Leopold Bloom's Mathematical Musings

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Across the Waters

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Outline

Abstract (Not for Display)

Leopold Bloom's Mathematical Musings

Joyce's performance in school and university mathematics ranged from indifferent to abysmal. However, he took a keen interest in scientific matters. Leopold Bloom, despite his limited formal schooling, also loved to muse about matters mathematical.

Advanced mathematics is not much in evidence in *Ulysses*, but high-school arithmetic, algebra, geometry and trigonometry all feature. The *ithaca* episode contains the greatest number of mathematical concepts and allusions. While writing *ithaca*, Joyce worked through Bertrand Russell's *Introduction to Mathematical Philosophy*, and made extensive notes on it.

At that time Joyce was living in Zurich, and mathematicians such as Ernst Zermelo and and Hermann Weyl were also working there, on the foundations of mathematics. It is not impossible that Joyce met some of them. He surely had an acquaintance with their work; he also had some knowledge of non-Euclidean geometry, at least in general terms.

Bloom was preoccupied by the ancient problem of squaring the circle. In 1882, the year of Joyce's (and Stephen's) birth, Ferdinand Lindemann proved that π (pi, the ratio of a circle's circumference to its diameter) is a transcendental number, which implied the impossibility of squaring the circle. But this did not deter Bloom (or many other amateurs) from trying.

Elsewhere in the *ithaca* episode, Bloom speculates on the largest number that can be written using only three decimal digits, concluding that its value was "the 9th power of the 9th power of 9", which has 78 decimal digits. However, closer examination of the text makes it clear that he must have meant "9 to the 9th power of 9", which has about 370 million digits. He estimated that 33 volumes, each of 1000 pages, would be required to write the number down explicitly. This is quite accurate, considering Joyce's dismal performance in his mathematical studies.

Outline

James Joyce (1915). *Ulysses 1st Ed.* 1922

ULYSSES

by

JAMES JOYCE

SHAKESPEARE AND COMPANY
12, Rue de l'Odéon, 12
PARIS
1922

Mathematical Themes in *Ulysses*

Mathematical themes occur throughout the book, most notably in the Ithaca episode.

Exchanges between Leopold and Stephen frequently touch on weighty scientific matters.

All areas of elementary mathematics are touched upon: Arithmetic, Geometry, Algebra.

There are allusions to more advanced topics, but no deep or weighty discussions of them.

⋆ ⋆ ⋆

Outline

Joyce's Tower, Sandycove

Joyce's Tower in Sandycove, Co. Dublin [Photo PL]

The Lightning Calculatrix

In the opening scene, the old lady bringing the milk performs a bravura feat of mental arithmetic:

"Well, it's seven mornings a pint at twopence
is seven twos is a shilling and twopence over
and these three mornings a quart at fourpence
is three quarts is a shilling
and one and two is two and two, sir."

To readers who remember the old pounds, shillings and pence, this can be decoded easily enough.

As time passes, the meaning becomes obscure.

Algebra in Telemachus

Higher mathematics is not evident in Ulysses, but algebra gets a curious mention when Buck Mulligan expounds on Stephen's theory of Hamlet:

"It's quite simple. He proves by algebra that Hamlet's grandson is Shakespeare's grandfather and that he himself is the ghost of his own father."

Abu 'Abdallah Muhammad ibn Musa al-Khwarizmi, the founder of algebra, would have been baffled!

Sirens — An Arithmetical Enigma

There's an example of musical mathematics in *Sirens*, where Bloom's internal monologue goes thus:

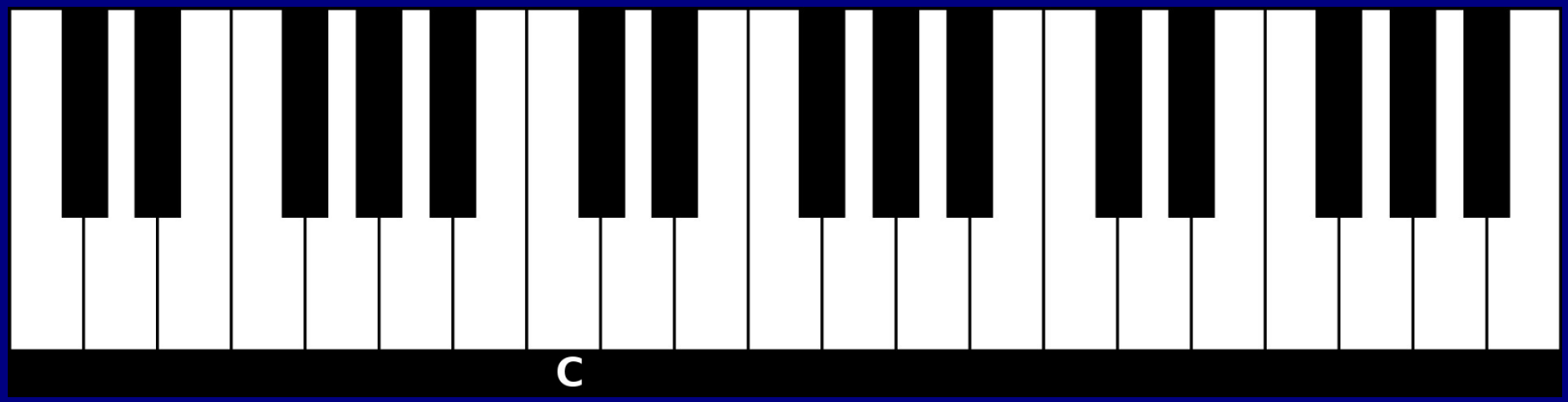
"Numbers it is. All music when you come to think.
Two multiplied by two divided by half is twice one."

This makes sense if we take "divided by half" to mean "halved". Bloom continues:

"Vibrations: chords those are.
One plus two plus six is seven.
Do anything you like with figures juggling.
Always find out this equal to that . . . ".

How can we have 1+2+6=7?

These numbers actually represent musical intervals, which are counted inclusively.



- From C to C itself is a unison (count 1).
- From C to D is a second (count 2).
- From D to B is a sixth (count 6).

The three intervals combine to make an interval of a seventh (count 7). So, 1 + 2 + 6 = 7.

Outline

Mathematical Musings in *Ithaca*

Joyce — like Bloom — was intrigued by mathematical and scientific themes.

The narrator of **Ithaca** takes every opportunity to express ideas in mathematical terms.

Leopold and Stephen converse as they make their way from nighttown to Bloom's home in Eccles St.

The opening question of this catechetical episode is:

What parallel courses did Bloom and Stephen follow returning?

evoking **Euclid's Fifth Postulate** on parallel lines.

Short-Cut at St George's Church

Bloom and Stephen take a diametrical route . . .

"the chord in any circle being
less than the arc which it subtends."

Non-Euclidean Geometry

When Joyce was writing Ithaca, the mathematicians Ernst Zermelo and Hermann Weyl were also working in Zurich, on the foundations of mathematics.

It is not impossible that Joyce met them or, at least, was acquainted with their work.

In his working notes for Ithaca, Joyce refers to Bernhard Riemann and Nikolai Lobachevskii, who developed forms of geometry that are fundamentally different from that of Euclid.

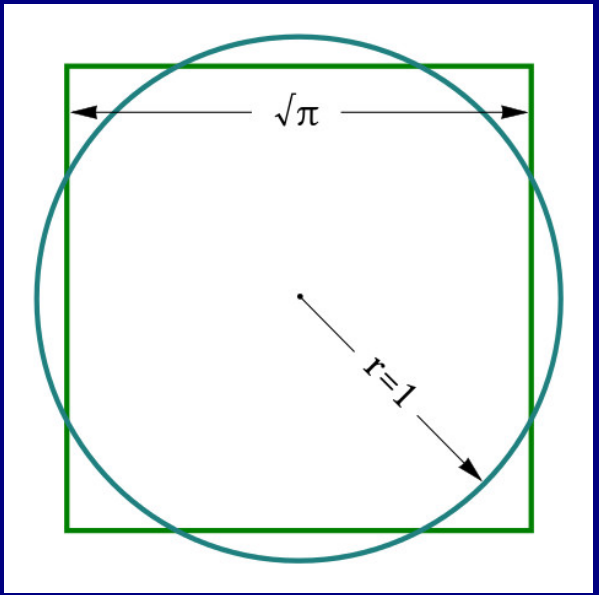
This suggests that Joyce had some knowledge of non-Euclidean geometry, at least in general terms.

Outline

Squaring the Circle

Challenge:

Construct a square with area equal to a circle using only the methods of classical Euclidean geometry.



Squaring the circle has attracted attention from outstanding mathematicians for millennia.

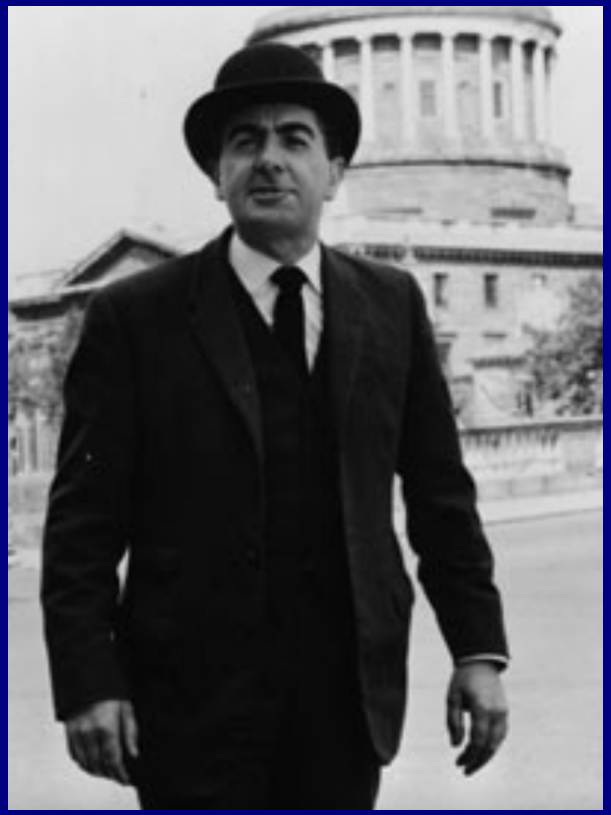
The phrase "squaring the circle" has become a metaphor for attempting the impossible.

Squaring the Circle

Leopold Bloom had a keen interest in the matter: there are at least three references in *Ulysses.*

In the Ithaca episode, we read of how Bloom planned to devote the summer of 1886 to square the circle and "win that million".

In reality, no such prize was ever on offer.



Milo O'Shea

1882: a Fateful Year

Ferdinand von Lindemann

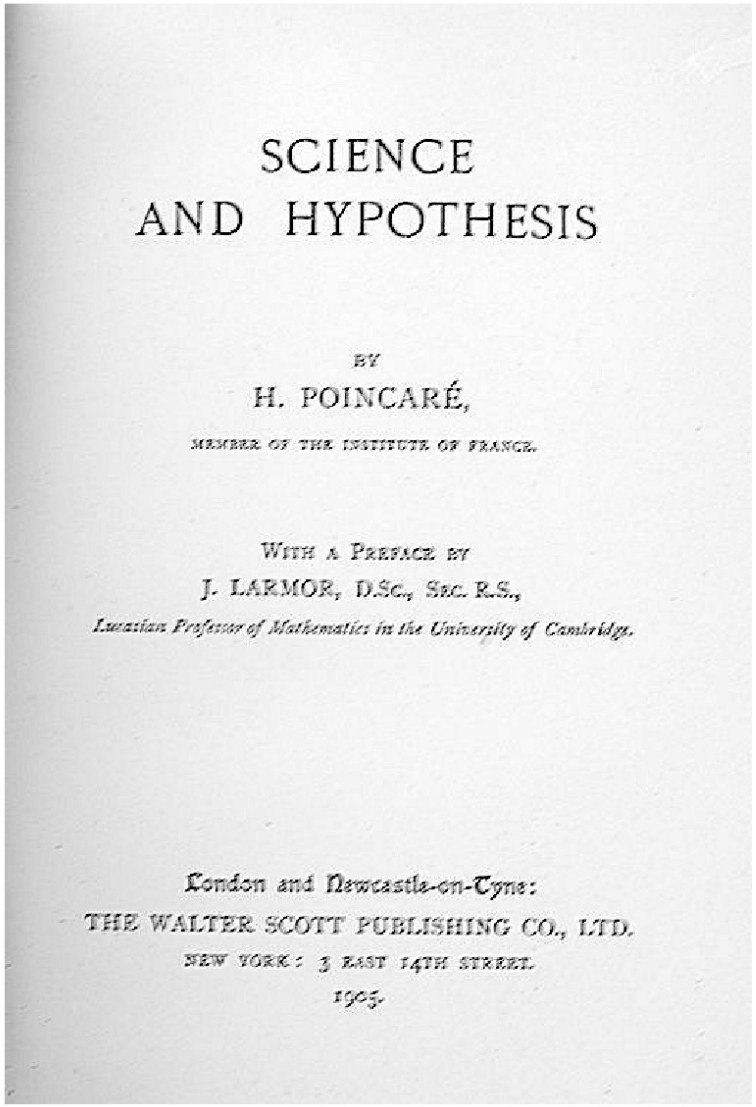
In 1882 . . .

- James Joyce was born.
- Stephen Dedalus was born.
- Ferdinand Lindemann proved that π is transcendental.

This means that the problem of squaring the circle within the framework of Euclidean Geometry is impossible.

Henri Poincaré

[Point Carré = Square Point]

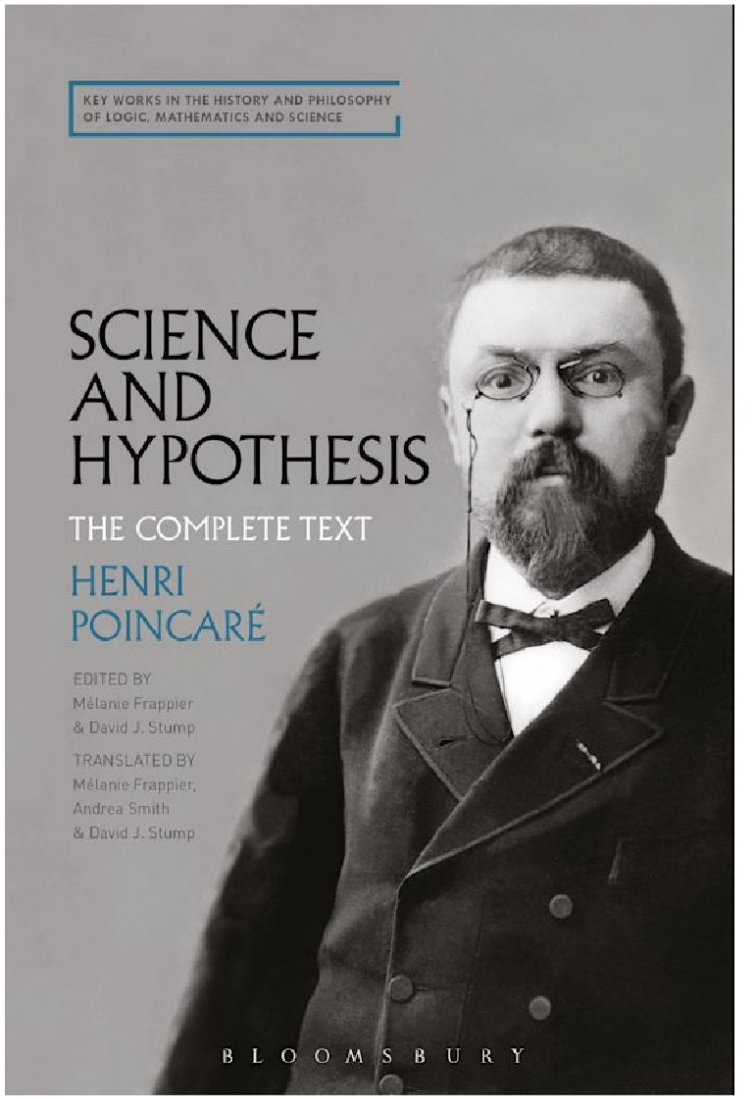


SCIENCE
AND HYPOTHESIS

BY
H. POINCARÉ,
MEMBER OF THE INSTITUTE OF FRANCE.

WITH A PREFACE BY
J. LARMOR, D.Sc., SEC. R.S.,
Lucasian Professor of Mathematics in the University of Cambridge.

London and Newcastle-on-Tyne:
THE WALTER SCOTT PUBLISHING CO., LTD.
NEW YORK: 3 EAST 14TH STREET.
1905.



Thanks eversore much, Pointcarried! **[FW, p. 304]**

Outline

Joyce's Number

What is the largest number that can be written using only three decimal digits?

An initial guess might be 999. But soon we realize that factorials permit much greater numbers, like 999!

999! has 2565 decimal digits

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The question is not really well-posed.

To remove ambiguity, we disallow the use of any symbols other than the three digits, and formulate the problem more precisely.

The Largest Three-digit Number

Problem:

What is the largest number that can be written in standard mathematical notation, using only three decimal digits and no other symbols?

Powers, written as superscripts, are allowed.

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Clearly, we should use only the digit 9.
we can consider numbers like

$$999 \qquad 99^9 \qquad (9^9)^9 = 9^{81} \qquad 9^{99} \qquad 9^{9^9}$$

These are in increasing order, and the last one seems to be the greatest possible with three digits.

Joyce's Number

In the Ithaca episode of *Ulysses*, Joyce had Leopold Bloom contemplating a large number:

"the 9th power of the 9th power of 9"

He indicated that to print the result would require "33 closely printed volumes of 1000 pages each".

The number Joyce specified (literal reading) was

$$G = (9^9)^9 .$$

Joyce's Number

The number $G = (9^9)^9$ can easily be bounded above:

$$(9^9)^9 < (10^{10})^{10} = 10^{100} \equiv \textbf{1 googol}\,.$$

So, it is a quite unremarkable number.

Is it the smallest unremarkable number?!!!!

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Is it the smallest unremarkable number?!!!!

Did Joyce really mean

$$(9^9)^9 \qquad \textbf{or} \qquad 9^{9^9}\,?$$

We must make allowances: his performance in mathematics ranged from indifferent to abysmal.

Digression: Joyce's Mathematical Studies

Joyce entered University in September 1898.

His examination marks are summarised in Richard Ellmann's biography.

The marks fluctuate widely, suggesting some lack of focus and dedication.

In his matriculation year, Joyce managed only an abysmal 22% in mathematics.

His marks improved the following year when he got an impressive 60%.

Digression: Joyce's Mathematical Studies

In his third year he dropped mathematics and studied logic, achieving a miserable 27%.

For his final examinations in 1902, he took English, French and Italian.

One might have expected a spectacular performance in English, but he was awarded only 43%!

Ellmann remarked that Joyce had done enough to pass, but had not bothered to excel . . .

. . . definitely a case of "Could Do Better!".

Joyce's Examination Marks

	1898–1899		1899–1900		1900–1901		1901–1902	
	Grade	Maximum	Grade	Maximum	Grade	Maximum	Grade	Maximum
Latin	725	1200	756	1200	353	1200	—	
French	416	800	—		489	900	465	800
English	490	800	358	800	313	900	344	800
Mathematics	220	1000	715	1200	—		—	
Natural Philosophy	183	500	373	800	—		—	
Italian	—		373	800	295	900	417	800
Logic	—		—		240	900	—	

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$$J = 9^{9^9} \qquad \textbf{IS MUCH BIGGER THAN} \qquad G = (9^9)^9.$$

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G has no more than 78 decimal digits.

Using only the result $2^{10} > 10^3$, we can show that:

$$J = 9^{9^9} > 10^{3.42 \times 10^8}$$

Therefore, J has more than 340 million digits.

A Weighty Tome

The number of digits in $J = 9^{9^9}$ is 369,693,100.
In round numbers, about 370 million digits.

How big a book would be needed to display it?

A Weighty Tome

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How big a book would be needed to display it?

To see Joyce's number in all its glory,
we need to print an enormous tome.

Assuming 100 digits per line and 100 lines per page,
this implies that something like 37 volumes, each
of 1000 pages, would be required to write down J.

Joyce's estimate in *Ulysses* was 33 volumes; not bad,
considering his dismal performance in mathematics.

Outline

Calculus / Analysis

Joyce had only a vague understanding of mathematical analysis (calculus) but he was intrigued with the concept of limit.

This is shown at the end of the Ithaca episode:

"dividends and divisors ever diminishing without actual division till, if the progress were carried far enough, nought never was nowhere reached."

This is clearly based on the definition of the derivative (differential coefficient) of a function.

Infinitesimals baffled some great mathematicians.

Reference: Mines and Dasenbrock

Outline

Foundations of Mathematics

Foundations of Mathematics

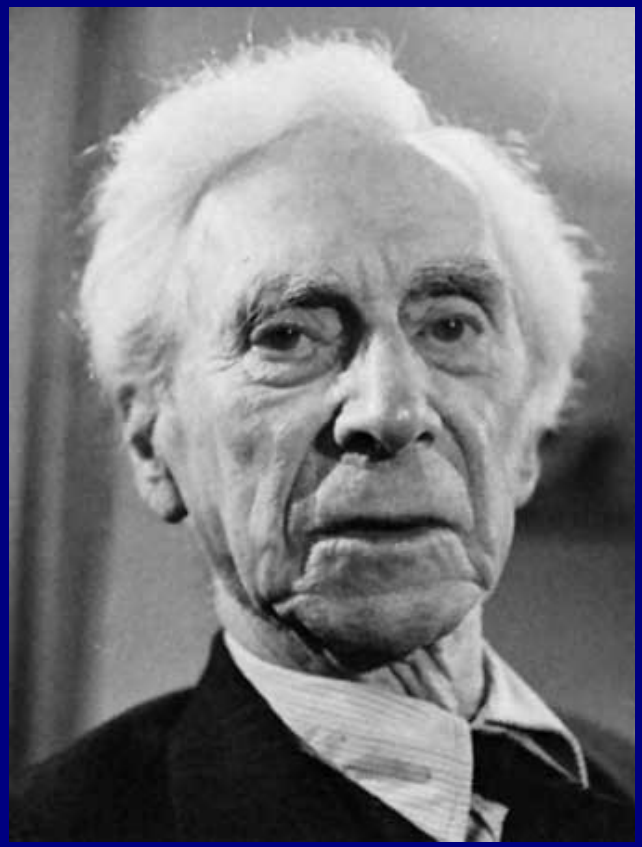
While writing the Ithaca episode, Joyce studied several mathematical texts, searching for terms and themes that would be of use to him.

Poincaré's book, *Science & Hypothesis* is one he was likely to have read.

Joyce also read Bertrand Russell's 1919 book
Introduction to Mathematical Philosophy
and made extensive notes on it.

This work would have introduced Joyce to contemporary thinking in mathematics.

Russell's Paradox



Bertrand Russell's book examined the foundations of mathematics.

He discovered that the concept of the set of all sets led to profound contradictions.

$$\text{Let } X := \{x : x \notin x\}.$$
$$\text{Then } X \in X \iff X \notin X.$$

The Paradox:
A barber shaves all the men in the village who don't shave themselves. Who shaves the barber?

Foundations of Mathematics

While Joyce was in Zurich, from 1915 to 1919, there was frenzied activity there to find firm foundations for mathematics.

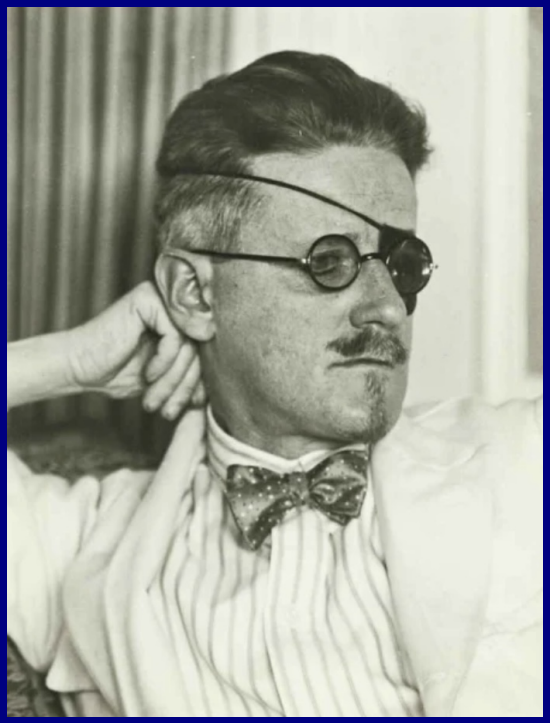
Ernst Zermelo held the Chair of Mathematics in Zurich from 1910 to 1916.

Russell's Paradox was actually discovered a few years earlier by Zermelo.

This impelled Zermelo and others to develop a consistent set of axioms for set theory.

The result was the **Zermelo-Fraenkel axioms,** still the standard system for set theory.

Foundations of Mathematics



Could Joyce have met Zermelo in Zurich?

Was he aware of the dramatic mathematical developments under way?

We may never know!

Outline

Summary

Joyce remarked in a letter to Harriet Shaw Weaver that the Ithaca episode should be read by someone who is "a physicist, mathematician and astronomer . . .

Talk about chutzpah! Joyce was asking here for qualifications that he himself did not possess!

Still: He made great use of the material he had!

He described the catechetical Ithaca as

"a mathematico-astronomico-physico-mechanico-geometrico-chemico sublimation . . . ",
an overture to the
"amplitudinously curvilinear" climax of Penelope.

Thank you