life of the Universe.

Even if a billion arrangements were chosen every second, there would be no chance of a duplicate.

So, the arrangement is unique:
it has never been seen before,
and it will never again be seen.



Bang! Bang! Bang!



Typical Comic-book 'bang' mark [Image from [vectorstock.com](https://www.vectorstock.com/)].



Bang! Bang! Bang!

There are many ways to generate enormous numbers.
One of the simplest is by using repeated factorials.

We can read the number $52!$ aloud as

“Fifty-two Bang”.

By taking factorials of factorials, we encounter
numbers large beyond our wildest imaginings.
The double factorial of 52 is utterly vast:

$52!!$ or “Fifty-two Bang Bang”.



Bang! Bang! Bang!

Let us consider a small beginning:

“3 Bang” is $3 \times 2 \times 1 = 6$.

“3 Bang Bang” is $6! = 720$.

“3 Bang Bang Bang” is $720!$.

But $720!$ is ineffable: we have already seen the vastness of $52!$, and $720!$ is incomparably larger:

$$3!!! = 6!! = 720! \approx 2.6 \times 10^{1746}$$



Bang! Bang! Bang!

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A curiosity:

$$4! = 24$$

$$4!! = 24! = 6.2 \times 10^{23}$$

$$\text{Avogadro's Number} = 6.02 \times 10^{23}$$



Title Slide in English

The Enduring Wonders of Enormous Numbers

Peter Lynch

**School of Mathematics & Statistics
University College Dublin**

India-Ireland Friendship Lecture Series

Indian Embassy, Dublin, 3 November 2022



Title Slide in Hindi

विशाल संख्या के
स्थायी चमत्कार

पीटर लिंग

स्कूल ऑफ मैथमेटिक्स एंड स्टैटिस्टिक्स
यूनिवर्सिटी कॉलेज डबलिन

भारत-आयरलैंड मैत्री व्याख्यान श्रृंखला

भारतीय दूतावास, डबलिन, 3 नवंबर 2022



Mathematics in India

- ▶ **Harappa Civilization.**
- ▶ **Vedic & Jaina Maths. Large Numbers.**
- ▶ **Classical Period.**
- ▶ **The Number System. Zero.**
- ▶ **Astronomy. Trigonometry.**
- ▶ **The Kerala School.**
- ▶ **Srinivasa Ramanujan.**



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Slides on my website at
<https://maths.ucd.ie/~plynch/Talks>



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The Art of Googology

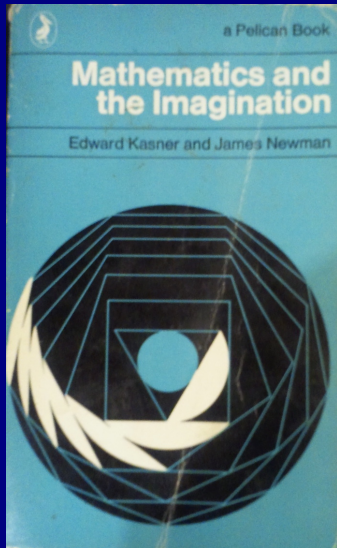
The Enigma of Infinity

Foucault Pendulum in Dublin

Fermi Resonance and Climate Change



A Googol



In 1938, a 9 year old boy coined the name **googol** for an enormous number.

Edward Kasner and James Newman popularised the name in this book, published in 1940.





The boy called the number a **googol**, defining it as
1 followed by a hundred zeros:

10 000 000 000 000 000 000 000 000 000 000 000 000 000
000 000 000 000 000 000 000 000 000 000 000 000 000
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This is long-winded to write. There is a better way:

$$1 \text{ googol} = 10^{100}.$$





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$$1 \text{ googol} = 10^{100}.$$

The company name **Google** is derived from **googol**.



A Googolplex

Kasner and Newman also defined a **googolplex**:
That is 1 followed by a googol zeros.

We could never write it without superscript notation.

$$1 \text{ googolplex} = 10^{\text{googol}} = 10^{(10^{100})}$$

This is a vast quantity, far beyond any of the
constants appearing in the physical sciences.



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The corporate headquarters of Google Inc.
in California is called the **Googleplex**.

The search for huge numbers is called **Googology**.



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The Enigma of Infinity

- ▶ The enigma of infinity has a long history.
- ▶ The stars in the sky seem beyond number.
- ▶ It has preoccupied and perplexed philosophers.
- ▶ It has mystified mathematicians for millennia.
- ▶ Children counting find **no largest number**.
- ▶ Given n , we have a larger number, $n + 1$.



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**Preoccupation with the infinite was originally in the domains of philosophy and theology.
Today, mathematics plays a central role.**



Euclid and Infinity

The word “infinite” does not appear anywhere in Euclid’s *Elements*.

Euclid proved that there is **no limit** to the number of primes, but he never stated that there is an infinity of prime numbers.

“Given any prime number, there is a greater prime”.

Parallel straight lines are straight lines which, being produced **indefinitely**, do not meet.



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“Given any prime number, there is a greater prime”.

Parallel straight lines are straight lines which, being produced **indefinitely**, do not meet.

This resonates with Aristotle’s **potential infinity**.

“You can keep going but you can never arrive.”



Galileo's Number Paradox

Galileo discovered that the counting numbers can be matched, one for one, with the perfect squares.

$$n \longleftrightarrow n^2 \quad \text{or} \quad \begin{array}{ccccccc} 1 & 2 & 3 & 4 & & & \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & & & \dots \\ 1 & 4 & 9 & 16 & & & \end{array}$$

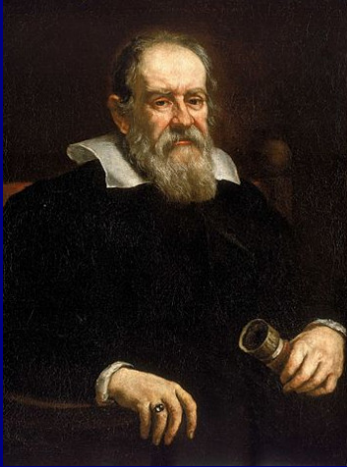
This result is in direct conflict with Euclid:

The whole is greater than the part.

Galileo argued that concepts like “**equal to**” and “**greater than**” are meaningless for infinite sets.



Galileo's Circle Paradox



Another paradoxical result of Galileo is that:

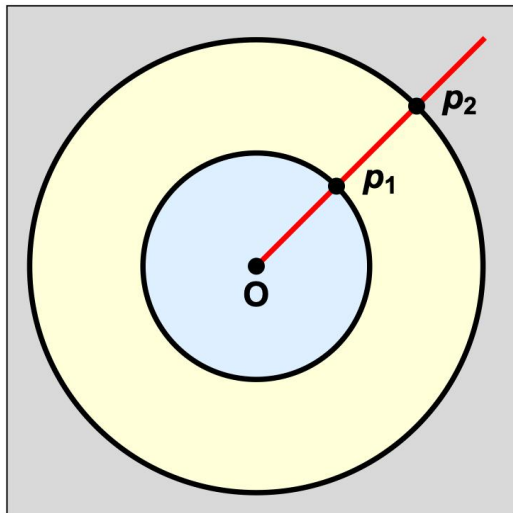
Two concentric circles of different radii contain the same number of points.

A simple diagram illustrates this paradox.



Galileo's Circle Paradox

Galileo's Circle Paradox



Georg Cantor



Inventor of **Set Theory**

**Born in St. Petersburg,
Russia in 1845.**

**Moved to Germany in
1856 at the age of 11.**

**His main career was at
the University of Halle.**



Georg Cantor (1845–1918)

- ▶ Invented Set Theory.
- ▶ One-to-one Correspondence.
- ▶ Infinite and Well-ordered Sets.
- ▶ Cardinal and Ordinal Numbers.
- ▶ Proved Rational Numbers are countable.
- ▶ Proved Real Numbers are uncountable.
- ▶ Infinite Hierarchy of Infinities.



A Passionate Mathematician

In 1874, Cantor married Vally Guttman.
They had six children.

According to Wikipedia:

“During his honeymoon in the Harz mountains, Cantor spent much time in mathematical discussions with Richard Dedekind.”

[Cantor had met the renowned mathematician Dedekind two years earlier while he was on holiday in Switzerland.]



Potential versus Actual Infinity

Cantor argued that a **potentially infinite** concept always arises from, and depends upon, a pre-existing **actually infinite** concept.



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Cantor argued that a **potentially infinite** concept always arises from, and depends upon, a pre-existing **actually infinite** concept.

Cantor's contributions were revolutionary. He showed that there is an unlimited **hierarchy of infinite numbers**.

Infinite sets do not all have the same size:
Some infinities are more infinite than others.

Cantor's ideas have had an enormous impact, on both mathematics and philosophy.



The Zermelo-Fraenkel Axioms

1.1. Axiom of Extensionality. If X and Y have the same elements, then $X = Y$.

1.2. Axiom of Pairing. For any a and b there exists a set $\{a, b\}$ that contains exactly a and b .

1.3. Axiom Schema of Separation. If P is a property (with parameter p), then for any X and p there exists a set $Y = \{u \in X : P(u, p)\}$ that contains all those $u \in X$ that have property P .

1.4. Axiom of Union. For any X there exists a set $Y = \bigcup X$, the union of all elements of X .

1.5. Axiom of Power Set. For any X there exists a set $Y = P(X)$, the set of all subsets of X .

1.6. Axiom of Infinity. There exists an infinite set.

1.7. Axiom Schema of Replacement. If a class F is a function, then for any X there exists a set $Y = F(X) = \{F(x) : x \in X\}$.

1.8. Axiom of Regularity. Every nonempty set has an $\in$ -minimal element.

1.9. Axiom of Choice. Every family of nonempty sets has a choice function.



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Infinity in Mathematics and Physics

Mathematics

- ▶ The sequence of natural numbers, 1, 2, 3, ...
- ▶ The points on a continuous line.
- ▶ The continuum of real numbers.
- ▶ Sets of sets, and the class of all sets.



Infinity in Mathematics and Physics

Mathematics

- ▶ The sequence of natural numbers, 1, 2, 3, ...
- ▶ The points on a continuous line.
- ▶ The continuum of real numbers.
- ▶ Sets of sets, and the class of all sets.

Physics

- ▶ Is there an infinitude of stars?
- ▶ Is space infinite in extent?
- ▶ Is space indefinitely divisible?
- ▶ Did time have a beginning?
- ▶ Does it endure forever?



The Shape of Space

The ultimate size of space is unknown:

We do not know whether space is limited in extent or whether it is unbounded.

We do not know whether time had a beginning or was “always there”.



Archytas and the size of the Universe

In a finite universe, what lies “beyond it”?



David Hilbert on “The Infinite!”

*“No other question has ever moved
so profoundly the spirit of man;*

*“no other idea has so fruitfully
stimulated his intellect;*

*“yet no other concept stands in
greater need of clarification
than that of the infinite ... ”.*



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Replication of Léon Foucault's Pendulum Experiment in Dublin

Peter Lynch
School of Mathematics & Statistics
University College Dublin

Irish History of Mathematics Conference
Edward Worth Library, Dublin
9 June 2017



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Setting the Scene: Is the Earth Spinning?

Despite evidence “staring us in the face”, most people believe that the Earth rotates on its axis.

Until about 1850, the only evidence for this came from **astronomical observations**.

In 1851, a way was found to demonstrate by **mechanical means** that the Earth is spinning.

We examine the remarkable pendulum experiment of **Léon Foucault**, and some of its consequences.



“You are Invited to See the Earth Turn ...”

In March 1851, a large crowd gathered in the Panthéon in Paris to witness the Earth spinning.

With simple apparatus comprising a heavy ball swinging on a wire, Léon Foucault showed how the Earth rotates on its axis.

His pendulum demonstration caused a sensation, and Foucault achieved instant and lasting fame.



Watch for a While: Swing-plane Rotates.



Figure: Foucault pendulum in the Panthéon, Paris.



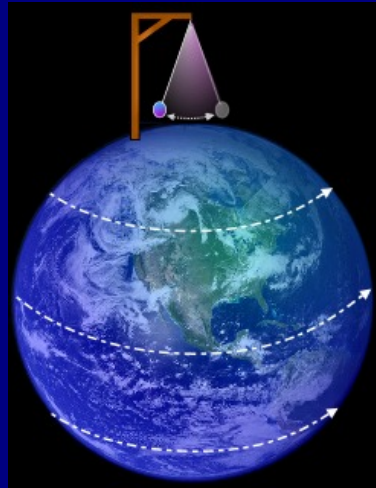
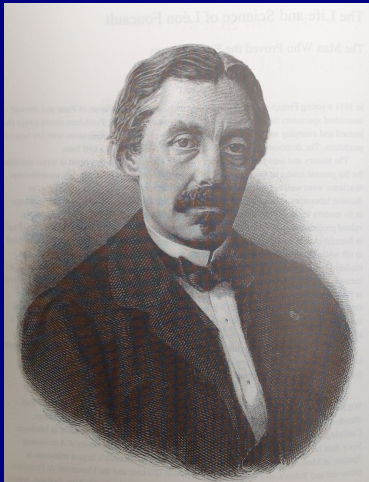


Figure: Simplistic explanation of pendulum precession.

Widespread International Interest

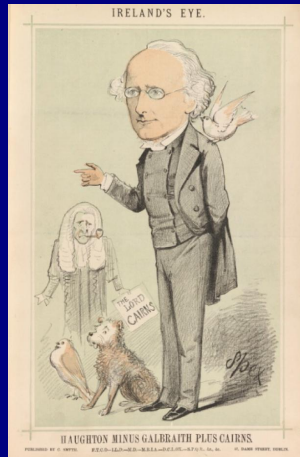
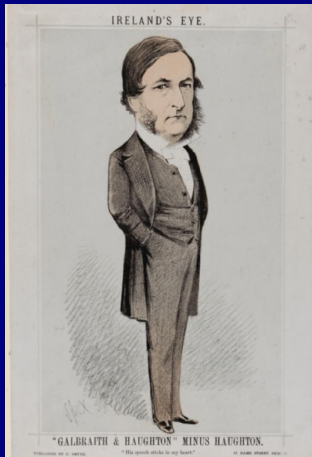
In the aftermath of Foucault's demonstration, **pendulum mania** raged across Europe and USA.

The experiment was repeated hundreds of times.

By June 1851 it had been repeated in many cities in Europe and America, including Dublin.

Many of these attempts were done without due care.





Cartoons from *Ireland's Eye*, October 1874.
Left: 'Galbraith & Haughton minus Haughton'.
Right: 'Haughton minus Galbraith plus Cairns'.



Experiments in Dublin

Experiments were done in April and May 1851.

Objective of the Experiments:

“To determine the azimuthal motion of the plane of vibration of a freely suspended pendulum”.

The experiments were at the engine-factory of the **Dublin & Kingstown Railway**, where Samuel's cousin Wilfred Haughton was Chief Engineer.

Six experiments were carried out, generally lasting between fifteen and twenty hours, with the azimuth being recorded every twenty minutes.



Observed Precession Rate

Foucault had derived the theoretical result:

$$\omega = \Omega \sin \theta$$

For Dublin, this implies precession through a **full circle in about 30 hours.**

The theoretical precession rate at the latitude of Dublin is $\omega = 12.07^\circ/\text{hr}$.

In the final experiment a full rotation of 360° was achieved **after 28 hours 26 minutes.**



International Reaction

In September 1851, the *American Journal of Science* surveyed several pendulum demonstrations in Europe and America.

These included the experiments carried out in Dublin.

According to the *Philosophical Magazine*,
'Messrs. Galbraith and Haughton ... have pursued their research with all imaginable precautions'.

Their impressive results confirm this assessment.



A complete account, with extracts from the Diary of Joseph Galbraith, is available online:

In Retrospect

Replication of Foucault's pendulum experiment in Dublin

PETER LYNCH*

University College Dublin, Belfield, Dublin 4

[Accepted 13 October 2015. Published 2 June 2016.]

Peter Lynch, 2016: *Proc. Roy. Irish Acad.*, 116C, 297–311.

<https://maths.ucd.ie/~plynch/Publications>



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How CO₂ Absorbes Radiative Energy

Recently a key property of CO₂ has been elucidated.

Global warming is tied to how CO₂ molecules vibrate.

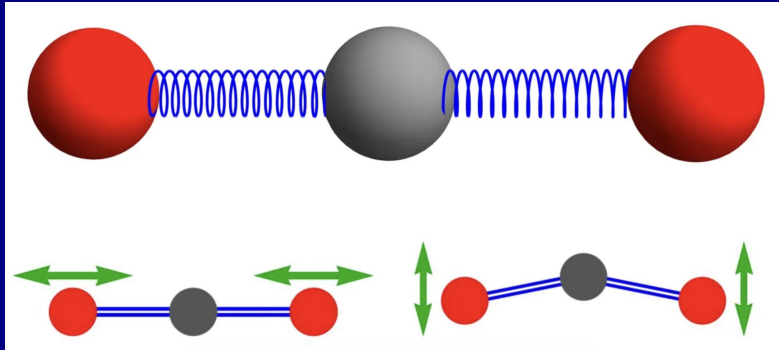
CO₂ absorbs energy at wavelengths over a broad band around 15 microns ($\nu = 20$ THz).

CO₂ traps far more infrared radiation than a simple analysis would indicate.



Vibrations of a CO₂ Molecule

This diagram shows a CO₂ molecule and its principal vibration modes.



Stretching Mode

Bending Mode



Fermi Resonance

The frequency of the stretching vibration is **almost exactly twice that of the bending mode.**

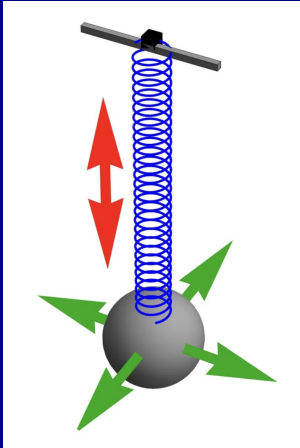
Bending mode vibrates 20 trillion times per sec.
Stretching mode vibrates 40 trillion times per sec.

These two types of oscillation are coupled:
the interaction is called Fermi resonance.
The complete explanation for how CO₂ behaves lies in quantum mechanics, but ...

... we can explain the essence of the phenomenon using the **elastic pendulum or *Swinging Spring*.**



The Swinging Spring



A heavy mass suspended from a spring

Two kinds of behaviour:

- ▶ **Spring up and down.**
- ▶ **Swinging back and forth.**

Resonant Motion: $\omega_Z = 2\omega_H$

If the Springing mode has **twice the frequency** of the swinging mode, the modes interact strongly:

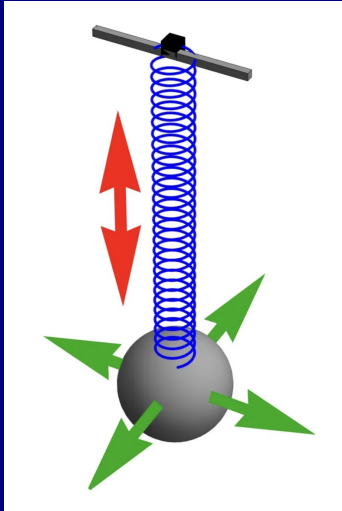
$$\omega_Z = 2\omega_H.$$

This nonlinear interaction is an example of Fermi Resonance, just like that of CO_2 .

It was discovered by Enrico Fermi in 1931.



The Swinging Spring



Demonstrate the SS or run the movie



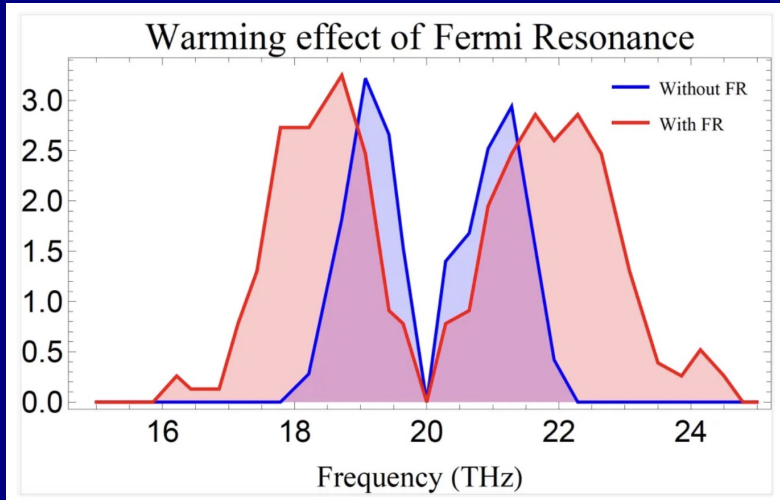
Relevance of Fermi Resonance

Fermi resonance might appear to be somewhat arcane, but it is relevant in many contexts:

- ▶ **Water waves,**
- ▶ **Rossby triad interactions in the atmosphere,**
- ▶ **Phonons (sound waves) in crystals,**
- ▶ **Resonance in plasmas and**
- ▶ **Nonlinear optics.**
- ▶ **Climate change.**



Consequences for Climate Change



Without FR

With FR



In Conclusion

Fermi resonance (FR) has profound consequences for climate change.

FR broadens the spectrum of CO₂ absorption, contributing about half the total warming.

Without FR, CO₂ would absorb radiation only in restricted bands around 15 microns (20 THz).

With FR , the absorption band is broadened over a greater range of wavelengths.

As a result, additional CO₂ will force the equilibrium temperature to ever greater heights.



Putting Athlone at the Centre of Ireland

Of course, Athlone is already at the centre ...
... but you could make others aware of this.

Why not build a **BIG SWINGING SPRING?**

The project could involve engineers, physicists,
mathematicians, technologists, computer scientists,
and many, many others in the arts and sciences.



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I would be happy to help!



Thank You

धन्यवाद

