The Emergence of Numerical Weather Prediction:

Fulfilment of a Dream & Realization of a Fantasy

Peter Lynch
School of Mathematics & Statistics
University College Dublin

Mathematics of Planet Earth Jamboree
Reading University, 22–23 March 2016.

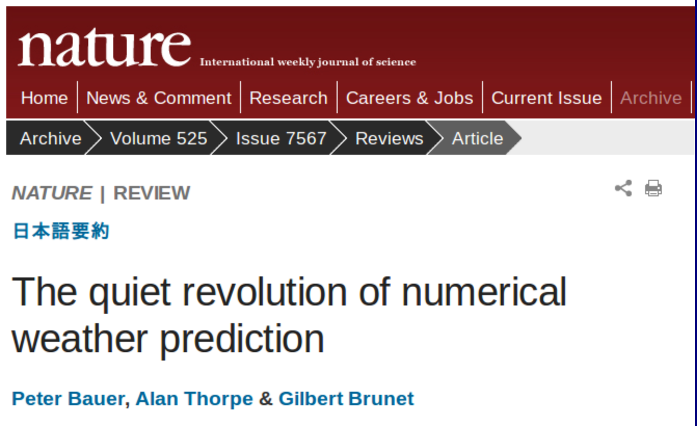
Outline

23 March: World Meteorological Day

WMO: Hotter, Drier, Wetter. Face the Future

Outline

A Recent Paper in Nature



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NATURE | REVIEW

日本語要約

The quiet revolution of numerical weather prediction

Peter Bauer, Alan Thorpe & Gilbert Brunet

A recent review of NWP by scientists from ECMWF.

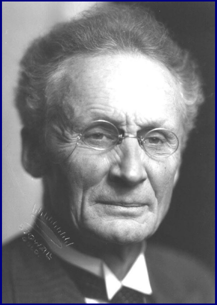
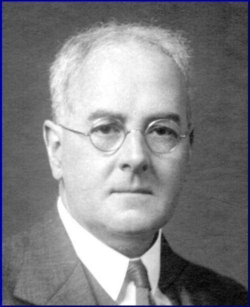
***Nature*, 3 September 2015 Vol 525 p.47**

The Quiet Revolution of NWP [Abstract]

- Advances in NWP represent a quiet revolution.
- Steady accumulation of technological advances.
- Among the greatest impacts of physical science.
- NWP is a computational problem comparable to:
  - Modelling the behaviour of the human brain
  - Simulating the evolution of the early universe.

Outline

Pioneers of Scientific Forecasting

Cleveland Abbe, Vilhelm Bjerknes, Lewis Fry Richardson

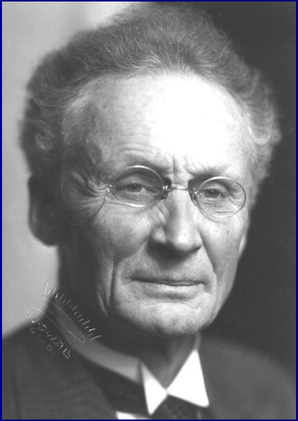
Cleveland Abbe

By 1890, the American meteorologist Cleveland Abbe had recognized that:

Meteorology is essentially the application of hydrodynamics and thermodynamics to the atmosphere.

Abbe proposed a mathematical approach to forecasting.

Vilhelm Bjerknes



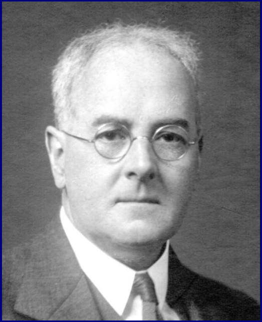
A more explicit analysis of weather prediction was undertaken by the Norwegian scientist Vilhelm Bjerknes

He identified the two crucial components of a scientific forecasting system:

- Analysis
- Integration

Vilhelm Bjerknes (1862–1951)

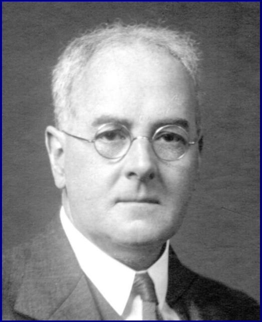
Lewis Fry Richardson



The English Quaker scientist Lewis Fry Richardson attempted a direct solution of the equations of motion.

He dreamed that numerical forecasting would become a reality 'one day in the distant future'.

Lewis Fry Richardson

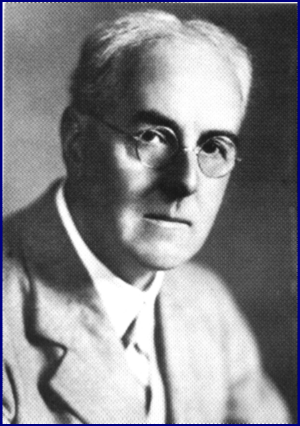


The English Quaker scientist Lewis Fry Richardson attempted a direct solution of the equations of motion.

He dreamed that numerical fore-casting would become a reality 'one day in the distant future'.

Today, forecasts are prepared routinely using his method . . . his dream has indeed come true.

Lewis Fry Richardson, 1881–1953.



During WW I, Richardson computed by hand the pressure change at a single point.

It took him two years !

Lewis Fry Richardson, 1881–1953.

During WW I, Richardson computed by hand the pressure change at a single point.

It took him two years !

His ‘forecast’ was a catastrophic failure:

$$\Delta p = 145 \text{ hPa in 6 hrs}$$

But Richardson’s method was scientifically sound.

Initialization of Richardson's Forecast

Richardson's Forecast was repeated on a computer.

The atmospheric observations for 20 May, 1910,
were recovered from original sources.

- **ORIGINAL:** $$\frac{\partial p_s}{\partial t} = +\mathbf{145}\,\mathrm{hPa}/6\,\mathrm{h}$$

Initialization of Richardson's Forecast

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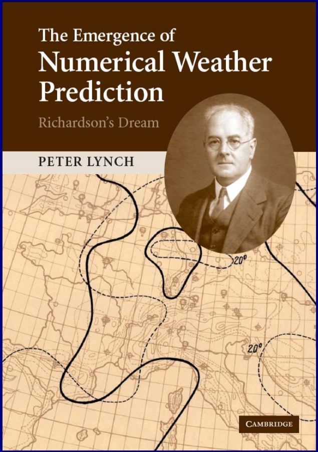
were recovered from original sources.

- **ORIGINAL:** $$\frac{\partial p_s}{\partial t} = +\mathbf{145}\,\mathrm{hPa}/6\,\mathrm{h}$$
- **INITIALIZED:** $$\frac{\partial p_s}{\partial t} = -\mathbf{0.9}\,\mathrm{hPa}/6\,\mathrm{h}$$

Observations: The barometer was steady!

Full Account of the Forecast



Richardson's Forecast and the Emergence of NWP are described in this book.

Cambridge Univ. Press, 2006

[Recently issued in paperback form]

Richardson’s Forecast Factory

64,000 Computers: the first Massively Parallel Processor

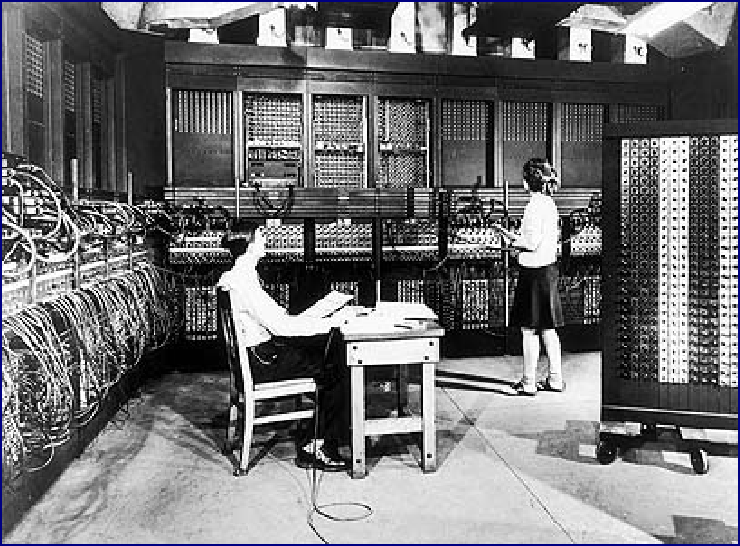
UCD DUBLIN

Outline

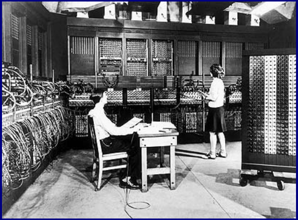
Crucial Advances, 1920–1950

- Dynamic Meteorology
  - Quasi-geostrophic Theory
- Numerical Analysis
  - CFL Criterion
- Atmopsheric Observations
  - Radiosondes
- Electronic Computing
  - ENIAC

The ENIAC



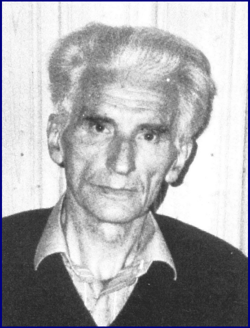
The ENIAC



The **ENIAC** was the first multi-purpose programmable electronic digital computer:

- 18,000 vacuum tubes
- 70,000 resistors
- 10,000 capacitors
- 6,000 switches
- Power: 140 kWatts

Charney Fjørtoft von Neumann

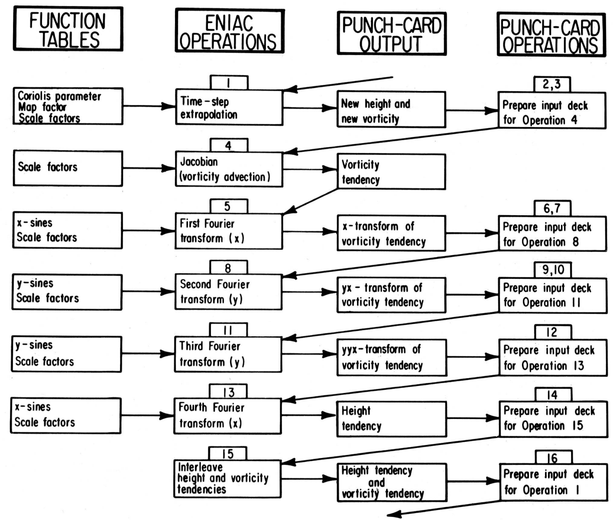

Numerical integration of the barotropic vorticity equation
***Tellus*, 2, 237–254 (1950).**

Charney, et al., *Tellus*, 1950.

- The atmosphere is treated as a single layer.
- The flow is assumed to be nondivergent.
- Absolute vorticity is conserved.

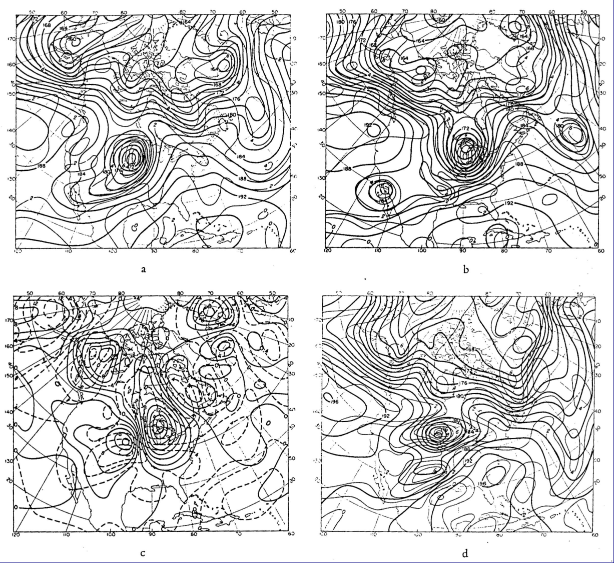
$$\frac{d(\zeta + f)}{dt} = 0.$$

The ENIAC Algorithm: Flow-chart



G. W. Platzman: The ENIAC Computations of 1950 — Gateway to Numerical Weather Prediction (BAMS, April, 1979).

ENIAC Forecast for Jan 5, 1949



NWP Operations

The Joint Numerical Weather Prediction Unit was established on July 1, 1954:

- Air Weather Service of US Air Force
- The US Weather Bureau
- The Naval Weather Service.

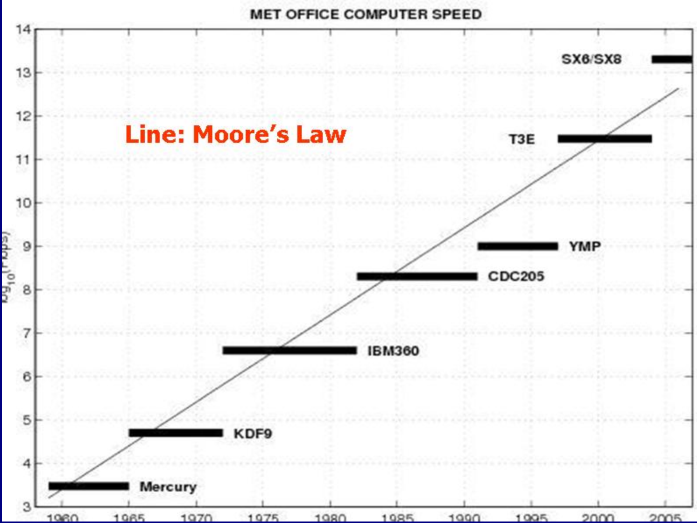
⋆ ⋆ ⋆

Operational numerical weather forecasting began in

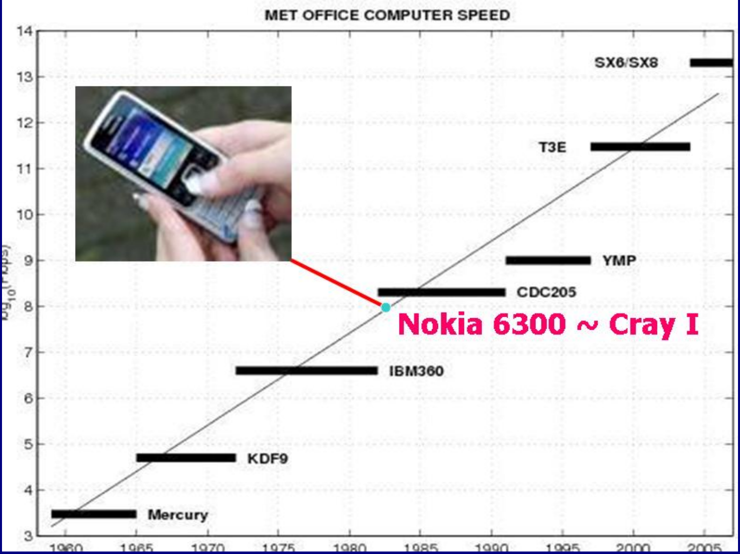
May 1955

using a 3-level quasi-geostrophic model.

An Order of Magnitude every 5 Years



An Order of Magnitude every 5 Years



Forecasts by PHONIAC

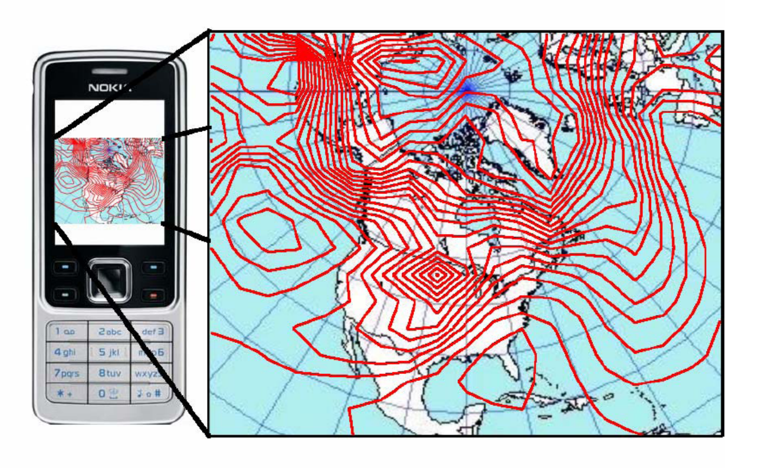
Peter Lynch & Owen Lynch

A modern hand-held mobile phone has far greater power than the ENIAC had.

We therefore decided to repeat the ENIAC integrations using a programmable mobile phone.

We wrote a program PHONIAC.JAR**, a J2ME application, and implemented it on a mobile phone.**

PHONIAC: Portable Hand Operated Numerical Integrator and Computer



Notices of the AMS

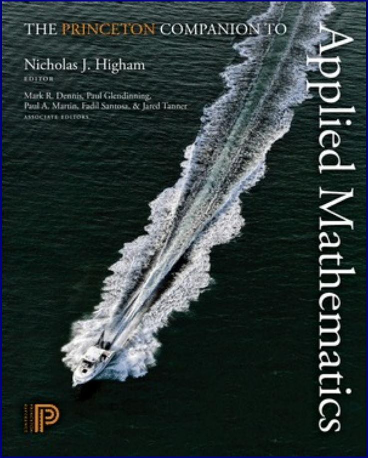
Forecasts by PHONIAC:
***Weather*, Nov. 2008.**

Cover of Sept. 2013
***Notices of the American Mathematical Society*.**

This technology has great potential for generation and delivery of operational weather forecast products.

Outline

Princeton Companion to Applied Maths



Numerical Weather Prediction
Peter Lynch

1 Introduction

The development of computer models for numerical simulation and prediction of the atmosphere and oceans is one of the great scientific triumphs of the past fifty years. Today, numerical weather prediction (NWP) plays a central and essential role in operational weather forecasting, with forecasts now having accuracy at ranges beyond a week. There are several reasons for this: enhancements in model resolution, better numerical schemes, more realistic parametrizations of physical processes, new observational data from satellites, and more sophisticated methods of determining the initial conditions. In this article we focus on the fundamental equations, the formulation of the numerical algorithms, and the variational approach to data assimilation. We present the mathematical principles of NWP and illustrate the process by considering some specific models and their application to practical forecasting.

Article available on my website

Reasons for Progress in Weather Forecasting

- Enhancements in model resolution;
- Faster computers;
- Better numerical schemes;
- More comprehensive physical processes;
- New observational data from satellites;
- More sophisticated methods of data assimilation;
- Paradigm shift to probabilistic forecasting.

The Equations of the Atmosphere

GAS LAW (Boyle's Law and Charles' Law.)

Relates the pressure, temperature and density

CONTINUITY EQUATION

Conservation of mass

WATER CONTINUITY EQUATION

Conservation of water (liquid, solid and gas)

EQUATIONS OF MOTION: Navier-Stokes Equations

Describe how the change of velocity is determined by the pressure gradient, Coriolis force and friction

THERMODYNAMIC EQUATION

Determines changes of temperature due to heating or cooling, compression or rarefaction, etc.

Seven equations; seven variables (u, v, w, ρ, p, T, q).

The Primitive Equations

$$\frac{du}{dt} - \left(f + \frac{u\tan\phi}{a}\right)v + \frac{1}{\rho}\frac{\partial p}{\partial x} + F_x = 0$$

$$\frac{dv}{dt} + \left(f + \frac{u\tan\phi}{a}\right)u + \frac{1}{\rho}\frac{\partial p}{\partial y} + F_y = 0$$

$$\frac{\partial p}{\partial z} + g\rho = 0$$

$$p = R\rho T$$

$$\frac{dT}{dt} + (\gamma - 1)T\nabla\cdot\mathbf{V} = \frac{Q}{c_p}$$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\rho\mathbf{V} = 0$$

$$\frac{\partial\rho_w}{\partial t} + \nabla\cdot\rho_w\mathbf{V} = [\textbf{Sources} - \textbf{Sinks}]$$

Scientific Forecasting in a Nut-Shell

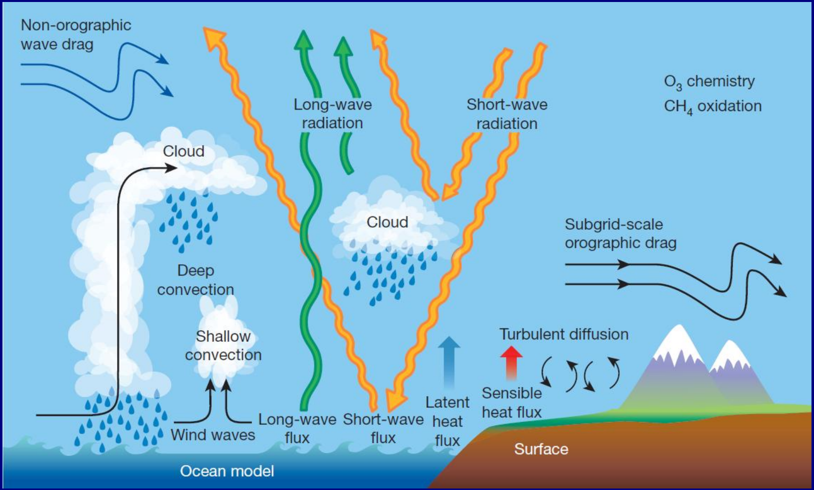
- The atmosphere is a physical system
- Its behaviour is governed by the laws of physics
- These laws are expressed quantitatively in the form of mathematical equations
- Using observations, we can specify the atmospheric state at a given initial time:
  "Today's Weather"
- Using the equations, we can calculate how this state will change over time:
  "Tomorrow's Weather"

Scientific Forecasting in a Nut-Shell

Problems:

- The equations are very complicated (non-linear): Powerful computer required to solve them.
- The accuracy decreases as the range increases; There is an inherent limit of predictibility.

Physical Processes in the Atmosphere



Time stepping schemes

Replace **continuous time by** $\{0, \Delta t, 2\Delta t, \ldots, n\Delta t\}$**:**

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = F(Q) \,.$$

Approximate **the time derivative by**

$$\frac{Q^{n+1} - Q^{n-1}}{2\Delta t} = F^n \,,$$

Compute **the forecast value** Q^{n+1} **from**

$$Q^{n+1} = Q^{n-1} + 2\Delta t \, F^n \,.$$

Repeat **until desired forecast range is reached.**

Spatial finite differencing

Consider the simple 1D wave equation

$$\frac{\partial Q}{\partial t} + c\frac{\partial Q}{\partial x} = 0,$$

Centered differences in space and time:

$$\frac{Q_m^{n+1} - Q_m^{n-1}}{2\Delta t} + c\left(\frac{Q_{m+1}^n - Q_{m-1}^n}{2\Delta x}\right) = 0,$$

Condition for stability of the solution:

$$\left|\frac{c\Delta t}{\Delta x}\right| \leq 1.$$

This is the Courant-Friedrichs-Lewy criterion (1928).

Spectral method

Fields expanded in series of spherical harmonics:

$$Q(\lambda,\phi,t)=\sum_{n=0}^{N}\sum_{m=-n}^{n}Q_n^m(t)Y_n^m(\lambda,\phi),$$

Coefficients $Q_n^m(t)$ depend only on time.

Fourier analysis on the sphere.

The model partial differential equations become a coupled set of nonlinear ODEs for $Q_n^m(t)$.

Variational assimilation

The model state is a high-dimensional vector X.

The cost function for 3D-Var is

$$J = J_{\mathrm{B}} + J_{\mathrm{O}}.$$

Background error term (Distance from first guess):

$$J_{\mathrm{B}} = \frac{1}{2}(\mathbf{X} - \mathbf{X}_{\mathrm{B}})^{\mathrm{T}}\mathbf{B}^{-1}(\mathbf{X} - \mathbf{X}_{\mathrm{B}})$$

Observational error term (Distance from obs):

$$J_{\mathrm{O}} = \frac{1}{2}(\mathbf{Y} - \mathbf{HX})^{\mathrm{T}}\mathbf{R}^{-1}(\mathbf{Y} - \mathbf{HX})$$

The minimum of J is attained at $\mathbf{X} = \mathbf{X}_{\mathrm{A}}$ where

$$\nabla_{\mathbf{X}} J = 0.$$

Computing this gradient, we get

$$\nabla_{\mathbf{X}} J = \mathbf{B}^{-1}(\mathbf{X} - \mathbf{X}_{\mathrm{B}}) + \mathbf{H}^{\mathrm{T}}\mathbf{R}^{-1}(\mathbf{Y} - \mathbf{H}\mathbf{X}).$$

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Setting this to zero we can deduce the expression

$$\mathbf{X} = \mathbf{X}_{\mathrm{B}} + \mathbf{K}[\mathbf{Y} - \mathbf{H}\mathbf{X}_{\mathrm{B}}]$$

where the *gain matrix*, is given by

$$\mathbf{K} = \mathbf{B}\mathbf{H}^{\mathrm{T}}(\mathbf{R} + \mathbf{H}\mathbf{B}\mathbf{H}^{\mathrm{T}})^{-1}.$$

Including the Time Dimension

Satellite data are distributed continuously in time.

Four-dimensional variational assimilation (4D-Var) uses all the observations in an interval $t_0 \leq t \leq t_N$.

Including the Time Dimension

Satellite data are distributed continuously in time.

Four-dimensional variational assimilation (4D-Var) uses all the observations in an interval $t_0 \leq t \leq t_N$.

The cost function now includes terms measuring the distance to observations at each time step t_n:

$$J = J_{\mathrm{B}} + \sum_{n=0}^{N} J_{\mathrm{O}}(t_n)$$

where $J_{\mathrm{O}}(t_n)$ is given by

$$J_{\mathrm{O}}(t_n) = (\mathbf{Y}_n - \mathbf{H}_n\mathbf{X}_n)^{\mathrm{T}}\mathbf{R}_n^{-1}(\mathbf{Y}_n - \mathbf{H}_n\mathbf{X}_n)$$

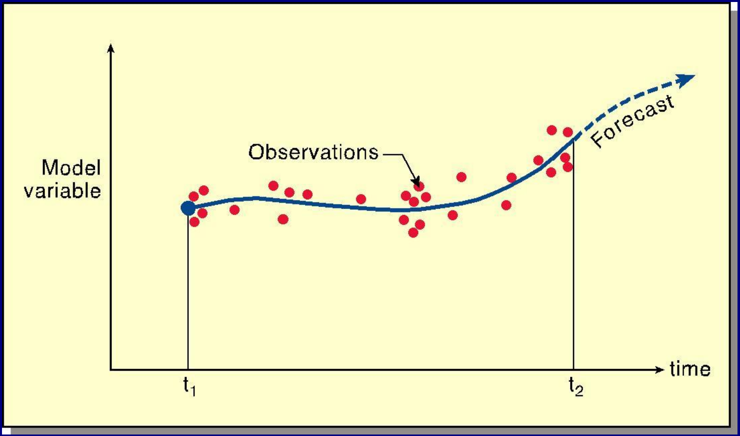


Figure : 4D Variational Assimilation Scheme

Some Technicalities

State vector $\mathbf{X}_n$ at time t_n generated by integrating the **forecast model**:

$$\mathbf{X}_n = \mathcal{M}_n(\mathbf{X}_0).$$

Model operator $\mathcal{M}_n$ linearized about trajectory from background field gives *tangent linear model* $\mathbf{M}_n$.

Minimization in 4D-Var involves transposition. The transpose of the tangent linear model, $\mathbf{M}_n^{\mathrm{T}}$, is called the *adjoint model*.

The *control variable* for the minimization of the cost function is $\mathbf{X}_0$. Model used as a **strong constraint.**

Benefits of 4D-Var

In OI and 3D-Var, all observations within a fixed time window —typically of six hours— assumed valid at the analysis time.

4D-Var finds initial conditions X_0 such that forecast best fits the observations within the assimilation window.

At ECMWF, 4D-Var has led to substantial improvements in operational forecasts.

Growth in Forecast Skill

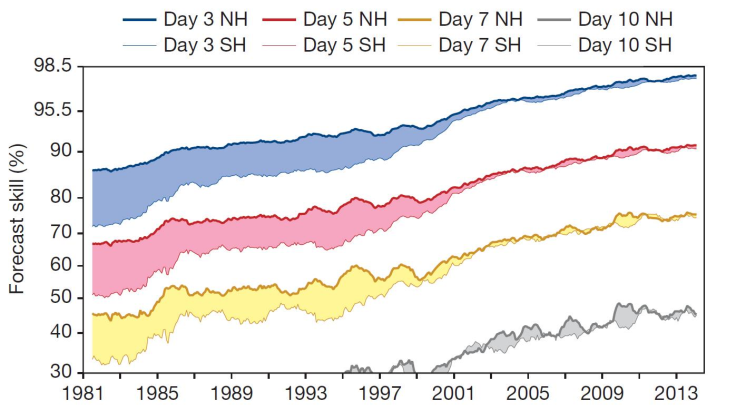


Figure : Anomaly correlation of 500 hPa geopotential height

Operational Forecasting: Suite of Models

Operational forecasting based on output from a suite of computer models.

Global models are used for predictions of several days ahead

Shorter-range forecasts are based on regional or limited-area models.

At many European NMSs:

- Short Range (LAM): HARMONIE Model
- Medium Range (GLOBAL): ECMWF Model

Outline

European Centre for Medium-Range Weather Forecasts (ECMWF, Reading, UK)

As an example of a global model, we consider the Integrated Forecast System (IFS) of ECMWF.

Forecast of Hurricane Sandy

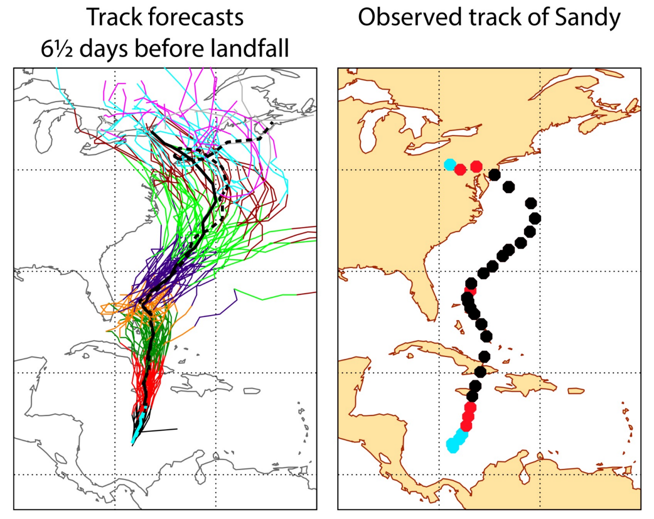
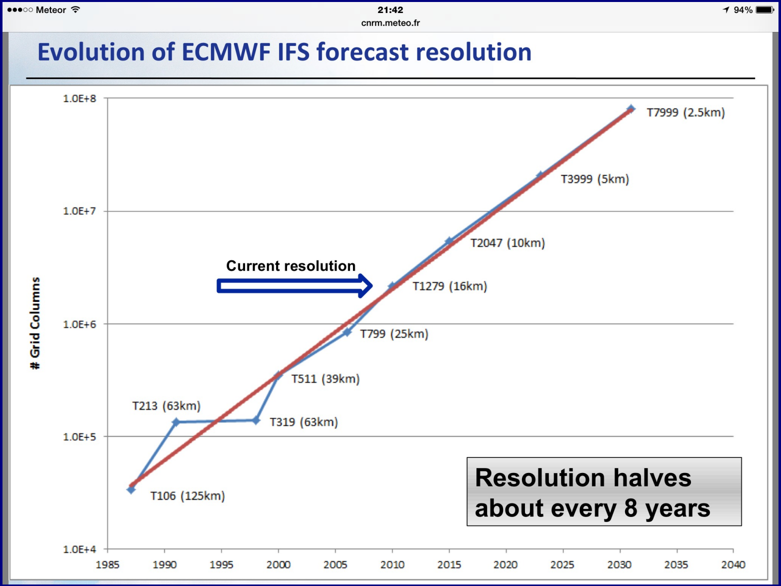


Figure : Landfall, New Jersey, 30 October 2012

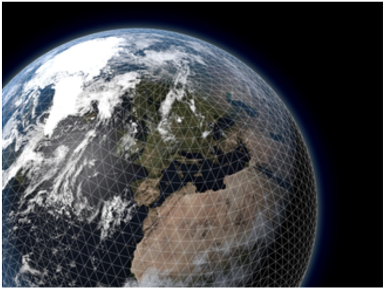
Resolution of the IFS System



Resolution of IFS System: 10 March 2016

New forecast model cycle brings highest-ever resolution

10 March 2016



ECMWF has launched a new model cycle bringing improved global weather forecasts at record-breaking resolution.

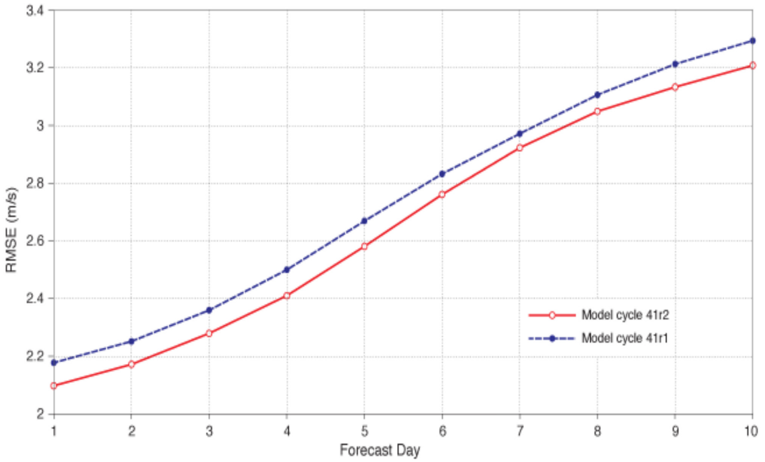
The new grid on which the forecasts are run comprises up to 904 million prediction points, three times as many as before.

Together with other upgrades to ECMWF's Integrated Forecasting System (IFS), the changes mean that Europe's weather can now be predicted with more detail, with greater accuracy and, as a result, up to half a day further ahead.

Table : ECMWF IFS Resolution

	2000	2006	2010	2016
Spectral Truncation	T511	T799	T1279	$T_{CO}1279$
Effective Resolution	39 km	25 km	16 km	$\sim$10 km
Model Levels	60	91	137	137

Resolution of the IFS System



Root-mean-square error of high-resolution 10-metre wind speed forecasts in Europe averaged over 12 UTC forecasts from 10 August 2015 to 25 February 2016. *Forecasts*

Some Details

- The basis of ECMWF operations is the IFS.
- *Spectral representation* of meteorological fields.
- The ENS system runs with a horizontal resolution half that of the deterministic model.
- Model has about a billion degrees of freedom.
- The computational task is formidable.
- The Centre has a Cray XC30 High Performance Computer, comprising some 160,000 processors,
- Sustained performance of over 200 Teraflops.

The OpenIFS Project

The ECMWF OpenIFS model

Filip Váňa1, Glenn Carver1
Walter Zwieflhofer1, Erland Källén1, Peter Bauer1, Umberto Modigliani1, Deborah Salmond1
Abdel Hannachi2, Joakim Kjellsson2, and Michael Tjernström2

1. ECMWF, Reading, UK
2. Dept. of Meteorology (MISU), Stockholm University, Sweden.

http://www.ecmwf.int/

Slide 1 © ECMWF

The ECMWF OpenIFS project

- **New Project for ECMWF.**
  - Started Dec 2011.
  - In development phase.

- **Key Objectives.**
  - Release version of IFS to academic & research users.
  - Increase scientific research undertaken using IFS.
  - Increase NWP training with IFS.

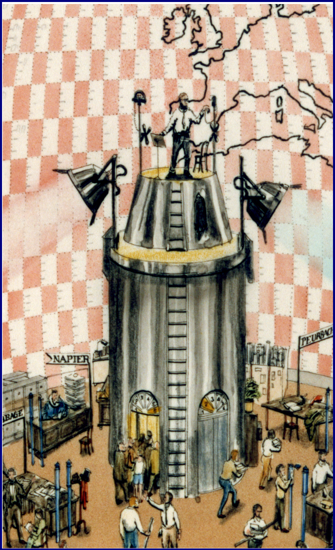
- **Other aims.**
  - Ease of use on external computer systems.
  - Identify user requirements.
  - Dedicated support.

Outline

Richardson’s Forecast Factory

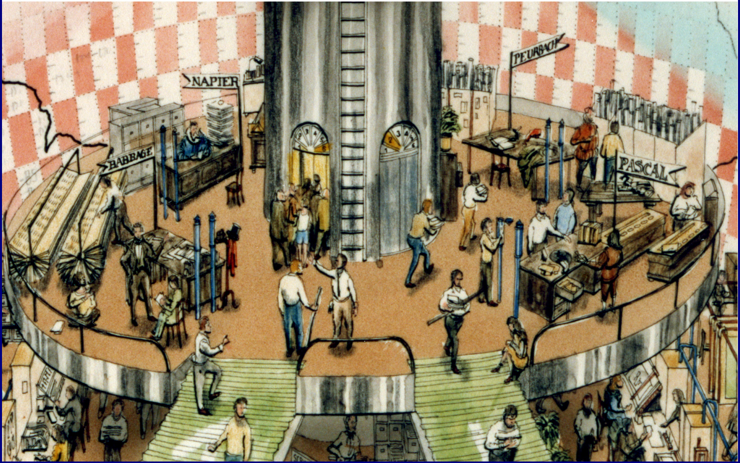
©Stephen Conlin, 1986

Zoom: Richardson Directing the Forecast



Lewis Fry Richardson conducting the forecast

Zoom: Historical Figures in Computing



Napier / Babbage / Pascal / Peurbach

Key to the Historical Figures

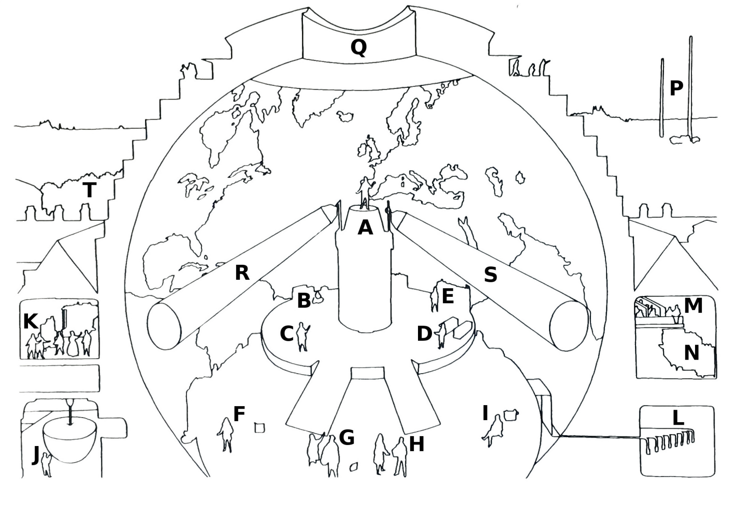


Table of Historical Figures

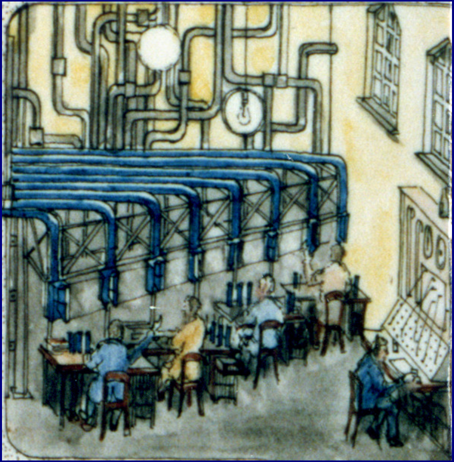
Table 1

Historical characters in the image (see Figure 6)

A	Lewis F. Richardson (1881–1953) in the pulpit, directing operations.
B	John Napier (1550–1617), inventor of logarithms, which had a profound influence on the course of astronomy and of science in general.
C	Charles Babbage (1791–1871), mathematician, inventor and mechanical engineer, originated the concept of a programmable computer and designed highly advanced mechanical calculating machinery.
D	Blaise Pascal (1623–1662), French mathematician, inventor, writer and philosopher. When only 18 years old, he constructed a mechanical calculator capable of addition and subtraction, called the *Pascaline*.
E	Georg von Peurbach (1423–1461), Austrian astronomer and instrument maker who arranged for the first printed set of sines to be computed. He also computed a set of eclipse tables, the *Tabulae Eclipsium*, which remained highly influential for many years.
F	Edmund Gunter (1581–1626), English clergyman and mathematician, inventor of the logarithmic ruler.
G	William Oughtred (1574–1660), English mathematician and Anglican minister, inventor of the slide rule. Walter Lilly (c. 1900), Lecturer in Mechanical Engineering, Trinity College Dublin, with his circular rule.
H	Gottfried Wilhelm von Leibniz (1646–1716), mathematician and philosopher who invented the first mass-produced mechanical calculator. His 'Stepped Reckoner', which performed addition, subtraction, multiplication and division, is illustrated on the table behind him, between Leibniz and George Fuller (one-time Professor of Engineering at Queen's College, Belfast) with his spiral rule.
I	Per Georg Scheutz (1785–1873), Swedish lawyer, translator, inventor and builder of the first practical difference engine. Scheutz's calculator was used for generating tables of logarithms.
J	Sir G. I. Taylor (1886–1975), distinguished hydrodynamicist, grandson of George Boole.
K	The Arithmetic Research Room. Left to right: Lord Kelvin (1824–1907) and his brother James Thomson (with a ball and disk integrator); Percy Ludgate (1883–1922), Irish inventor of an Analytical Engine; Ada Lovelace (1815–1852), daughter of Lord Byron and friend of Babbage; George Boole (1815–1864), inventor of Boolean algebra.
L	Tube Room, or 'quiet room', in which weather information is communicated within the forecast factory by pneumatic tube and to and from the outside world by wireless telegraphy.
M	Hollerith Machines in the research department.
N	Scheutz Difference Engine in the research department.
P	Radio masts for reception of observations and transmission of forecasts.
Q	Public viewing gallery.
R	A rosy light – shone on computers who are forward in their computations.
S	A blue light – shone on computers who are behind in their computations.
T	Recreation area, since *those who compute the weather should breathe of it freely*.

Weather – January 2016, Vol. 71, No. 1

Zoom: Communications & Computing



The Tube Room

Zoom: Communications & Computing

The Computer Laboratory

Zoom: Experimentation & Research

Dish Pan Experiment (G. I. Taylor presiding)

Zoom: Experimentation & Research

Babbage's Analytical Engine

Richardson’s Forecast Factory

64,000 Computers: the first Massively Parallel Processor

The Fantastic Forecast Factory

An Artist's Impression of Richardson's Fantastic Forecast Factory. *Weather*, 71, 14–18.

[Reprint on my website]

High-res Image with Zoom on website of European Meteorological Society:

http://www.emetsoc.org/

Thank you

Growth in Forecast Skill

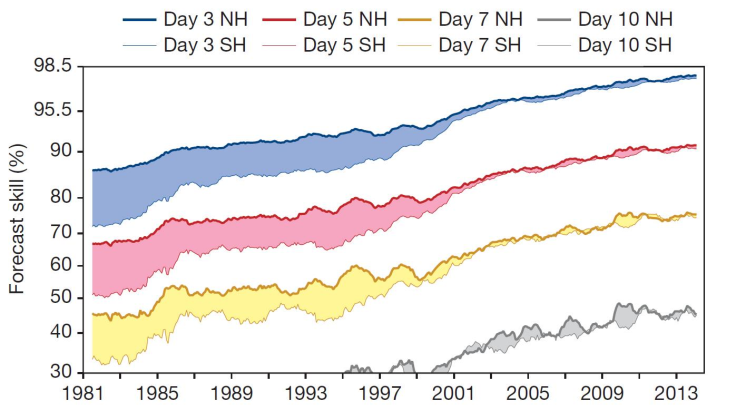


Figure : Anomaly correlation of 500 hPa geopotential height