Numerical Weather Prediction

A Laplace Transform Scheme for Integration of the Forecast Equations

Peter Lynch
School of Mathematics & Statistics
University College Dublin

Seminar, School of Mathematical Sciences, Universtiy College Cork, 8 March 2018

Outline

Introduction

Pioneers of NWP: The Dream

The Dynamical Core

ENIAC Integrations

NWP Today

ECMWF System

LTIS Scheme in PEAK Model

Forecast Factory: The Fantasy

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A Recent Paper in Nature

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NATURE | REVIEW

日本語要約

The quiet revolution of numerical weather prediction

Peter Bauer, Alan Thorpe & Gilbert Brunet

A recent review of Numerical Weather Prediction

***Nature*, 3 September 2015 Vol 525 p.47**

The Quiet Revolution of NWP [Abstract]

- Advances in NWP represent a quiet revolution.
- Steady accumulation of technological advances.
- Among the greatest impacts of physical science.
- NWP is a computational problem comparable to:
  - Modelling the behaviour of the human brain.
  - Simulating the evolution of the early universe.

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- NWP is a computational problem comparable to:
  - Modelling the behaviour of the human brain.
  - Simulating the evolution of the early universe.
- Performed daily at operational weather centres.

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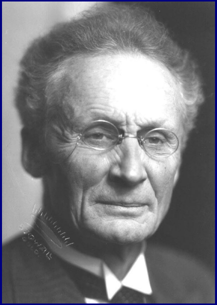
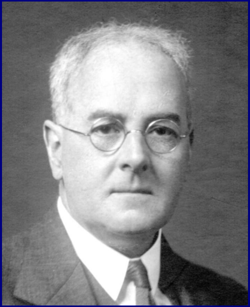
Pioneers of Scientific Forecasting





Cleveland Abbe, Vilhelm Bjerknes, Lewis Fry Richardson

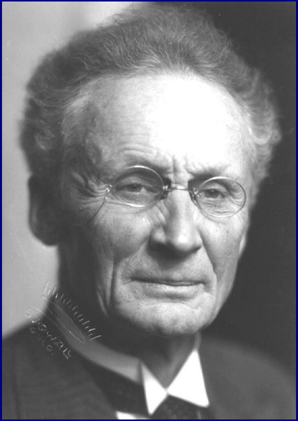
Cleveland Abbe

By 1890, the American meteorologist Cleveland Abbe had recognized that:

Meteorology is essentially the application of hydrodynamics and thermodynamics to the atmosphere.

Abbe proposed a mathematical approach to forecasting.

Vilhelm Bjerknes



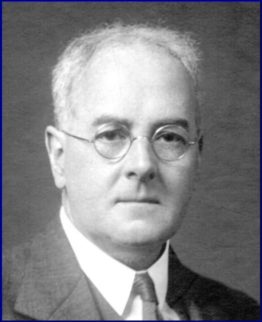
A more explicit analysis of weather prediction was undertaken by the Norwegian scientist Vilhelm Bjerknes

He identified the two crucial components of a scientific forecasting system:

- Analysis
- Integration

Vilhelm Bjerknes (1862–1951)

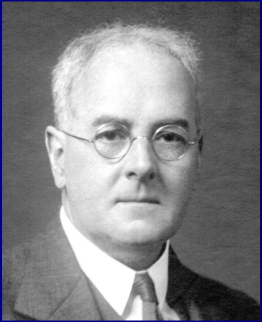
Lewis Fry Richardson



The English Quaker scientist Lewis Fry Richardson attempted a direct solution of the equations of motion.

He dreamed that numerical fore-casting would become a reality 'one day in the distant future'.

Lewis Fry Richardson

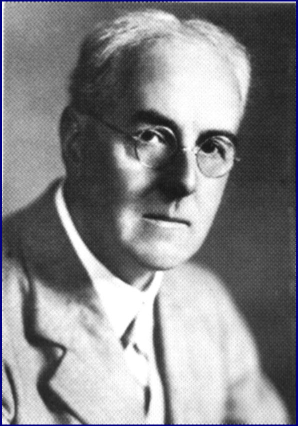


The English Quaker scientist Lewis Fry Richardson attempted a direct solution of the equations of motion.

He dreamed that numerical fore-casting would become a reality 'one day in the distant future'.

Today, forecasts are prepared routinely using his method . . . his dream has indeed come true.

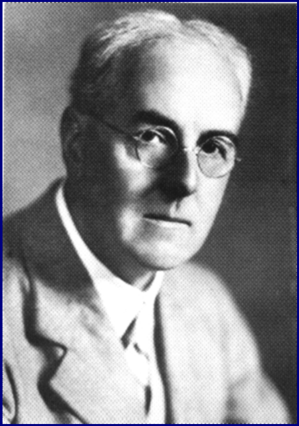
Lewis Fry Richardson, 1881–1953.



During WW I, Richardson computed by hand the pressure change at a single point.

It took him two years !

Lewis Fry Richardson, 1881–1953.



During WW I, Richardson computed **by hand** the pressure change at a single point.

It took him **two years** !

His ‘forecast’ was a catastrophic failure:

$$\Delta p = 145 \text{ hPa in 6 hrs}$$

But Richardson’s **method** was scientifically sound.

Initialization of Richardson's Forecast

Richardson's Forecast was repeated on a computer.

The atmospheric observations for 20 May, 1910,
were recovered from original sources.

- **ORIGINAL:** $$\frac{\partial p_s}{\partial t} = +\mathbf{145}\,\mathrm{hPa}/6\,\mathrm{h}$$

Initialization of Richardson's Forecast

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- **ORIGINAL:** $$\frac{\partial p_s}{\partial t} = +\mathbf{145}\,\mathrm{hPa}/6\,\mathrm{h}$$
- **INITIALIZED:** $$\frac{\partial p_s}{\partial t} = -\mathbf{0.9}\,\mathrm{hPa}/6\,\mathrm{h}$$

Observations: The barometer was steady!

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Weather and Climate Models

Computer models for simulating weather and climate are known as Earth System Models.

They are of great complexity.

At the heart of every model is a Dynamical Core.

At the kernel of the core lie the Navier-Stokes Equations.

George Gabriel Stokes

G. G. Stokes was born
in Skreen, Co. Sligo.

His equations for fluid flow underlie all atmospheric and ocean models.

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$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{V} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{V} - \mathbf{g}$$

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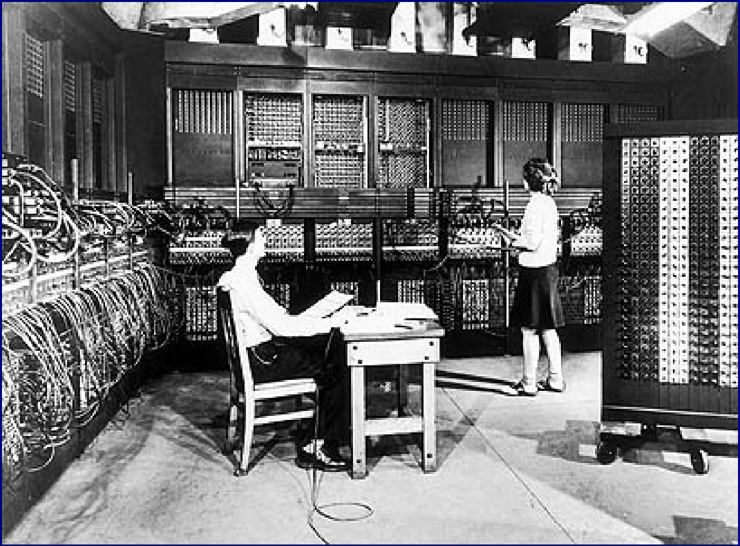
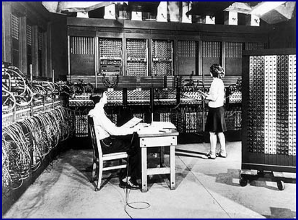
Crucial Advances, 1920–1950

- Dynamic Meteorology
  - Quasi-geostrophic Theory
- Numerical Analysis
  - CFL Criterion
- Atmopsheric Observations
  - Radiosondes
- Electronic Computing
  - ENIAC

The ENIAC



The ENIAC



The **ENIAC** was the first multi-purpose programmable electronic digital computer:

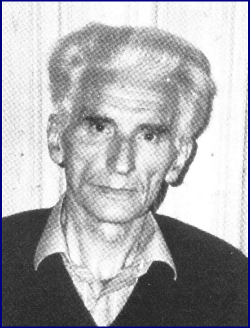
- 18,000 vacuum tubes
- 70,000 resistors
- 10,000 capacitors
- 6,000 switches
- Power: 140 kWatts

Charney, et al., *Tellus*, 1950.

- The atmosphere is treated as a single layer.
- The flow is assumed to be nondivergent.
- Absolute vorticity is conserved:

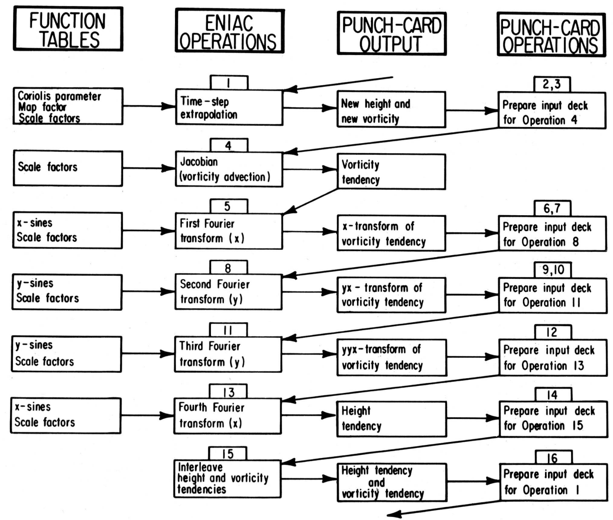
$$\frac{d(\zeta + f)}{dt} = 0.$$

Charney Fjørtoft von Neumann



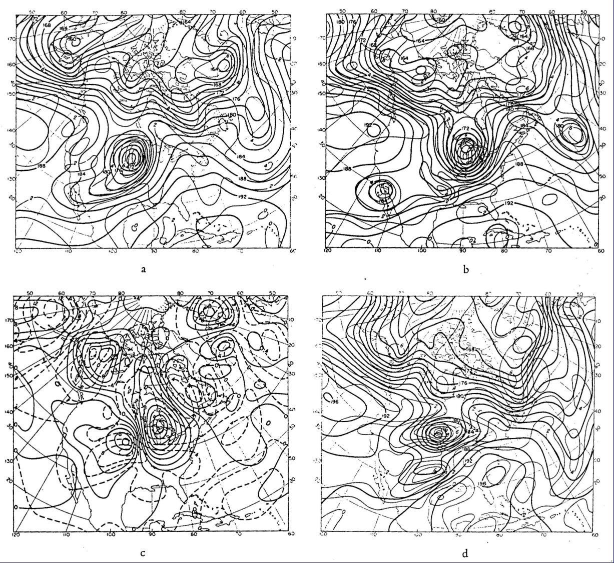
Numerical integration of the barotropic vorticity equation
***Tellus*, 2, 237–254 (1950).**

The ENIAC Algorithm: Flow-chart



G. W. Platzman: The ENIAC Computations of 1950 — Gateway to Numerical Weather Prediction (BAMS, April, 1979).

ENIAC Forecast for Jan 5, 1949



NWP Operations

The Joint Numerical Weather Prediction Unit was established on July 1, 1954:

- Air Weather Service of US Air Force
- The US Weather Bureau
- The Naval Weather Service.

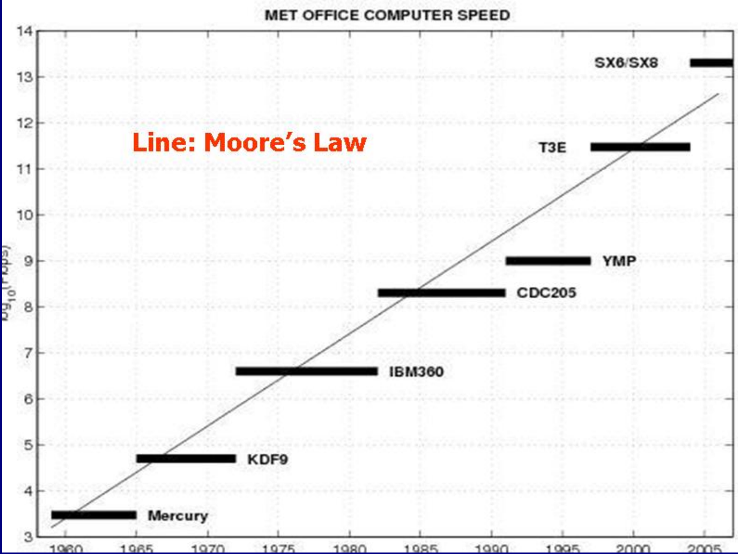
⋆ ⋆ ⋆

Operational numerical weather forecasting began in

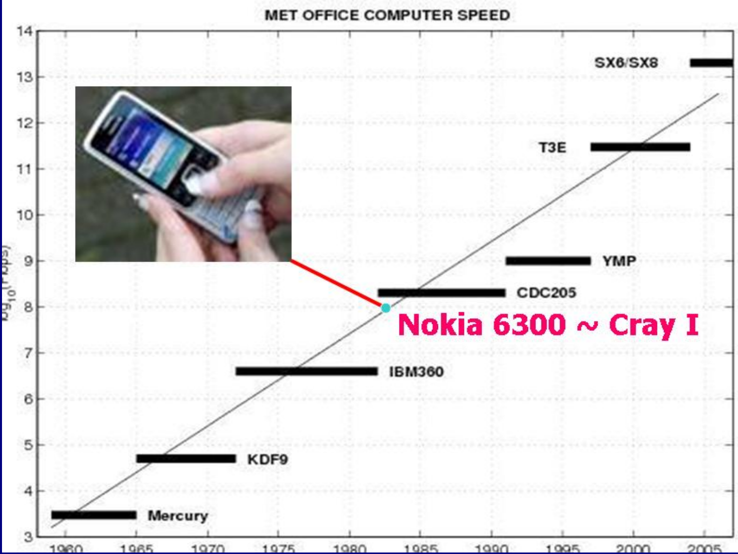
May 1955

using a 3-level quasi-geostrophic model.

An Order of Magnitude every 5 Years



An Order of Magnitude every 5 Years



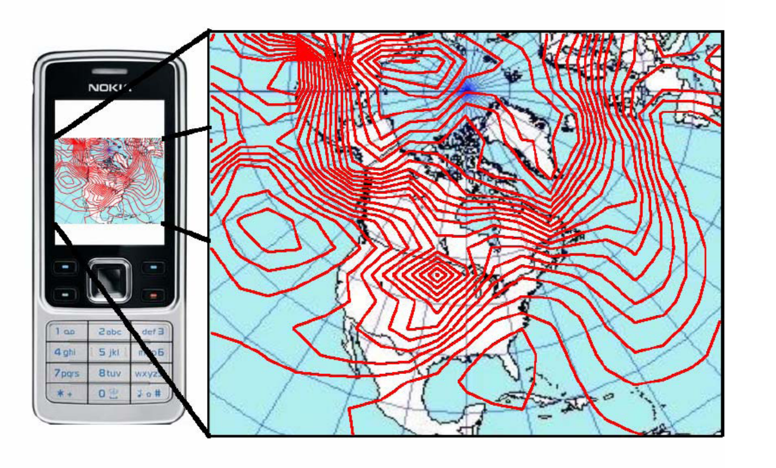
Forecasts by PHONIAC

A modern hand-held mobile phone has far greater power than the ENIAC had.

The ENIAC integrations were repeated using a programmable mobile phone.

A program PHONIAC.JAR**, a J2ME application, was written and implemented on a Nokia 6300.**

PHONIAC: Portable Hand Operated Numerical Integrator and Computer



Weather Magazine, November 2008

Forecasts by PHONIAC

Weather – November 2008, Vol. 63, No. 11

Peter Lynch[1] and Owen Lynch[2]

[1]*University College Dublin, Meteorology and Climate Centre, Dublin*
[2]*Dublin Software Laboratory, IBM Ireland*

The first computer weather forecasts were made in 1950, using the ENIAC (Electronic Numerical Integrator and Computer). The ENIAC forecasts led to operational numerical weather prediction within five years, and paved the way for the remarkable advances in weather prediction and climate modelling that have been made over the past half century. The basis for the forecasts was the barotropic vorticity equation (BVE). In the present study, we describe the solution of the BVE on a mobile phone (cell-phone), and repeat one of the ENIAC forecasts. We speculate on the possible applications of mobile phones for micro-scale numerical weather prediction.

The ENIAC Integrations

and John von Neumann (1950; cited below as CFvN). The story of this work was recounted by George Platzman in his Victor P. Starr Memorial Lecture (Platzman, 1979). The atmosphere was treated as a single layer, represented by conditions at the 500 hPa level, modelled by the BVE. This equation, expressing the conservation of absolute vorticity following the flow, gives the rate of change of the Laplacian of height in terms of the advection. The tendency of the height field is obtained by solving a Poisson equation with homogeneous boundary conditions. The height field may then be advanced to the next time level. With a one hour time-step, this cycle is repeated 24 times for a one-day forecast.

The initial data for the forecasts were prepared manually from standard operational 500 hPa analysis charts of the U.S. Weather Bureau, discretised to a grid of 19 by 16 points, with grid interval of 736 km. Centred spatial finite differences and a leapfrog time-scheme were used. The boundary conditions for height were held constant throughout each 24-hour integration. The forecast starting at 0300 UTC, January 5, 1949 is shown in

vorticity. The forecast height and vorticity are shown in the right panel. The feature of primary interest was an intense depression over the United States. This deepened, moving NE to the 90 °W meridian in 24 hours. A discussion of this forecast, which underestimated the development of the depression, may be found in CFvN and in Lynch (2008).

Dramatic growth in computing power

The oft-cited paper in *Tellus* (CFvN) gives a complete account of the computational algorithm and discusses four forecast cases. The ENIAC, which had been completed in 1945, was the first programmable electronic digital computer ever built. It was a gigantic machine, with 18,000 thermionic valves, filling a large room and consuming 140 kW of power. Input and output was by means of punch-cards. McCartney (1999) provides an absorbing account of the origins, design, development and destiny of ENIAC.

Advances in computer technology over the past half-century have been spectacular. The increase in computing power is encap-

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Reasons for Progress in Weather Forecasting

- Faster computers;
- Better numerical schemes;
- Enhancements in model resolution;
- New observational data from satellites;
- More comprehensive physical processes;
- Paradigm shift to probabilistic forecasting;
- More sophisticated methods of data assimilation.

The Equations of the Atmosphere

GAS LAWS

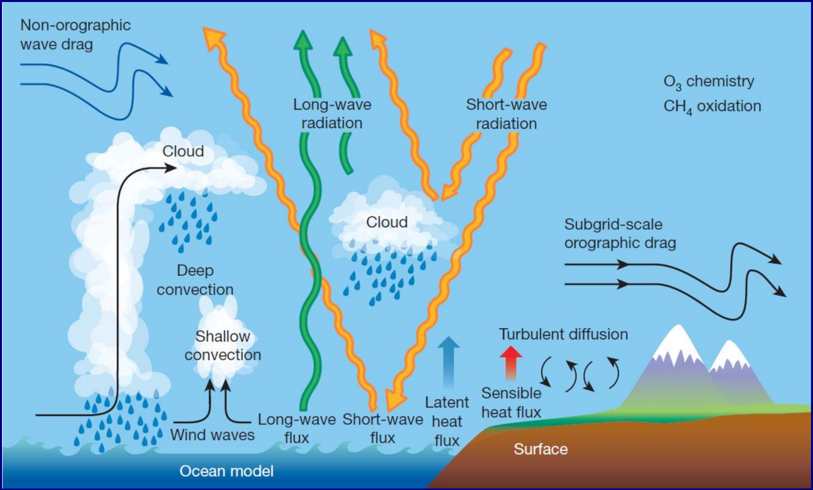
THERMODYNAMIC EQUATION

EQUATIONS OF MOTION: Navier-Stokes Equations

CONTINUITY EQUATION

WATER SUBSTANCE EQUATION

Physical Processes in the Atmosphere



Scientific Forecasting in a Nut-Shell

- The atmosphere is a physical system
- Its behaviour is governed by the laws of physics
- These laws are expressed quantitatively in the form of mathematical equations
- Using observations, we can specify the atmospheric state at a given initial time:
  "Today's Weather"
- Using the equations, we can calculate how this state will change over time:
  "Tomorrow's Weather"

Scientific Forecasting in a Nut-Shell

Problems:

- The equations are very complicated (non-linear): Powerful computer required to solve them.
- The accuracy decreases as the range increases; There is an inherent limit of predictibility.

Time stepping schemes

Replace continuous time by $\{0, \Delta t, 2\Delta t, \ldots, n\Delta t\}$:

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = F(Q)\,.$$

There are two ways to treat the time derivative:

- Eulerian
- Lagrangian

The first operational implementation of a model with a Lagrangian scheme was developed by Ray Bates.

Operational Forecasting: Suite of Models

Operational forecasting is based on the output from a **suite of computer models.**

Global models are used for predictions of several days ahead

Shorter-range forecasts are based on regional or **limited-area models.**

Met Éireann and several other European NMSs use:

- **HARMONIE** for Short Range Forecasting
- **ECMWF Model** for Medium Range Forecasting

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European Centre for Medium-Range Weather Forecasts (ECMWF, Reading, UK)

Forecast of Hurricane Sandy

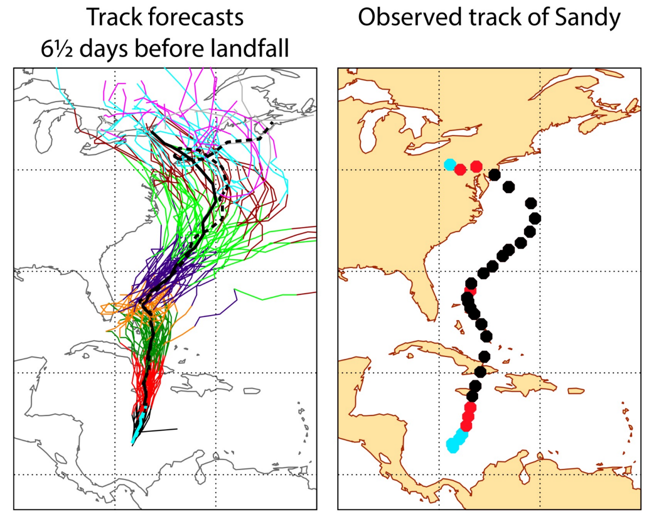
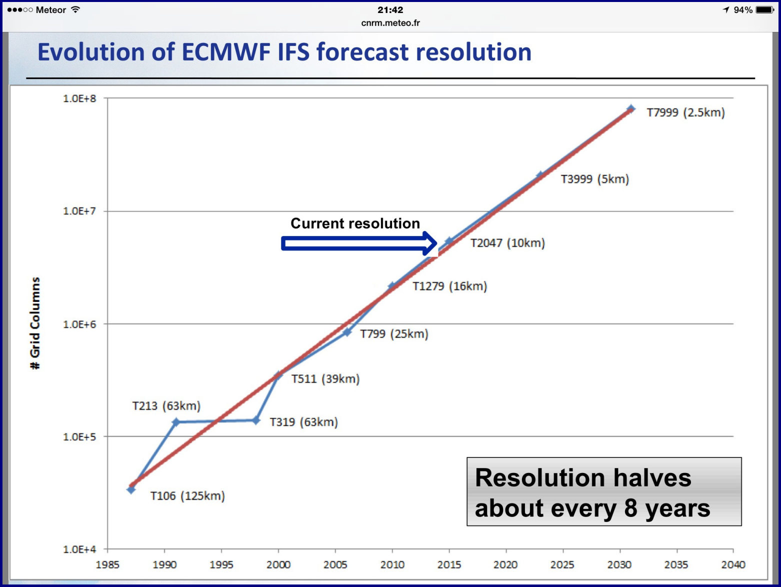
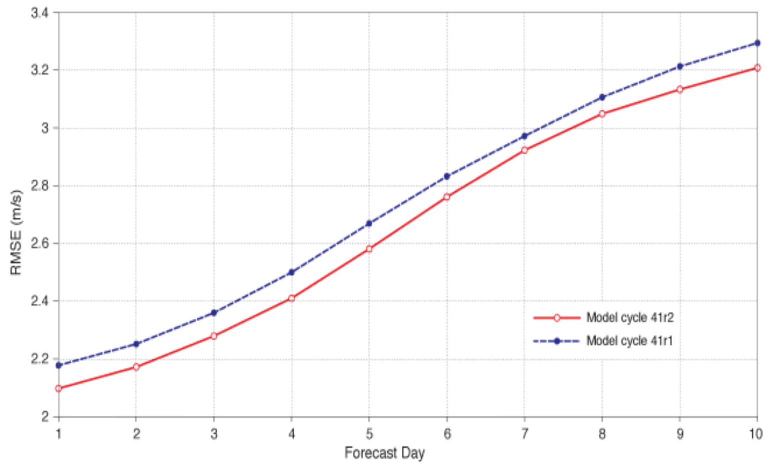


Figure : Landfall, New Jersey, 30 October 2012

Resolution of the IFS System



Resolution of the IFS System



Root-mean-square error of high-resolution 10-metre wind speed forecasts in Europe averaged over 12 UTC forecasts from 10 August 2015 to 25 February 2016. *Forecasts*

Growth in Forecast Skill

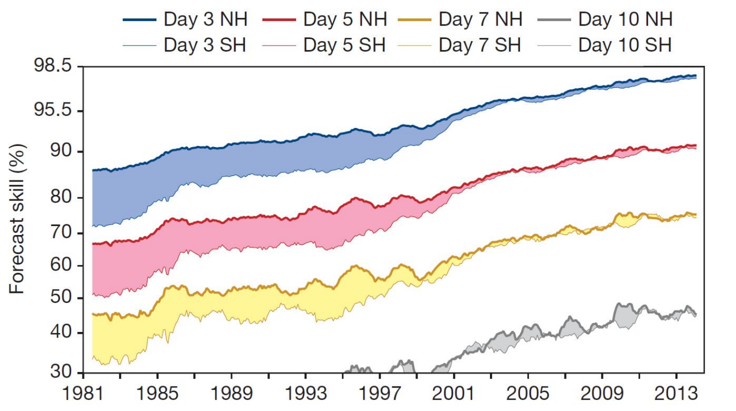


Figure : Anomaly correlation of 500 hPa geopotential height

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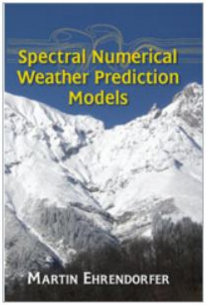
Laplace Transform Integration of Peak

A Laplace Transform Integration scheme, already tested in shallow water models, has been implemented in a Baroclinic Model.

The model is PEAK, a global spectral model written by Martin Ehrendorfer.



LTIS in PEAK

The LT scheme provides an attractive alternative to the popular semi-implicit (SI) scheme.

Analysis shows that LT is more accurate than SI for both linear and nonlinear terms of the equations.

Numerical experiments confirm the superior performance of the LT scheme.

The algorithmic complexity of the LT scheme is comparable to that of SI.

It gives the possibility of improving weather forecasts at comparable computational cost.

Accuracy Analysis

A simple nonlinear oscillation equation:

$$\dot{X} = i\omega X + N(X)$$

For this analysis we assume that N is constant.
The exact solution is then

$$X(t) = X^0 \exp(i\omega t) + \left[\frac{\exp(i\omega t) - 1}{i\omega}\right] N$$

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$$X(t) = X^0 \exp(i\omega t) + \left[\frac{\exp(i\omega t) - 1}{i\omega}\right] N$$

We can express the solution X^+ at time $(n+1)\Delta t$ as

$$X^+ = \Big[\exp(2i\theta)\Big] X^- + \left[\frac{\exp(2i\theta) - 1}{2i\theta}\right] 2\Delta t N$$

where X^- is the solution at time $(n-1)\Delta t$.
We have introduced the digital frequency $\theta = \omega\Delta t$.

SI Solution

The SI approximation for the equation is

$$\frac{X^+ - X^-}{2\Delta t} = i\omega \frac{X^+ + X^-}{2\Delta t} + N$$

SI Solution

The SI approximation for the equation is

$$\frac{X^+ - X^-}{2\Delta t} = i\omega \frac{X^+ + X^-}{2\Delta t} + N$$

Solving for the new value X^+, we have

$$X^+ = \left(\frac{1 + i\theta}{1 - i\theta}\right) X^- + \left(\frac{1}{1 - i\theta}\right) 2\Delta t N$$

Both the linear and nonlinear components of the solution are misrepresented.

SI Solution: Linear Term

For the exact solution , X^- is multiplied by $\exp(2i\theta)$. This has unit modulus and phase 2θ.

For the SI solution, the multiplier is

$$\rho = \left[\frac{1 + i\theta}{1 - i\theta}\right] ,$$

which has modulus and phase given by

$$|\rho| = 1 \qquad \textbf{and} \qquad \arg\rho = 2\arctan\theta \, .$$

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which has modulus and phase given by

$$|\rho| = 1 \qquad \textbf{and} \qquad \arg\rho = 2\arctan\theta\,.$$

Thus, there is no amplification, but a phase error depending on θ.

SI Solution: Nonlinear Term

The multiplier of the nonlinear term $2\Delta t N$ is

$$\rho = \left[\frac{\exp(2i\theta) - 1}{2i\theta}\right] ,$$

with modulus and phase of ρ given by

$$|\rho| = \frac{\sin\theta}{\theta} \qquad \textbf{and} \qquad \arg\rho = \theta$$

SI Solution: Nonlinear Term

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The corresponding factor for the SI scheme is

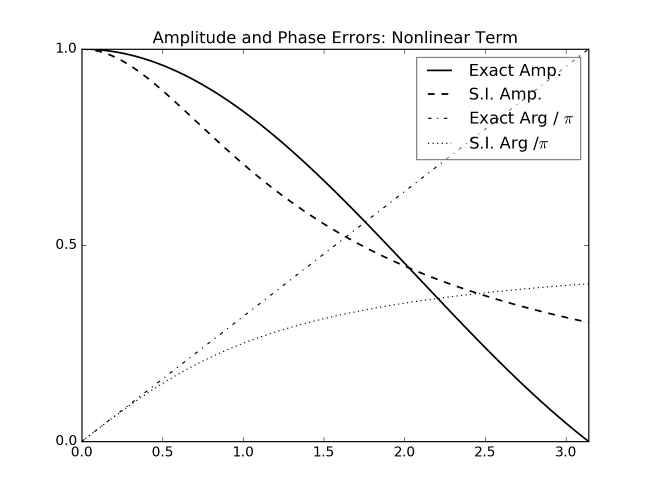
$$\rho = \left[\frac{1}{1 - i\theta}\right] ,$$

with modulus and phase of ρ given by

$$|\rho| = \frac{1}{\sqrt{1+\theta^2}} \qquad \textbf{and} \qquad \arg\rho = \arctan\theta .$$

The SI scheme has both modulus and phase errors in the NL term.

Errors in the SI Scheme



LT Solution

We now apply the Laplace Transform $\mathcal{L}$ to the equation, taking the origin of time at $(n-1)\Delta t$:

$$s\hat{X} - X^{-} = i\omega\hat{X} + \frac{N}{s}.$$

LT Solution

We now apply the Laplace Transform $\mathcal{L}$ to the equation, taking the origin of time at $(n-1)\Delta t$:

$$s\hat{X} - X^{-} = i\omega\hat{X} + \frac{N}{s}.$$

Solving for $\hat{X}$ gives

$$\hat{X} = \frac{X^{-}}{s - i\omega} + \frac{N}{s(s - i\omega)}.$$

LT Solution

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Solving for $\hat{X}$ gives

$$\hat{X} = \frac{X^{-}}{s - i\omega} + \frac{N}{s(s - i\omega)}.$$

The inverse Laplace transform $\mathcal{L}^{-1}$ at time $2\Delta t$ yields the exact solution at time $(n+1)\Delta t$.

The scheme filters high frequency components by using a modified inversion operator $\mathcal{L}^*$.

We invert the transform analytically, eliminating components with ω greater than a cut-off ω_c.

Assuming that $|\omega| \ll \omega_c$, the inverse Laplace transform at time $(n+1)\Delta t$ gives

$$X^+ = [\exp(2i\omega\Delta t)]\, X^- + \left[\frac{\exp(2i\omega\Delta t) - 1}{i\omega}\right] N\,.$$

This agrees with the exact analytic result for both the linear and nonlinear terms.

Thus, the LT scheme is free from error (to the extent that N can be regarded as constant).

The Five-day Wave

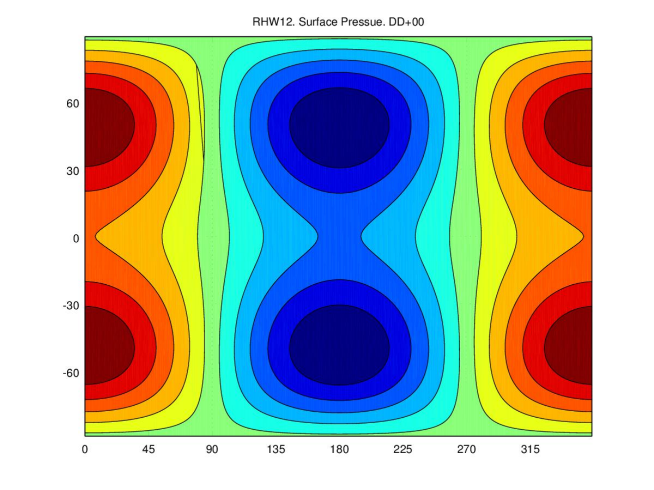

Figure : The Five-day Wave at day 0

The Five-day Wave

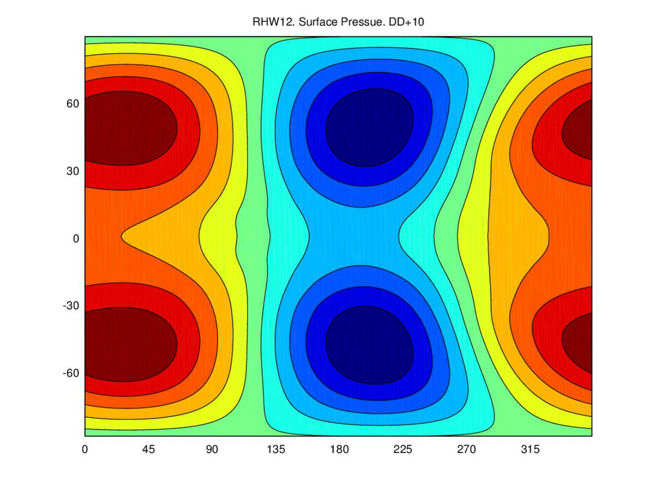

Figure : The Five-day Wave at day 10

L_∞-Score for Five-day Wave

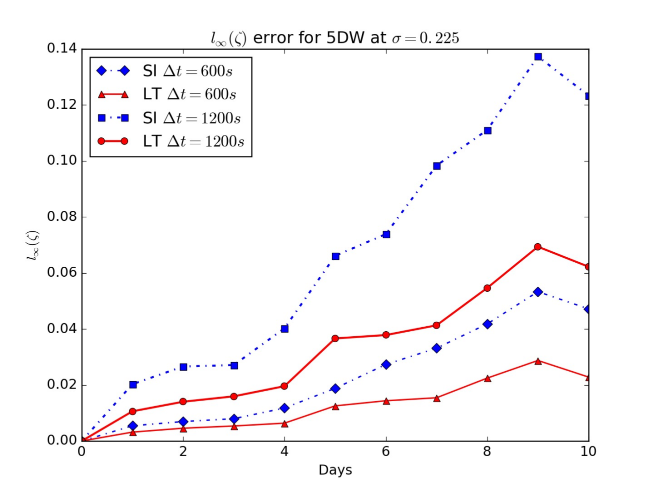

Figure : Scores for Five-day Wave

L_∞-Score for R-H Wave

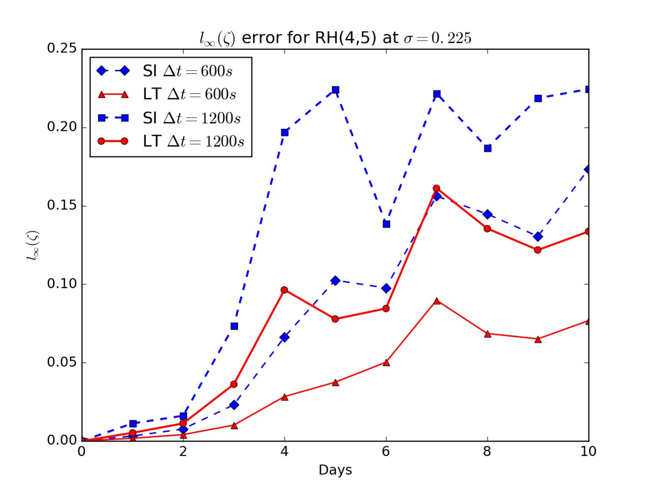

Figure : Scores for Rossby-Haurwitz Wave

L_∞-Score for Unstable Flow (Polvani)

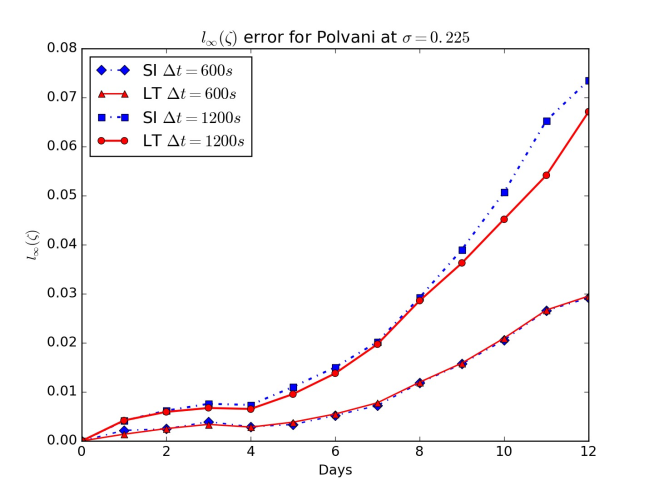

Figure : Scores for Baroclinically Unstable Wave

Conclusions

- LT is an attractive alternative to SI
- Algorithmic complexity is comparable
- Possibility of improving weather forecasts

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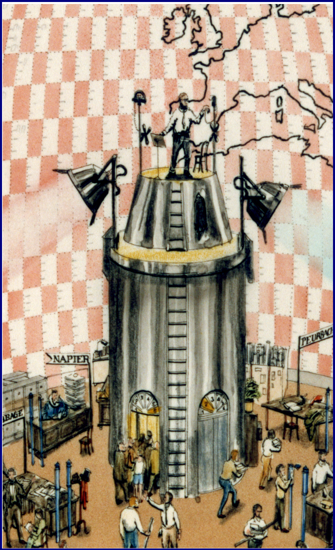
Richardson’s Forecast Factory

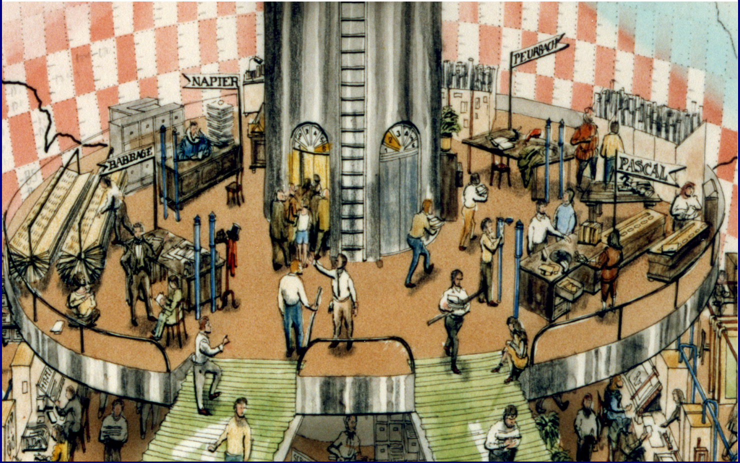
©Stephen Conlin, 1986

Zoom: Richardson Directing the Forecast



Lewis Fry Richardson conducting the forecast

Zoom: Historical Figures in Computing



Napier / Babbage / Pascal / Peurbach

Zoom: Experimentation & Research

Babbage's Analytical Engine
Kelvin on left. Boole on right.

Richardson’s Forecast Factory

64,000 Computers: the first Massively Parallel Processor

The Fantastic Forecast Factory

An Artist's Impression of Richardson's Fantastic Forecast Factory. *Weather*, 71, 14–18.

[Reprint on my website]

High-res Image with Zoom on website of European Meteorological Society:

http://www.emetsoc.org/

Thank you

Growth in Forecast Skill

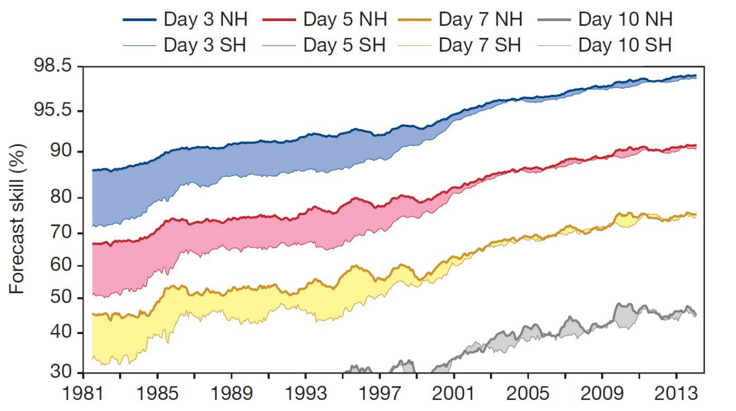


Figure : Anomaly correlation of 500 hPa geopotential height