"New Ways of Doing Sums"

The Mathematical Life of George Gabriel Stokes

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12 November 2026



Outline

- George Gabriel Stokes
- Maths and Physics Papers
- Stokes the Physicist
- Life, Science and Faith (OUP)
- Stokes and the Royal Society
- Navier-Stokes Equations
- Millennium Prize
- Ocean Waves
- Miscellany



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George Gabriel Stokes, 1819–1903



George Gabriel Stokes,
born in Skreen, Sligo on 13 August 1819.


Skreen
Sligo
Collooney

Childhood and Education

George Gabriel Stokes was born in Skreen on 13 August 1819 — about 207 years ago.

He was the youngest of seven children of Rev. Gabriel Stokes, Rector of the Church of Ireland.

From an early age, Stokes showed signs of brilliance:

His school-teacher wrote that

"Master George was working out
new ways of doing sums,
better than those in the book."



The ‘Old’ Rectory at Skreen (c. 1900)



Descendants of Gabriel Stokes (1682–1768), Great-grandfather of GGS

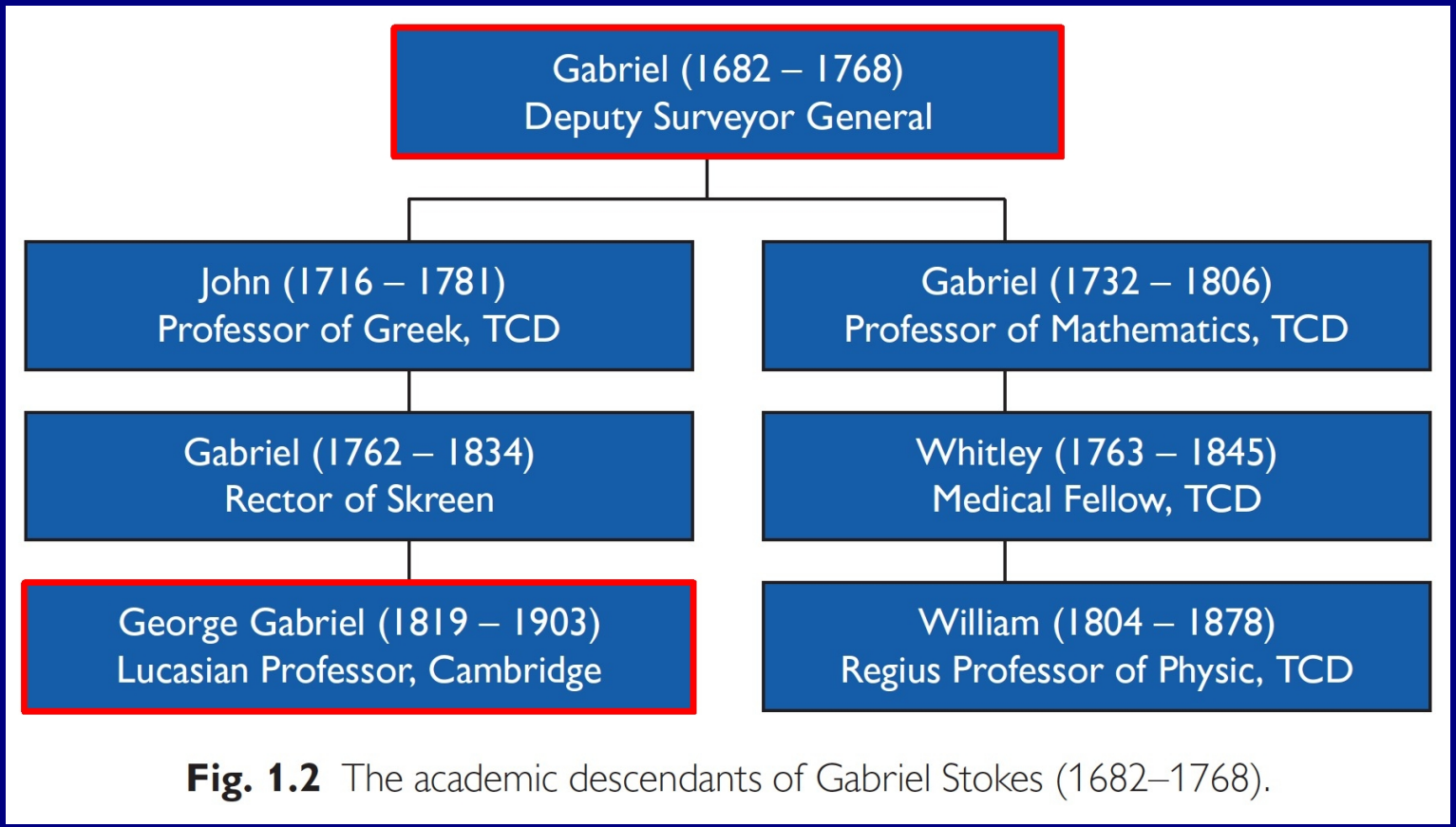


Fig. 1.2 The academic descendants of Gabriel Stokes (1682–1768).

Descendents of Stokes's great-grandfather Gabriel.

Childhood and Education

- Educated in Skreen, Dublin and Bristol.
- 1837: Pembroke College in Cambridge.
- 1841: Graduated as Senior Wrangler.

  First place in the Mathematical Tripos.
  Winner of the prestigious Smith's Prize.



Senior Wrangler

Success in the Tripos was a passport to a great career.

A relative wrote that Stokes had only

"...to decide whether he would be Prime Minister, Lord Chancellor or Archbishop of Canterbury."



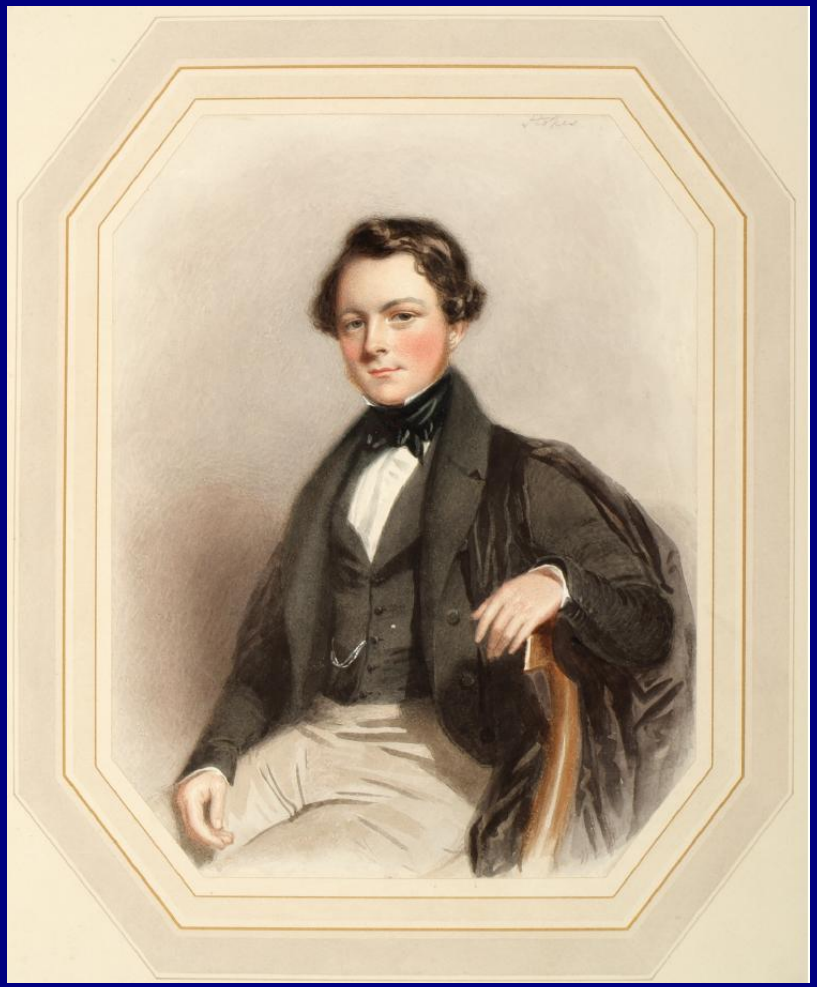
Wrangling and the Tripos (modern style)



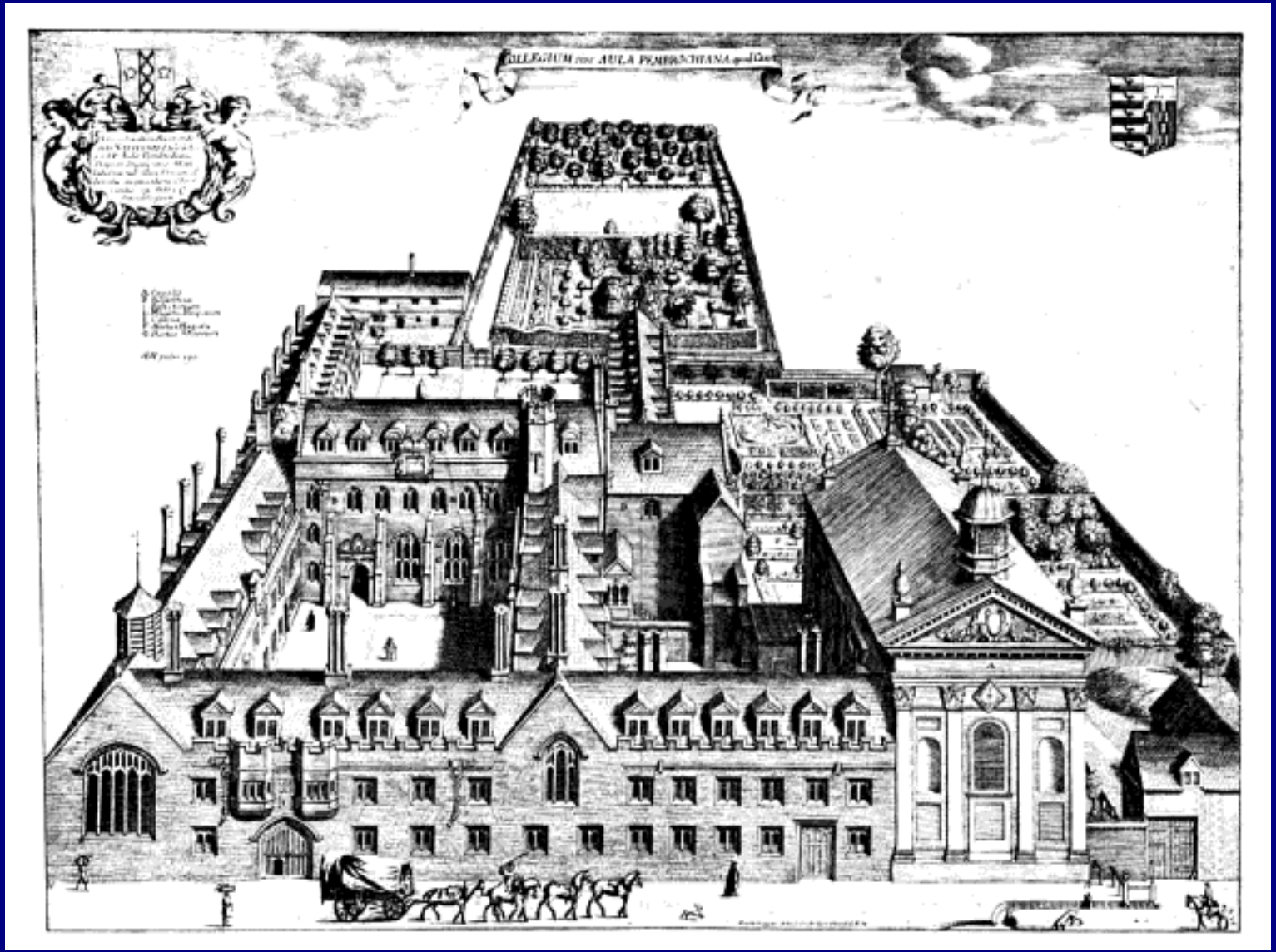
Giving out the results of the Mathematical Tripos.

Stokes as Senior Wrangler (1841)

Elected a Fellow of Pembroke College on the basis of his results in the Mathematical Tripos.



Pembroke College, 1690



Pembroke College, Victorian Era



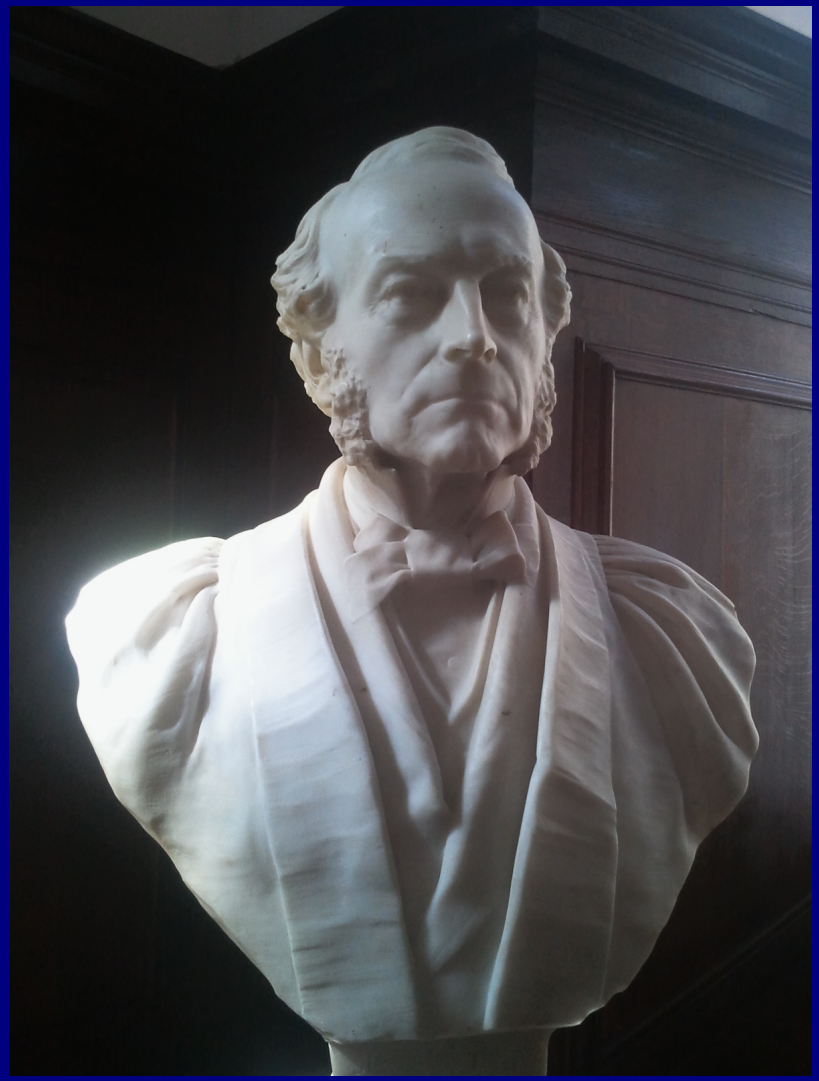
Pembroke College Today



Stokes's Progress

In 1849, Stokes was appointed **Lucasian Professor** of Mathematics.

He held this chair for over fifty years.



The Lucasian Chair of Mathematics

Table of Lucasian Professors of Mathematics

Isaac Barrow	1663–9	Charles Babbage	1828–39
Isaac Newton	1669–1702	Joshua King	1839–49
William Whiston	1702–10	**George Gabriel Stokes**	**1849–1903**
Nicholas Saunderson	1711–39	Joseph Larmor	1903–32
John Colson	1739–60	Paul Dirac	1932–69
Edward Waring	1760–98	James Lighthill	1969–80
Isaac Milner	1798–1820	Stephen Hawking	1980–2009
Robert Woodhouse	1820–2	Brian Green	2009–15
Thomas Turton	1822–6	Michael Cates	2015–
George Airy	1826–8		

The Lucasian Chair of Mathematics

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George Airy	1826–8		

Stokes's Wife Mary



In 1859 Stokes married Mary Susannah, daughter of **Thomas Romney Robinson**, astronomer at Armagh Observatory.

George and Mary had five children.

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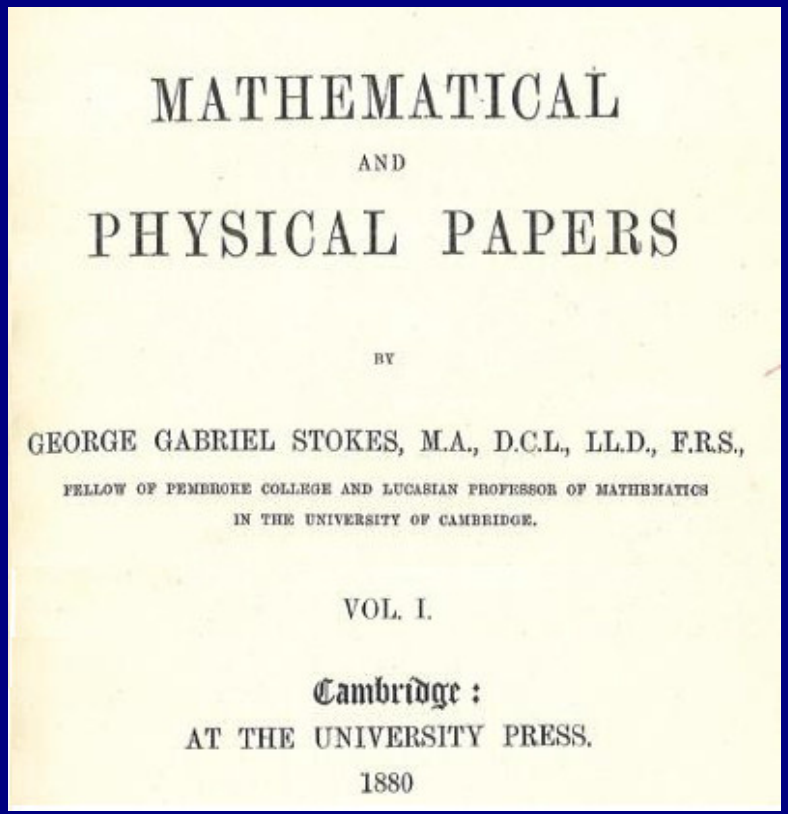
Miscellany



Mathematical & Physical Papers



MATHEMATICAL

AND

PHYSICAL PAPERS

BY

GEORGE GABRIEL STOKES, M.A., D.C.L., LL.D., F.R.S.,

FELLOW OF PEMBROKE COLLEGE AND LUCASIAN PROFESSOR OF MATHEMATICS IN THE UNIVERSITY OF CAMBRIDGE.

VOL. I.

Cambridge :

AT THE UNIVERSITY PRESS.

1880

Stokes's Collected Works, in 5 volumes, include some 140 papers.

Mathematical & Physical Papers

PAGE
Report on Recent Researches on Hydrodynamics 157
I. General theorems connected with the ordinary equations of Fluid Motion 158
II. Theory of waves, including tides 161
III. The discharge of gases through small orifices 176
IV. Theory of sound 178
V. Simultaneous oscillations of fluids and solids 179
VI. Formation of the equations of motion when the pressure is not supposed equal in all directions 182
Supplement to a Memoir on some cases of Fluid Motion 188
On the Theory of Oscillatory Waves 197
On the Resistance of a Fluid to two Oscillating Spheres 230
On the Critical Values of the Sums of Periodic Series 237
SECTION I.—Mode of ascertaining the nature of the discontinuity of a function which is expanded in a series of sines or cosines, and of obtaining the developments of the derived functions 239
SECTION II.—Mode of ascertaining the nature of the discontinuity of the integrals which are analogous to the series considered in Section I., and of obtaining the developments of the derivatives of the expanded functions 271
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SECTION IV.—Examples of the application of the formulæ proved in the preceding sections 286
Supplement to a paper on the Theory of Oscillatory Waves 314
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ERRATA.

P. 103, l. 14. *for* their *read* there.
P. 198, l. 3, *for* y^{s-b} *read* y_{s-b}.

ERRATUM.

P. 318, Equations (17) and (18). *For* − *read* + before the terms multiplied by $\sin 3\phi$ and $\cos 3\phi$.

UNIV. OF CALIFORNIA

MATHEMATICAL AND PHYSICAL PAPERS.

[From the *Transactions of the Cambridge Philosophical Society*, Vol. VII. p. 439.]

ON THE STEADY MOTION OF INCOMPRESSIBLE FLUIDS.

[Read April 25, 1842.]

IN this paper I shall consider chiefly the steady motion of fluids in two dimensions. As however in the more general case of motion in three dimensions, as well as in this, the calculation is simplified when $udx+vdy+wdz$ is an exact differential, I shall first consider a class of cases where this is true. I need not explain the notation, except where it may be new, or liable to be mistaken.

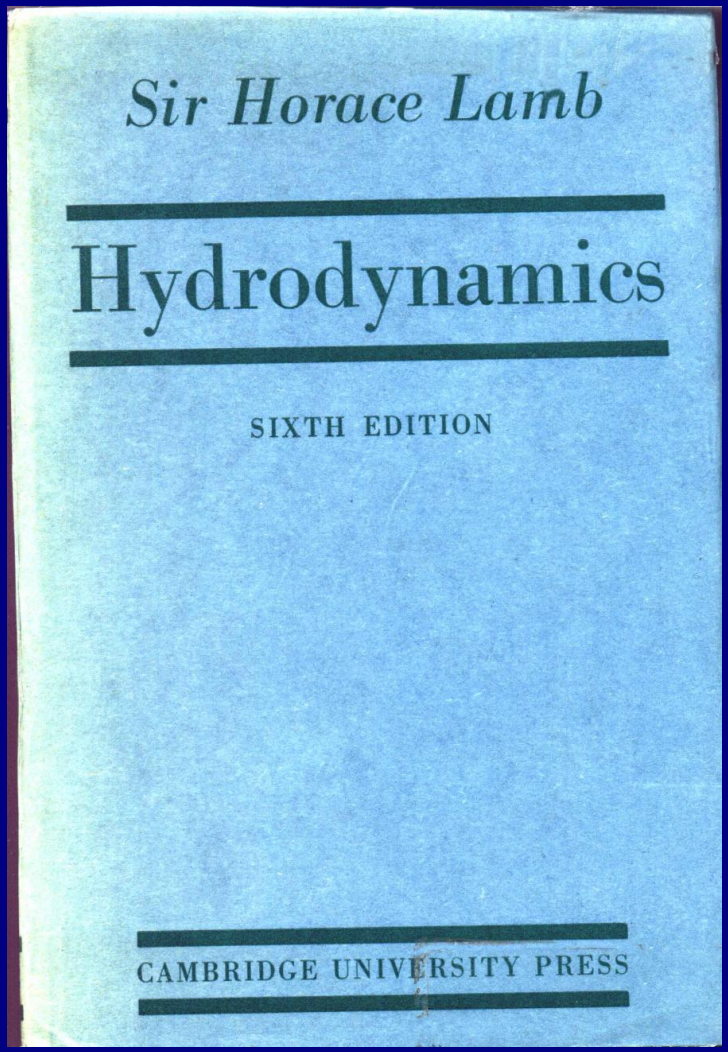
To prove that $udx+vdy+wdz$ is an exact differential, in the case of steady motion, when the lines of motion are open curves, and when the fluid in motion has come from an expanse of fluid of indefinite extent, and where, at an indefinite distance, the velocity is indefinitely small, and the pressure indefinitely near to what it would be if there were no motion.

By integrating along a line of motion, it is well known that we get the equation

$$\frac{p}{\rho} = V - \tfrac{1}{2}(u^2+v^2+w^2) + C \quad \ldots\ldots\ldots\ldots\ldots\ldots (1),$$

where $dV = Xdx + Ydy + Zdz$, which I suppose an exact differential. Now from the way in which this equation is obtained,

A Crude but indicative Metric



In his book *Hydrodynamics*, (6th edition), Horace Lamb has more than 50 page references to Stokes.

Some Contributions of Stokes

- Stokes Drag
- Stokes Drift
- Stokes's Law
- Stokes Waves
- Stokes's Theorem
- Stokes Parameters
- Stokes Phenomenon
- The Navier-Stokes Equations
- Campbell-Stokes Sunshine Recorder



Stokes's Theorem

An important theorem in vector calculus.

$$\oint_\Gamma \mathbf{V}\cdot d\mathbf{l} = \int\int_\Sigma \nabla \times \mathbf{V}\cdot \mathbf{n}\, da\,.$$

Stokes's Theorem was discovered by Kelvin in 1854.

It is of central importance in fluid dynamics.



Stokes Drag and Stokes's Law

A Child's Query:

Child: **Daddy, why don't clouds fall down?**

Daddy: **Clouds do fall, but very slowly!**



Stokes Drag and Stokes's Law

A Child's Query:

Child: **Daddy, why don't clouds fall down?**

Daddy: **Clouds do fall, but very slowly!**

Stokes formulated the drag law for small particles:

$$F = 6\pi\mu r v$$

This leads an expression for the terminal velocity:

$$v_s = \frac{2r^2\rho g}{9\mu}$$

A droplet of radius 5 microns falls with a terminal speed of about 3 mm/s (about four days for 1 km).



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A Talented Experimentalist

Stokes's combined mathematical sophistication with a great experimental facility.

He devised and performed many ingenious experiments in optics.

His work gave evidence supporting the wavelike nature of light.

He carried out a spectral analysis of blood, showing how oxygen is carried by haemoglobin.



Supernumerary Rainbows

Stokes evaluated Airy's integral

$$\mathrm{Ai}(x) = \frac{1}{\pi}\int_0^\infty \cos\left(\frac{t^3}{3} + xt\right)\mathrm{d}t$$

for large values of x.

Stokes developed new methods of exponential asymptotics which still have many important applications.



Fluorescence

Stokes elucidated the strange phenomenon of fluorescence with a radically new theory.

Objects are normally invisible in ultra-violet light, but a fluorescent body emits light at a lower frequency.

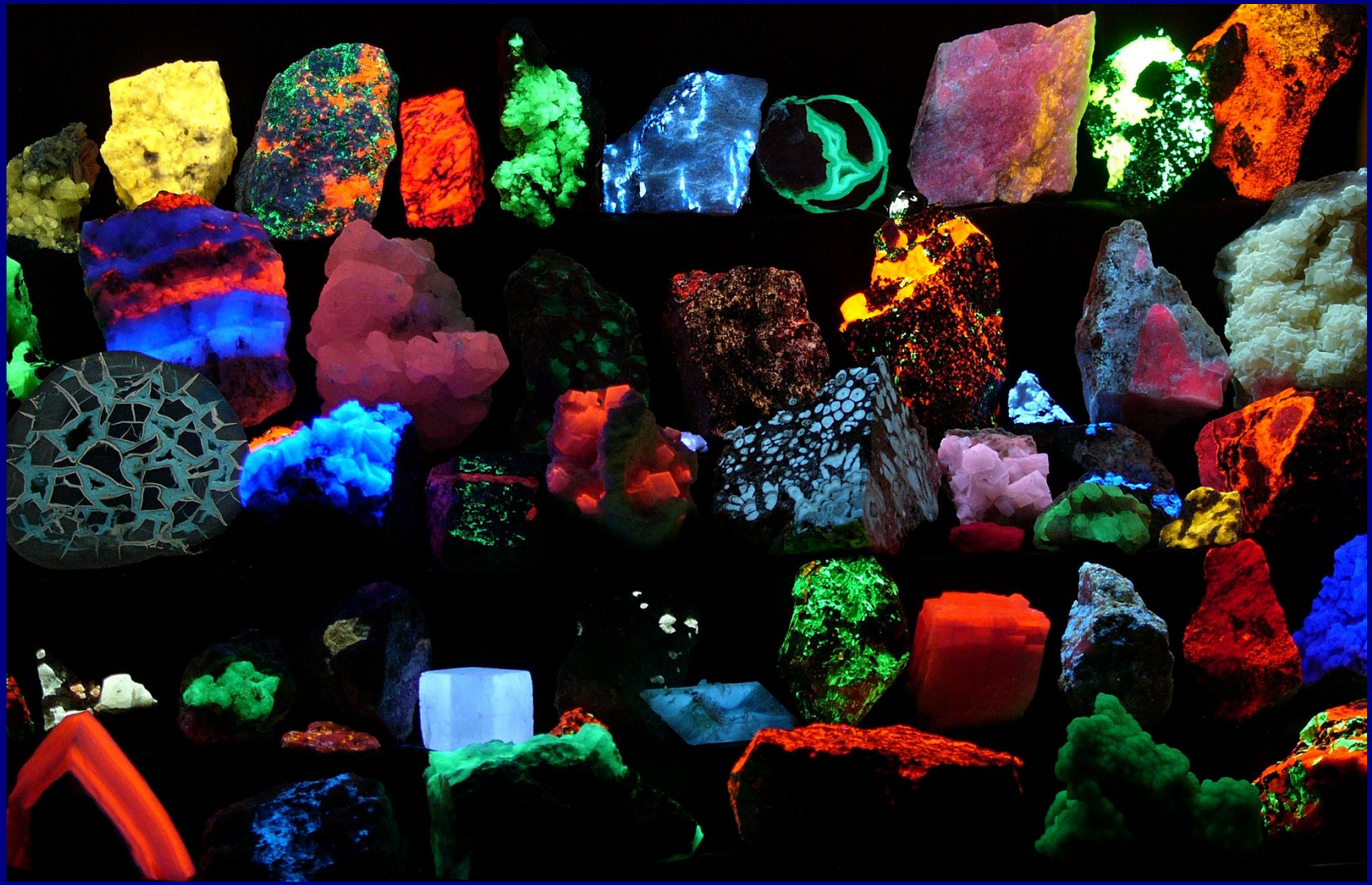
Stokes knew that fluorescence is found in many biological systems in the marine environment.

Stokes realised that his work on fluorescence offered a way to measure the UV spectrum of sunlight.

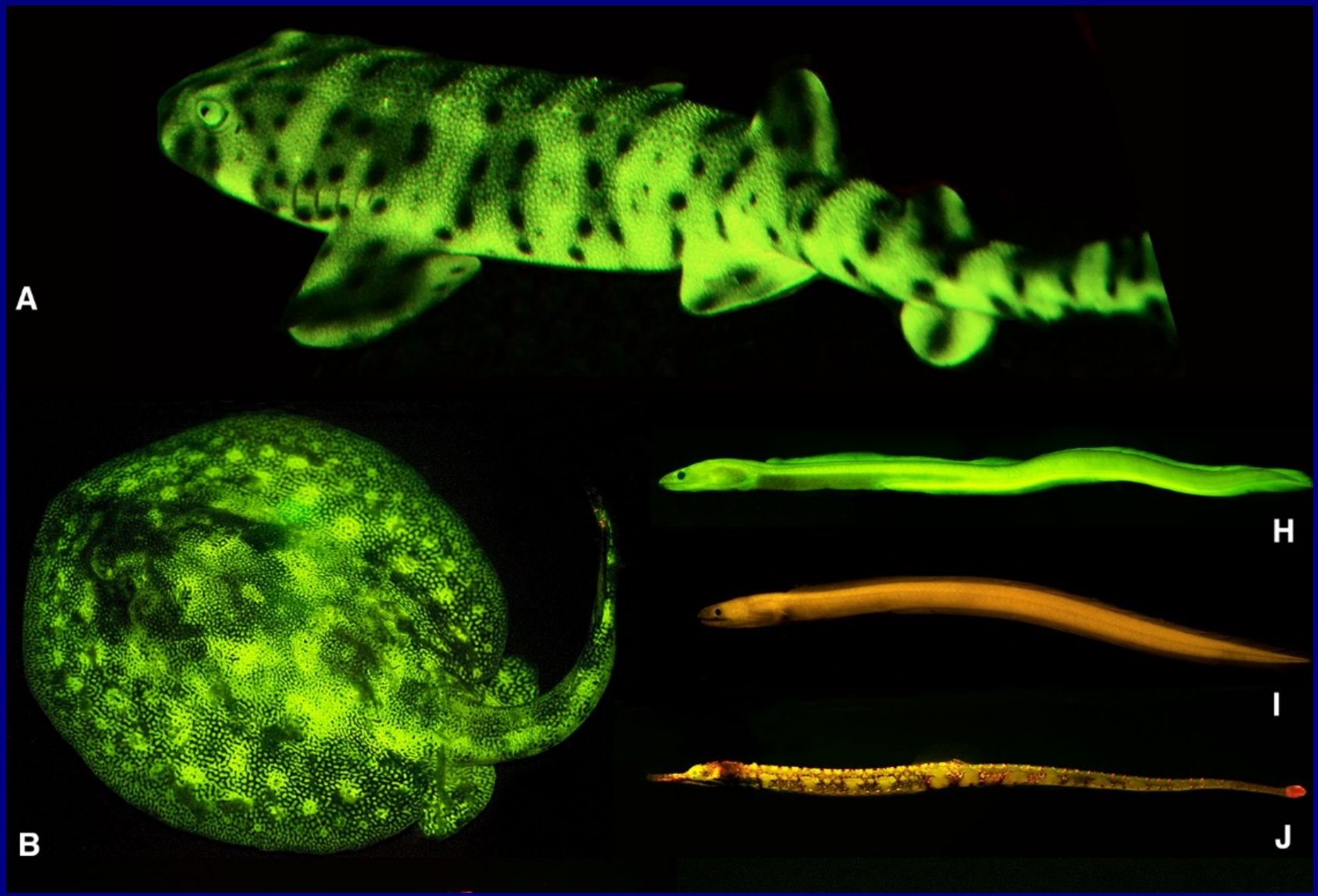
We benefit from his work through fluorescent lamps, where a phosphor coating fluoresces, emitting light.



Fluorescence in the Mineral World



Fluorescence in the Animal World



Fluorescence in the Social World



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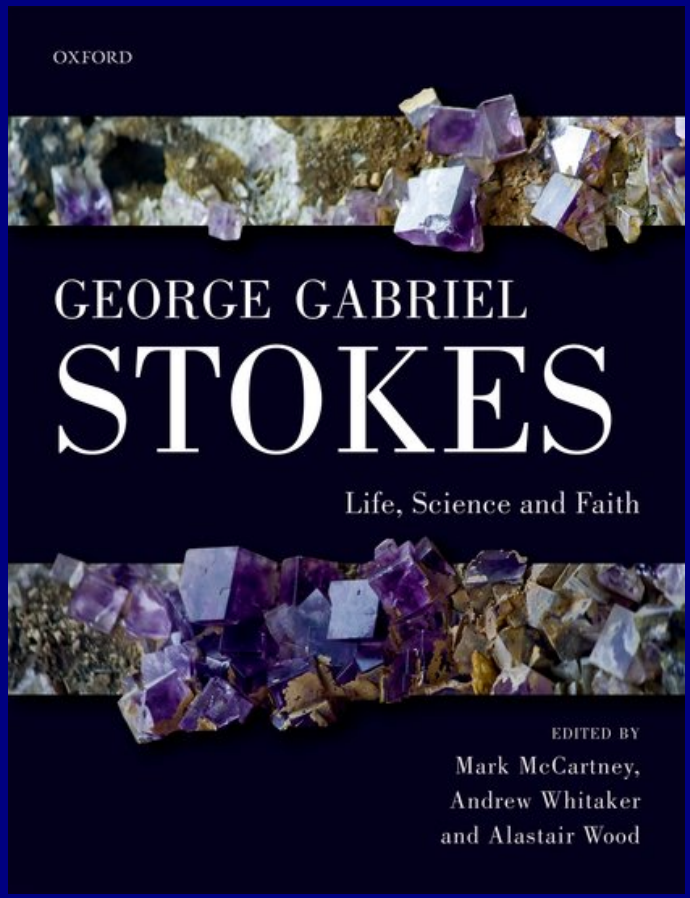
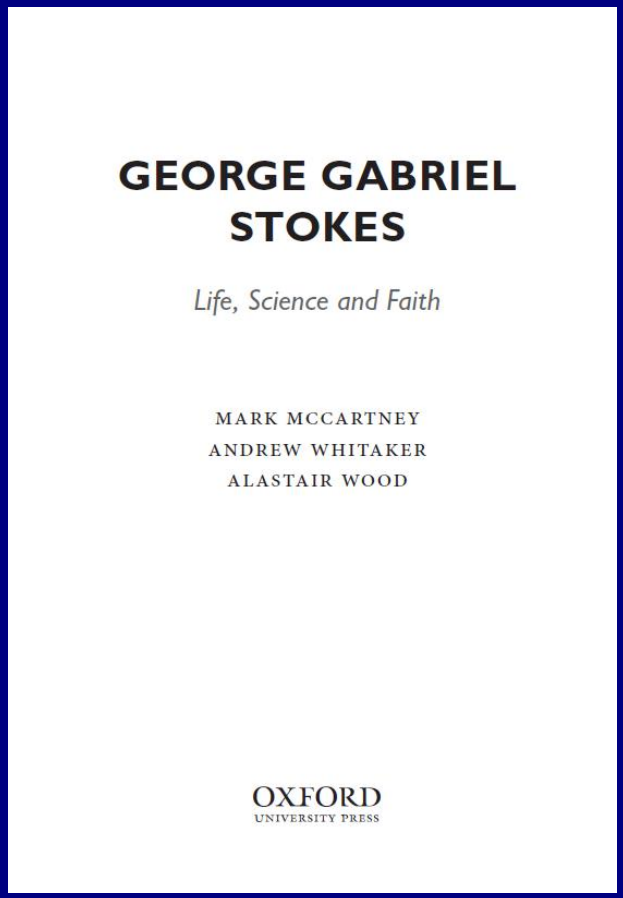
Stokes: Life, Science and Faith





George Gabriel Stokes: Life, Science and Faith.
Eds. Mark McCartney, Andrew Whitaker, and Alastair Wood,
Oxford University Press (2019). ISBN: 978-0-1988-2286-8

Stokes: Life, Science and Faith

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   The Mathematical Education of George Gabriel Stokes
   JUNE BARROW-GREEN
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   OLIVIER DARRIGOL
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Royal Society

1851: Stokes elected a Fellow of the Royal Society.

(Along with William Thomson, T. H. Huxley and John Tyndall.)

1854–1884: Stokes Secretary of the Royal Society

President from 1885 to 1890.

> **"I am naturally of rather a retiring character,
> and should feel not a little out of my element
> in being brought so prominently forward."**
>
> **Stokes to Th. R. Robinson.**

Thomas H. Huxley criticised Stokes for his "ultra-conservative and theological viewpoint."



Stokes as President of Royal Society

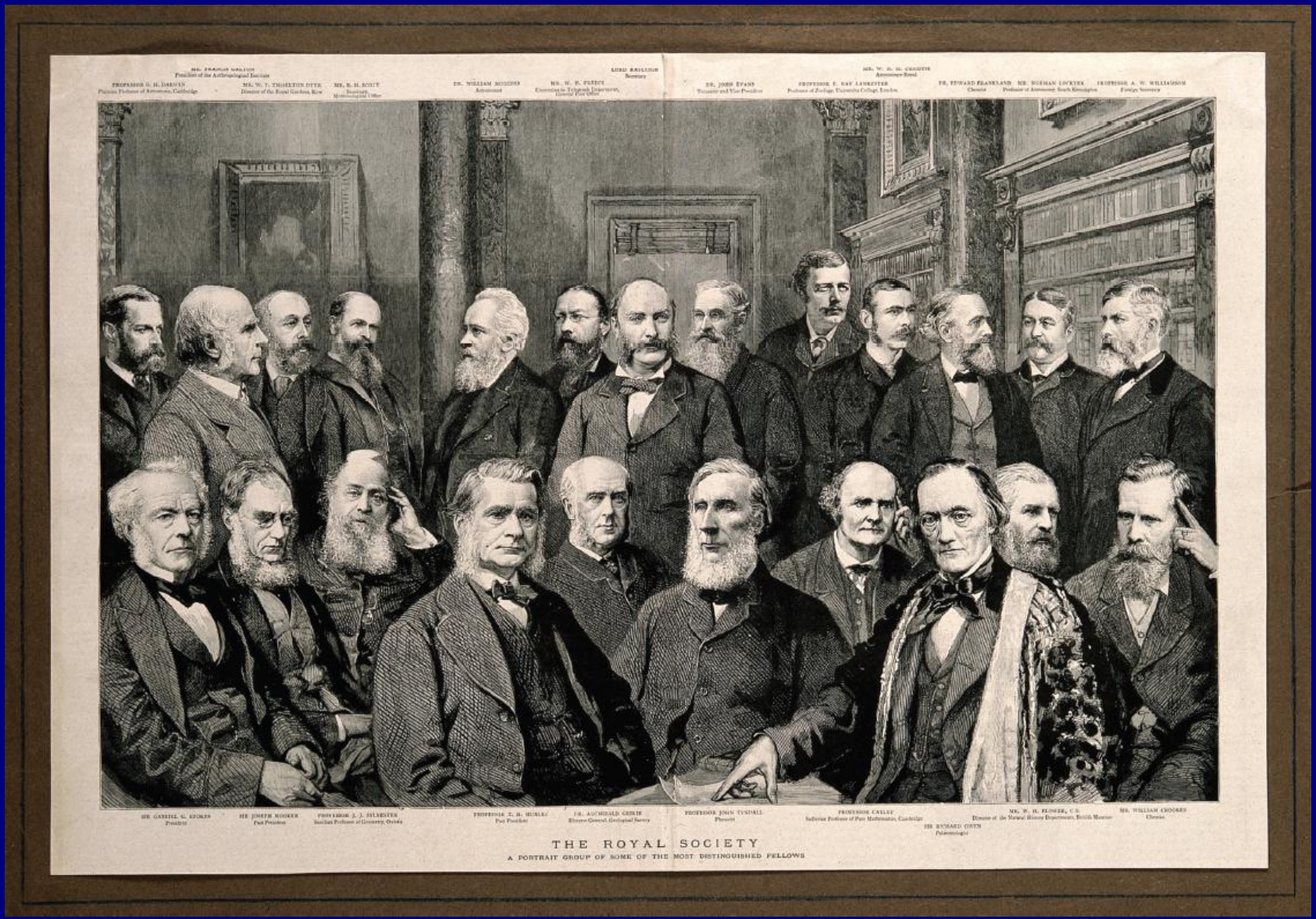


President of Royal Society



According to his daughter, “Stokes was apt to look bored when being painted and to draw down the corners of his mouth. Thus, the portrait by Herkomer at the Royal Society is not satisfactory to those who knew him best.”

Prominent Members of the Royal Society



Royal Society

Stokes was awarded the Copley Medal in 1893.



Fig. 8.4 The Royal Society Copley Medal awarded to Stokes in 1893. Image courtesy Nick Lefebvre.

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Euler's Fluid Equations



Leonhard Euler, born on 15 April, 1707 in Basel. Died on 18 September, 1783 in St Petersburg.

Euler formulated the equations for incompressible, **inviscid fluid flow:**

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} + \frac{1}{\rho} \nabla p = \mathbf{g} \,.$$

$$\nabla \cdot \mathbf{V} = 0$$

Some Paradoxes in Hydrodynamics

- D'Alembert's Paradox
- The Reversibility Paradox
- Paradoxes of Airfoil Theory
- The Rayleigh Paradox
- Von Neumann's Paradox
- Kopal's Paradox
- The Eiffel Paradox
- The Rising Bubble Paradox
- The Magnus Effect Paradox
- Stokes's Paradox



Jean Le Rond d'Alembert



A body moving at constant speed through a gas or a fluid does not experience any resistance (d'Al. 1752).

Jean Le Rond d'Alembert

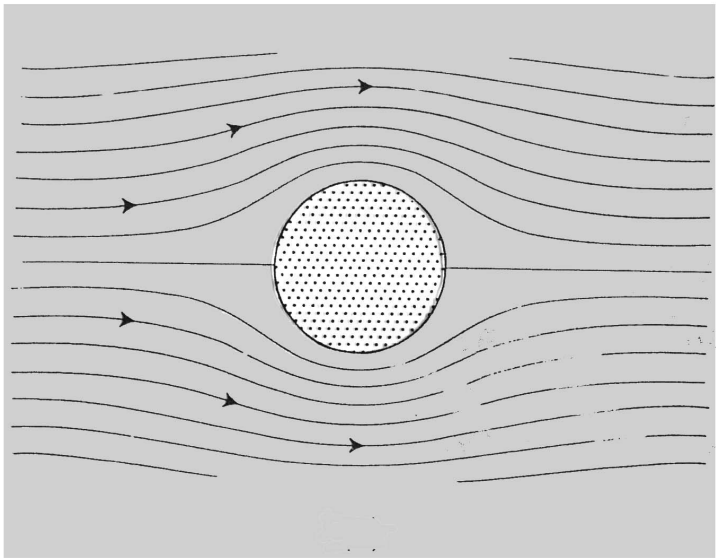


D'Alembert expressed his concerns thus:

"I do not see how one can satisfactorily explain, by theory, the resistance of fluids."

He remarked that the theory leads to "a singular paradox which I leave to future geometers for elucidation."

Resolution of d'Alembert's Paradox



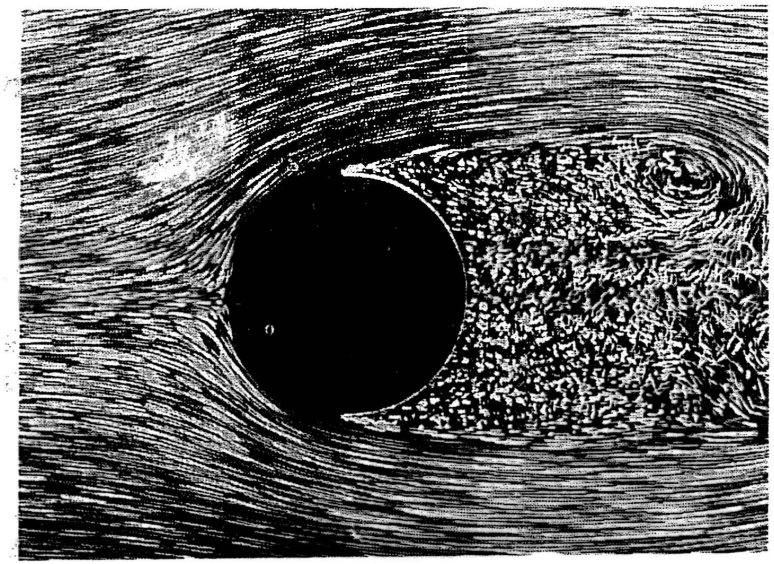


Fig. 9.1 Flow past a circular cylinder for (a) a hypothetical fluid with zero viscosity, (b) a real fluid with very small viscosity μ (from van Dyke 1982).

The minutest amount of viscosity has a profound qualitative impact on the character of the solution.

The N-S equations incorporate the effect of viscosity.

C. L. M. H. Navier, 1785–1836



Claude Louis Marie Henri Navier, French engineer and physicist who specialized in mechanics.

Basic Publications and Review

Navier, C. L. M. H., 1822:
Mémoire sur les lois du mouvement des fluides.
Mém. Acad. Sci. Inst. France, **Vol. 6, 389–440.**

Stokes, G. G., 1845:
On the theories of the internal friction of fluids in motion.
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Between Hydrodynamics and Elasticity Theory:
The First Five Births of the Navier-Stokes Equation.
Olivier Darrigol, 2002:
***Arch. Hist. Exact Sci.*, Vol. 56, (2), 95–150.**



The Navier-Stokes Equations

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} + \frac{1}{\rho} \nabla p = \nu \nabla^2 \mathbf{V} \, .$$

The Navier-Stokes Equations describe how the change of velocity (the acceleration) is determined by the pressure gradient force and frictional force.



The Navier-Stokes Equations

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} + \frac{1}{\rho} \nabla p = \nu \nabla^2 \mathbf{V} \, .$$

The Navier-Stokes Equations describe how the change of velocity (the acceleration) is determined by the pressure gradient force and frictional force.

For motion relative to the rotating earth, we include the Coriolis force and gravity:

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} + 2\Omega \times \mathbf{V} + \frac{1}{\rho} \nabla p = \nu \nabla^2 \mathbf{V} + \mathbf{g} \, .$$



Some Applications of the Navier-Stokes Equations

- Designing aircraft
- Modelling blood-flow
- Studying propulsion or lubrication
- Constructing wind turbines
- Forecasting the weather
- Ocean modelling
- Fundamental studies of turbulence.



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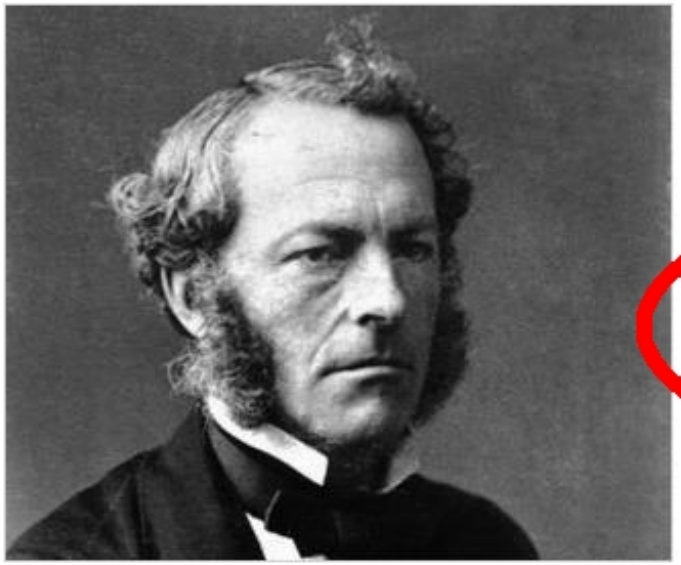
Clay Maths Institute Millennium Prize

CMI

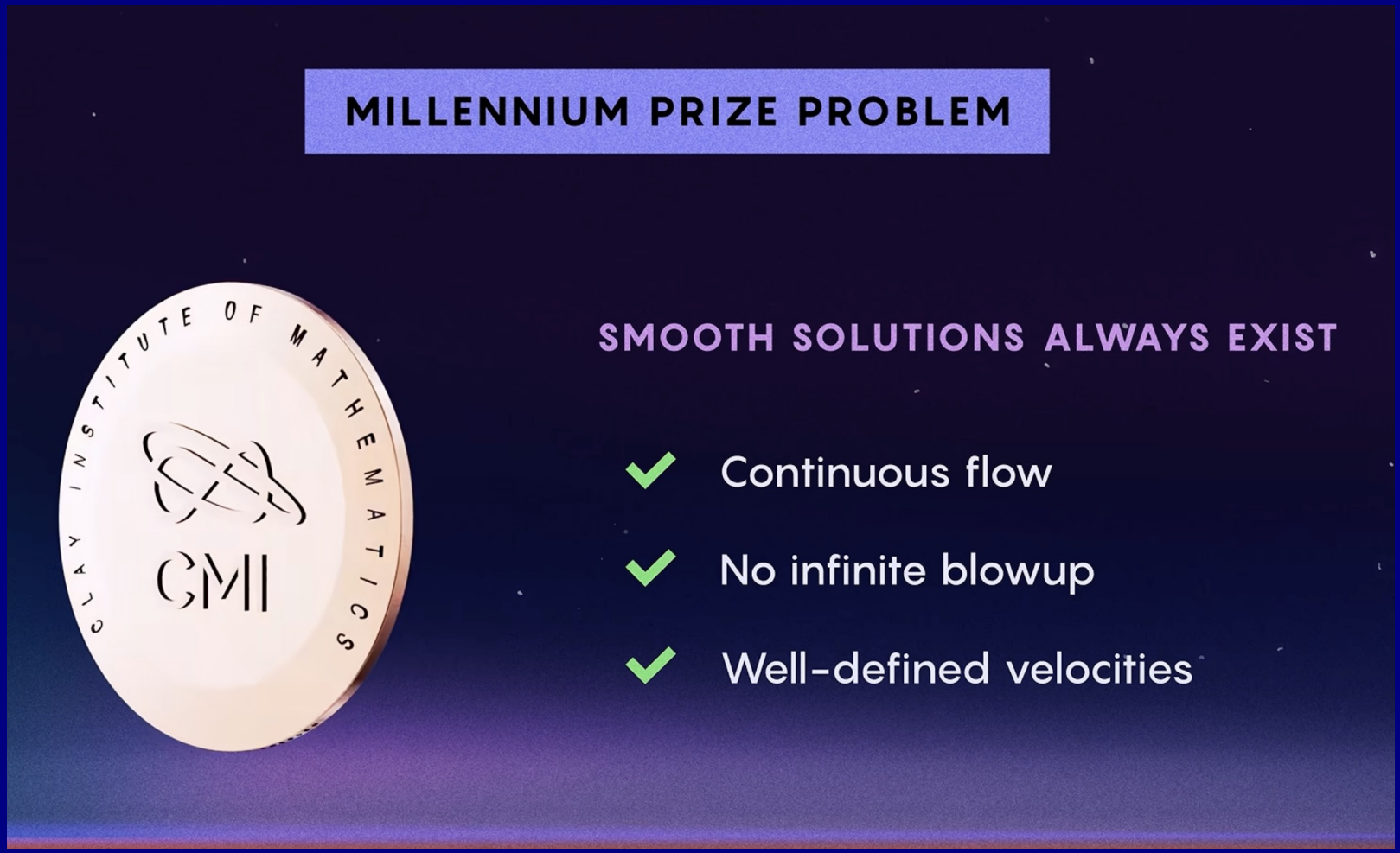
ABOUT PROGRAMS MILLENNIUM PROBLEMS PEOPLE PUBLICATIONS

Navier–Stokes Equation

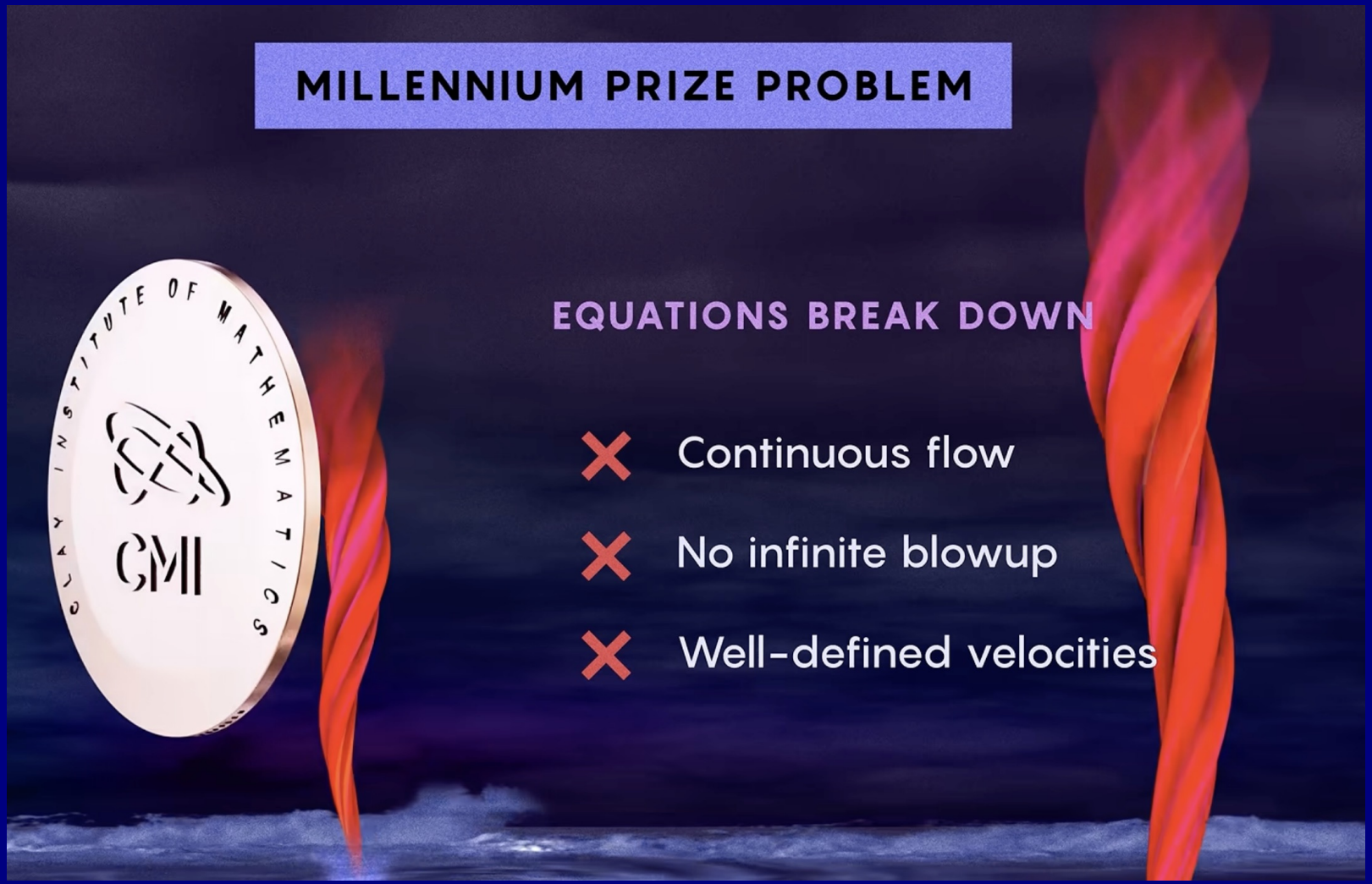


Waves follow our boat as we meander across the lake, and turbulent air currents follow our flight in a modern jet. Mathematicians and physicists believe that an explanation for and the prediction of both the breeze and the turbulence can be found through an understanding of solutions to the Navier-Stokes equations. Although these equations were written down in the 19th Century, our understanding of them remains minimal. The challenge is to make substantial progress toward a mathematical theory which will unlock the secrets hidden in the Navier-Stokes equations.

Clay Maths Institute Millennium Prize



Clay Maths Institute Millennium Prize

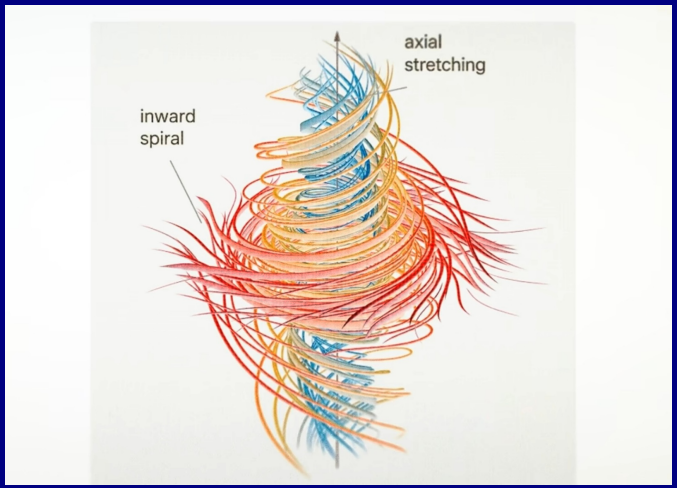


OpenAI Paper, 8 September 2026

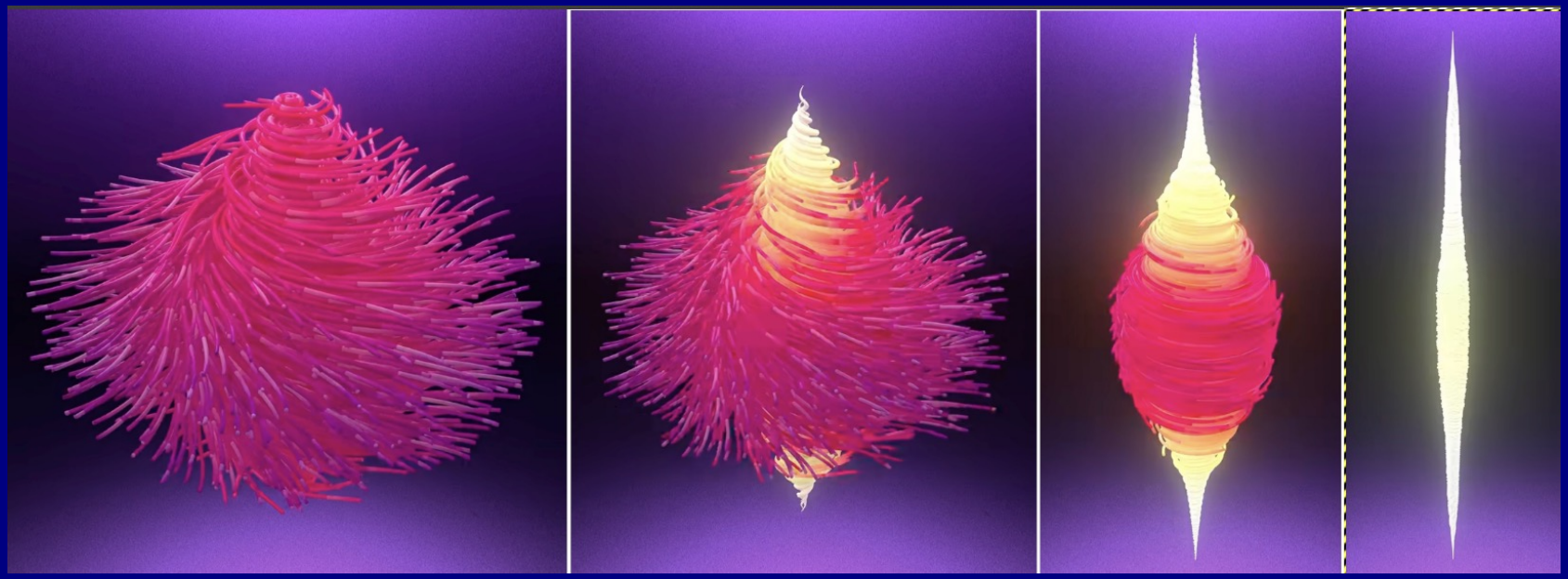
FINITE TIME BLOWUP FOR NAVIER–STOKES

OPENAI

ABSTRACT. For every positive viscosity, we construct a solution of the three-dimensional incompressible Navier–Stokes equations that starts from rest and develops unbounded velocity in finite time while maintaining uniformly bounded kinetic energy.



Blow-up of Solution of NS Equations



Quanta Magazine Video:

https://www.youtube.com/watch?v=PsWR1rQEWYo&t=705s

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WILD ATLANTIC WAY
SLÍ AN ATLANTAIGH FHIÁIN
NORTHERN HEADLANDS
MALIN HEAD
CIONN FHÁNADA
Buncrana
Muff
City of Derry Airport
Donegal Airport
Dungloe
Letterkenny
SLIABH LIAG
Donegal
MULLAGHMORE
SURF COAST
DOWNPATRICK HEAD
Sligo
Belmullet
Ballina
KEEM BAY
Westport
Ireland West Airport Knock

Ocean Waves

Stokes, growing up on Ireland's Wild Atlantic Way, was a skilled swimmer and a keen observer of nature.

During holidays in Skreen and elsewhere in Ireland, he made observations of waves and swell.

He considered the "highest possible periodic wave."

He showed that the wave of maximum height has a crest with an angle of 120°.





Surf off the Sligo Coast

A Sligo Man

Stokes never forgot his origins in Skreen.

He returned to Sligo and elsewhere in Ireland regularly for summer vacations.

In one of his mathematical papers he wrote of

"the surf that breaks upon the western coasts as a result of storms out in the Atlantic",

the majestic rollers thundering towards the shore as he strolled as a boy along Dunmoran Strand.



Thank you



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Stokes Honoured in Ireland?

Surely there must be an
Irish postage stamp
featuring Stokes?

In a quick web-search,
no such stamp was found.



Stokes Honoured in Ireland?

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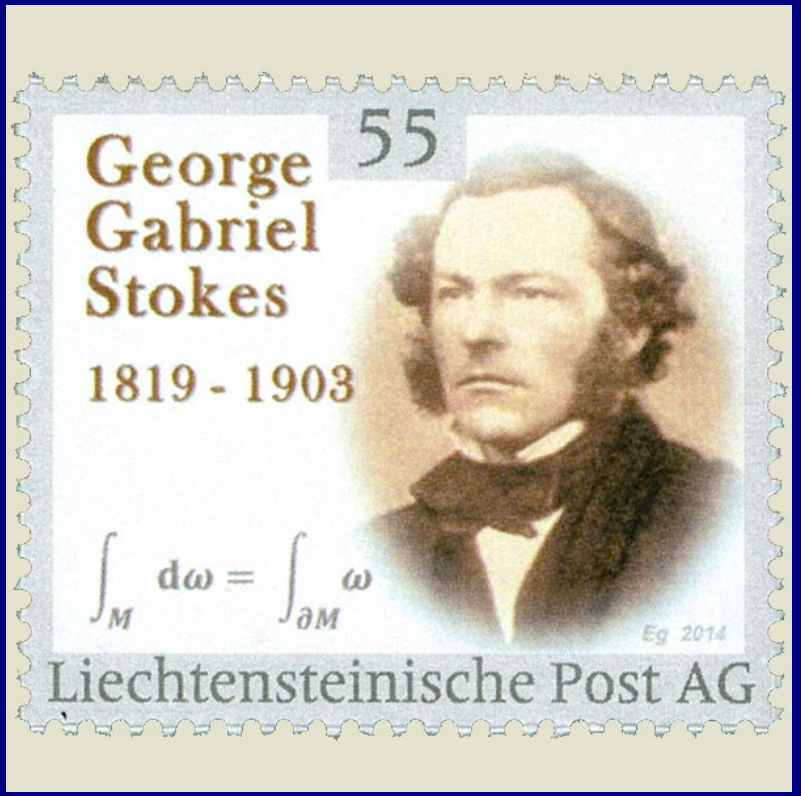
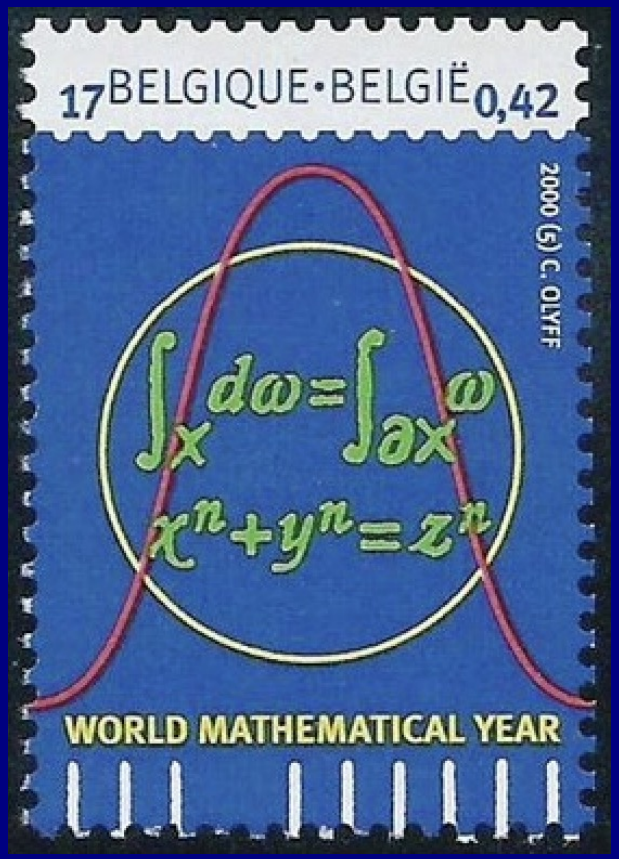


More Stokes Postage Stamps






UCD
DUBLIN

Modelling the Changing Climate

Amongst the most pressing problems facing humanity today.

Enormous uncertainties exist concerning the future climate.

The best means we have for reducing these is by means of computer simulations.

At the heart of every climate model lie the Navier-Stokes equations.

The same models are used regularly for short and medium range weather forecasts.



Equations of the Atmosphere

Gas Law: (Boyle's Law and Charles' Law.)

Conservation of Mass for Air

Conservation of Mass for Water

Equations of Motion: Navier-Stokes Equations

Thermodynamic Equation

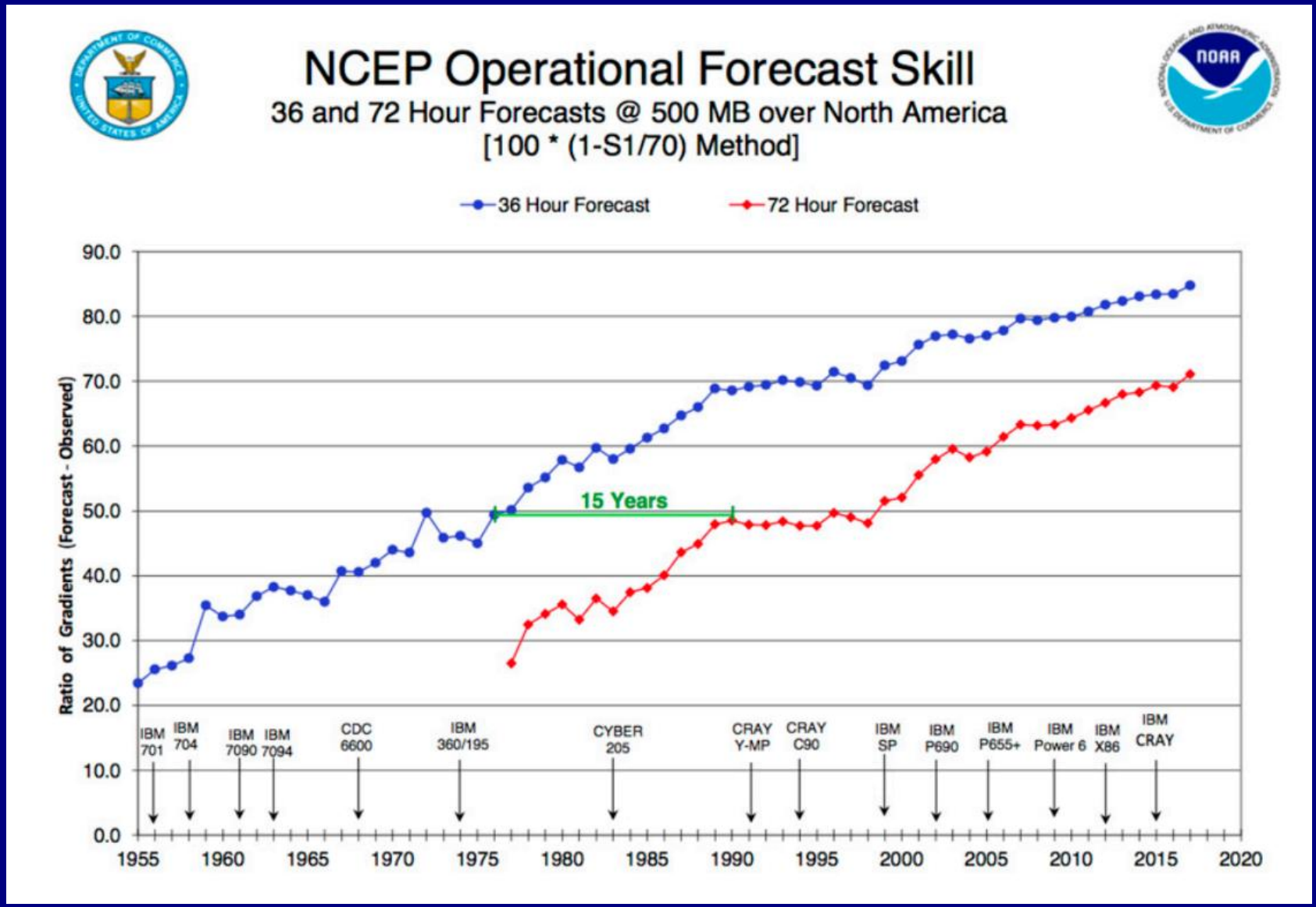


Weather Forecasting in a Nut-Shell

- The atmosphere is a physical system
- Its behaviour is governed by the laws of physics
- These laws are expressed quantitatively as mathematical equations
- Using observations, we can specify the atmospheric state: "Today's Weather"
- Using the equations, we can calculate how this state changes with time: "Tomorrow's Weather"



Long-term Skill Growth



Forecast of Hurricane Sandy

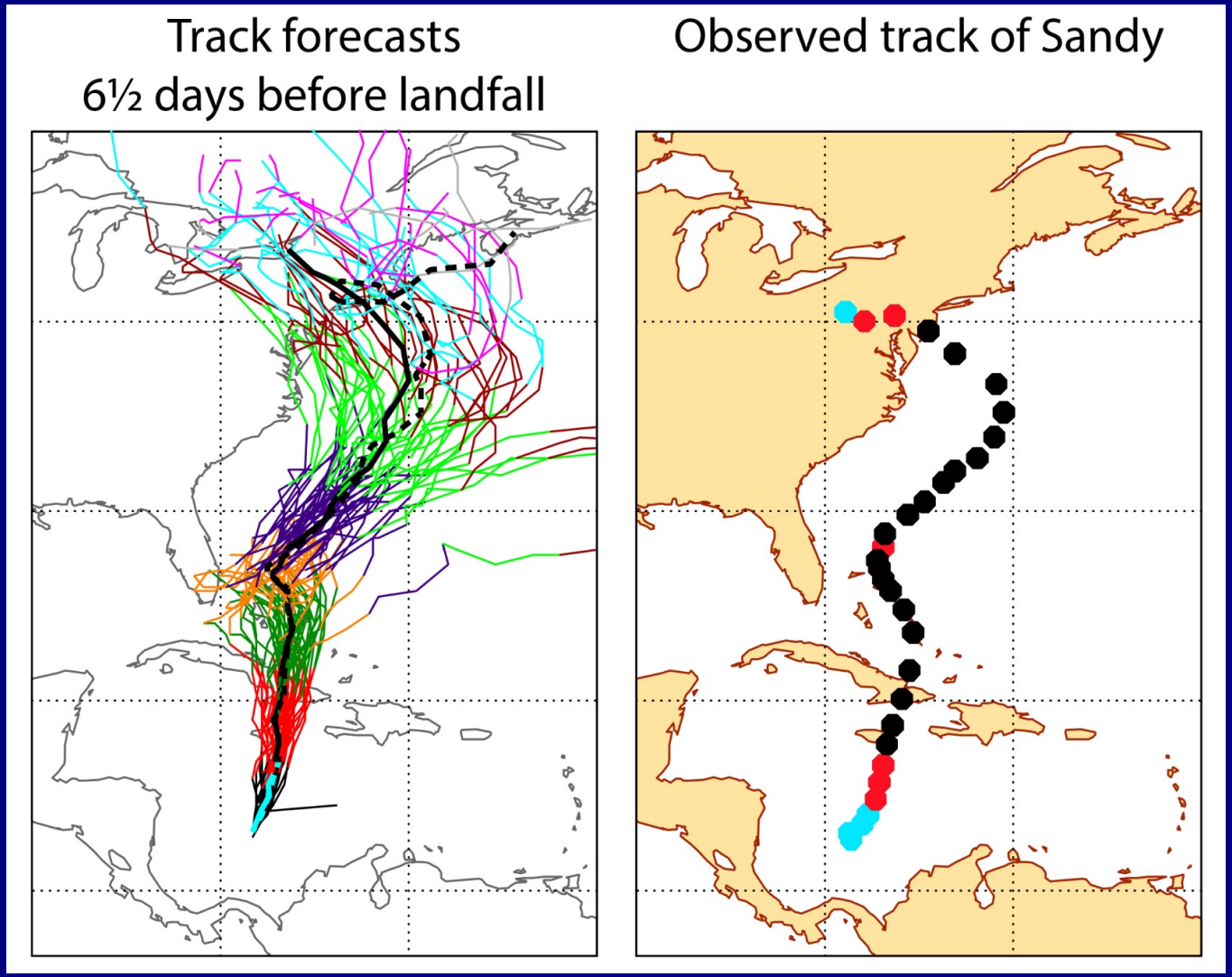


Figure: Landfall, New Jersey, 30 October 2012