Calculating the Weather: The Mathematics of Atmospheric Modelling

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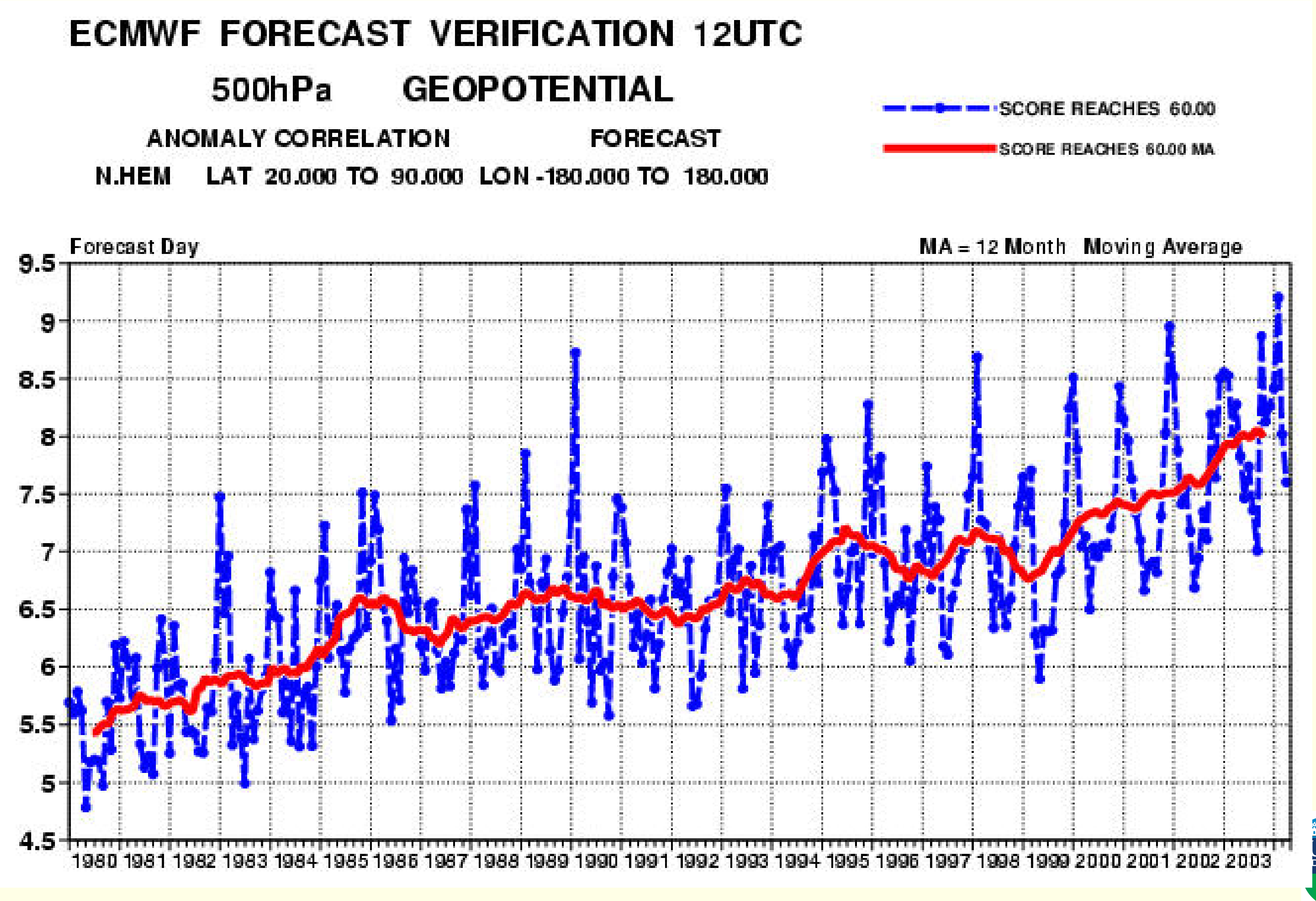
Outline of the Lecture

- *Pre-history of NWP*
- *Richardson's Forecast*
- *The ENIAC Integrations*

Interlude

- *Data Assimilation*
- *Ensemble Prediction*
- *Spherical Grids*

Increase in Forecasting Skill



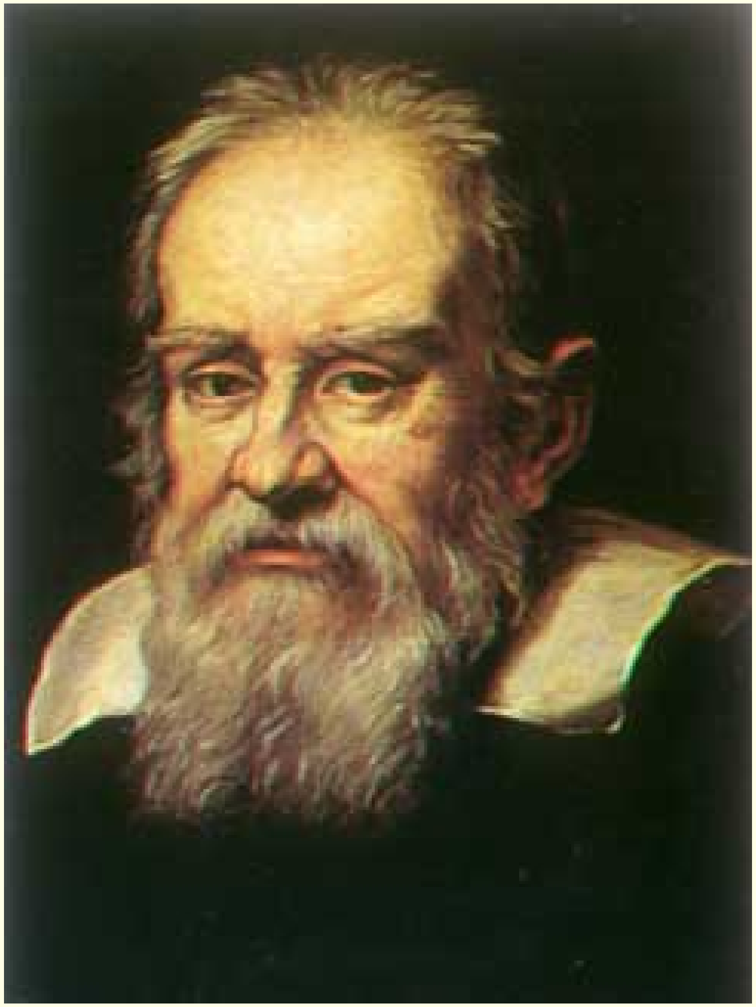
Relevant Mathematical Areas

- *Partial Differential Equations*
- *Numerical Analysis*
- *Linear Algebra*
- *Variational Methods*
- *Dynamical Systems*
- *Geometry of the Sphere*

Something for everyone!

Ancient Times

Galileo Galilei (1564–1642)



Galileo formulated the basic law of falling bodies, which he verified by careful measurements.

He constructed a telescope, with which he studied lunar craters, and discovered four moons revolving around Jupiter.

Galileo is credited with the invention of the Thermometer.

Thus began quantitative measurements of the atmosphere.

Evangelista Torricelli

Evangelista Torricelli (1608–1647), a student of Galileo, devised the first accurate barometer.

Torricelli's Theorem:

$$v = \sqrt{2gh}$$

Torricelli inventing the barometer

Newton's Law of Motion

The rate of change of momentum of a body is equal to the sum of the forces acting on the body.

If $\mathbf{F}$ is the total applied force, Newton's Second Law gives a differential equation:

$$\frac{d\mathbf{p}}{dt} = \mathbf{F}\,.$$

The acceleration $\mathbf{a}$ is the rate of change of velocity, that is, $\mathbf{a} = d\mathbf{V}/dt$. If the mass m is constant, we have

$$\mathbf{F} = m\mathbf{a}\,.$$

$$\textbf{Force} = \textbf{Mass} \times \textbf{Acceleration}\,.$$

Euler's Equations for Fluid Flow

Leonhard Euler

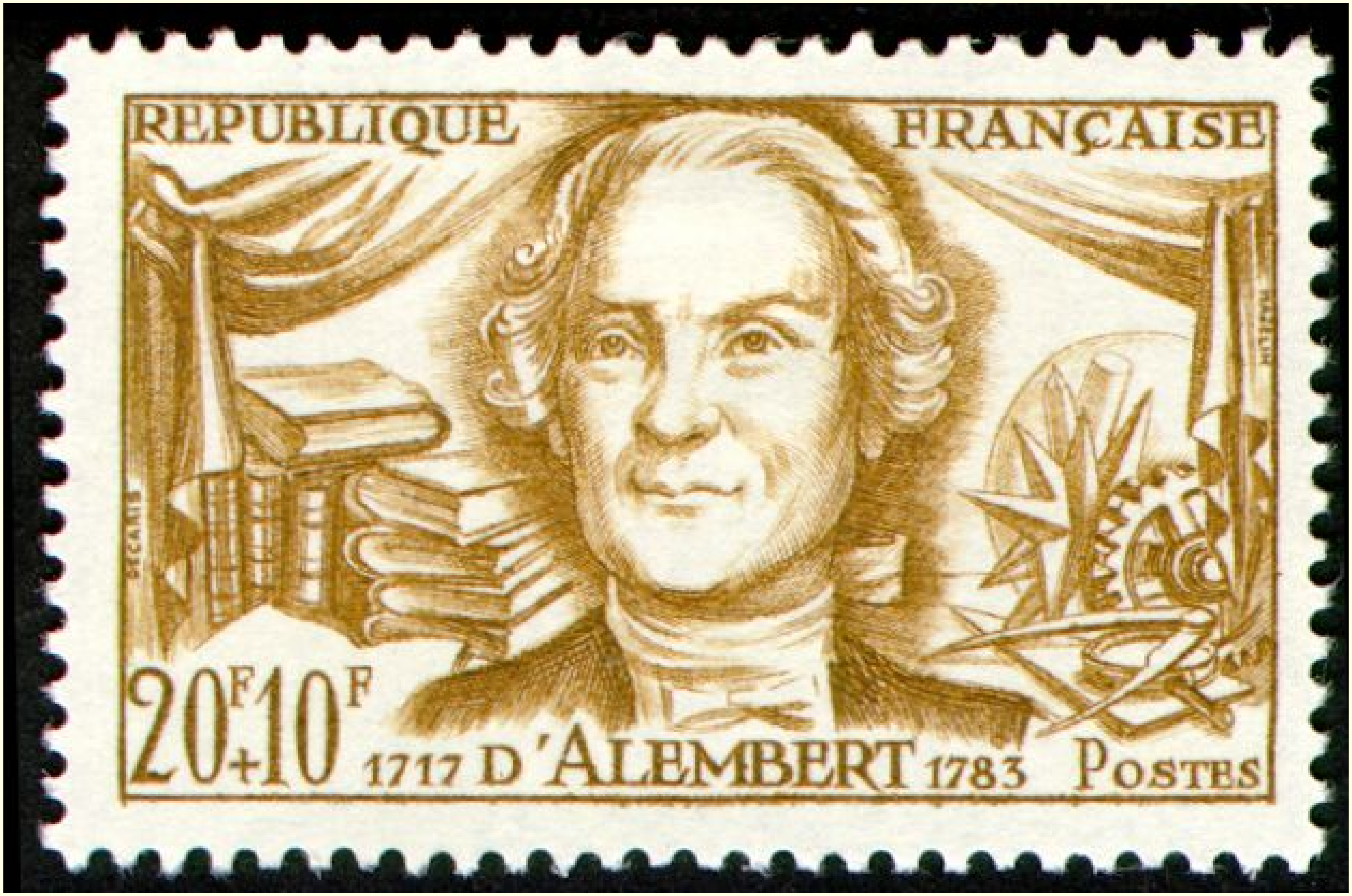
- **Born in Basel in 1707.**
- **Died 1783 in St Petersburg.**
- **Formulated the equations for incompressible, inviscid fluid flow:**

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} + \frac{1}{\rho} \nabla p = \mathbf{g} \,.$$

$$\nabla \cdot \mathbf{V} = 0$$

Partial differential equations.

Jean Le Rond d'Alembert



A body moving at constant speed through a gas or a fluid does not experience any resistance (D'Alembert 1752).

George G Stokes, 1819–1903

George Gabriel Stokes, founder of modern hydrodynamics.

ASIDE: Stokes' Theorem

$$\oint_{\Gamma} \mathbf{V} \cdot d\mathbf{l} = \iint_{\Sigma} \nabla \times \mathbf{V} \cdot \mathbf{n} \, da \, .$$

Stokes' Theorem was actually discovered by Kelvin in 1854. It is of central importance in fluid dynamics.

It leads on to Bjerknes' Circulation Theorem:

$$\frac{dC}{dt} = - \iint_{\Sigma} \nabla \frac{1}{\rho} \times \nabla p \cdot d\mathbf{a} = - \oint_{\Gamma} \frac{dp}{\rho} \, ,$$

which generalized Kelvin's Circulation Theorem to baroclinic fluids (ρ varying independently of p), and ushered in the study of Geophysical Fluid Dynamics.

Resolution of d'Alembert's Paradox

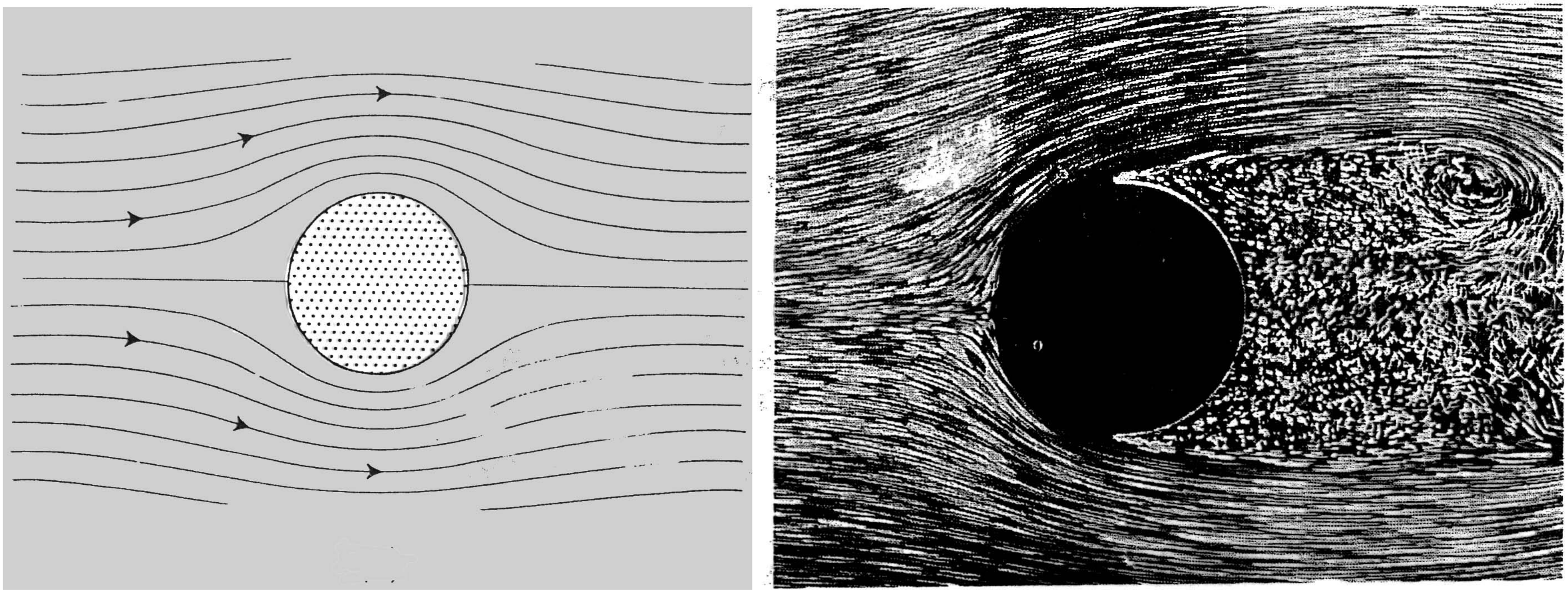


Fig. 9.1 Flow past a circular cylinder for (a) a hypothetical fluid with zero viscosity, (b) a real fluid with very small viscosity μ (from van Dyke 1982).

The minutest amount of viscosity has a profound qualitative impact on the character of the solution.

The Navier-Stokes equations incorporate the effect of viscosity.

The Navier-Stokes Equations

Euler's Equations:

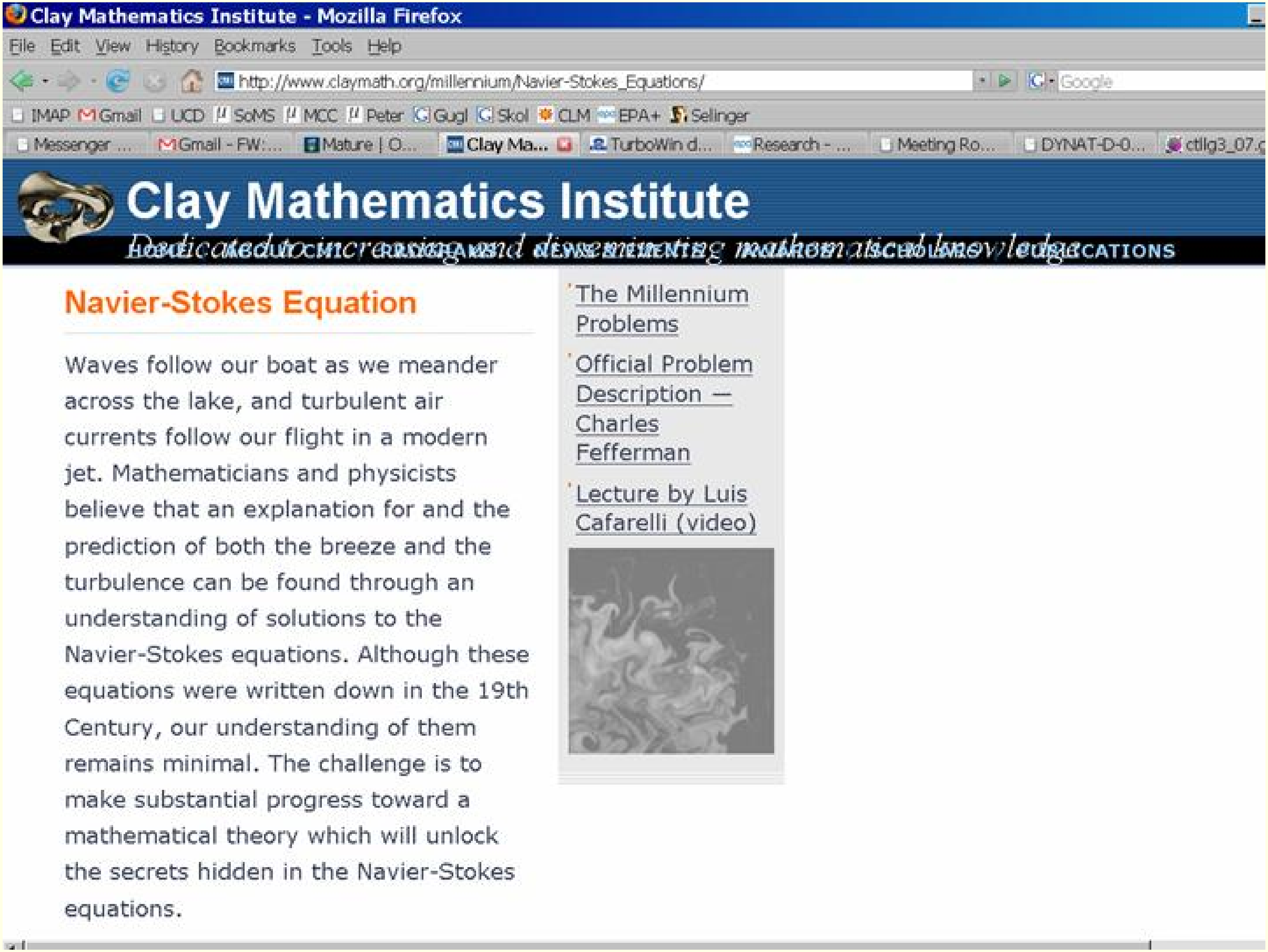
$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} + \frac{1}{\rho} \nabla p = \mathbf{g} \,.$$

The Navier-Stokes Equations

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} + \frac{1}{\rho} \nabla p = \nu \nabla^2 \mathbf{V} + \mathbf{g}^\star \,.$$

Motion on the rotating Earth:

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} + 2\boldsymbol{\Omega} \times \mathbf{V} + \frac{1}{\rho} \nabla p = \nu \nabla^2 \mathbf{V} + \mathbf{g} \,.$$



Clay Mathematics Institute - Mozilla Firefox

File Edit View History Bookmarks Tools Help

http://www.claymath.org/millennium/Navier-Stokes_Equations/

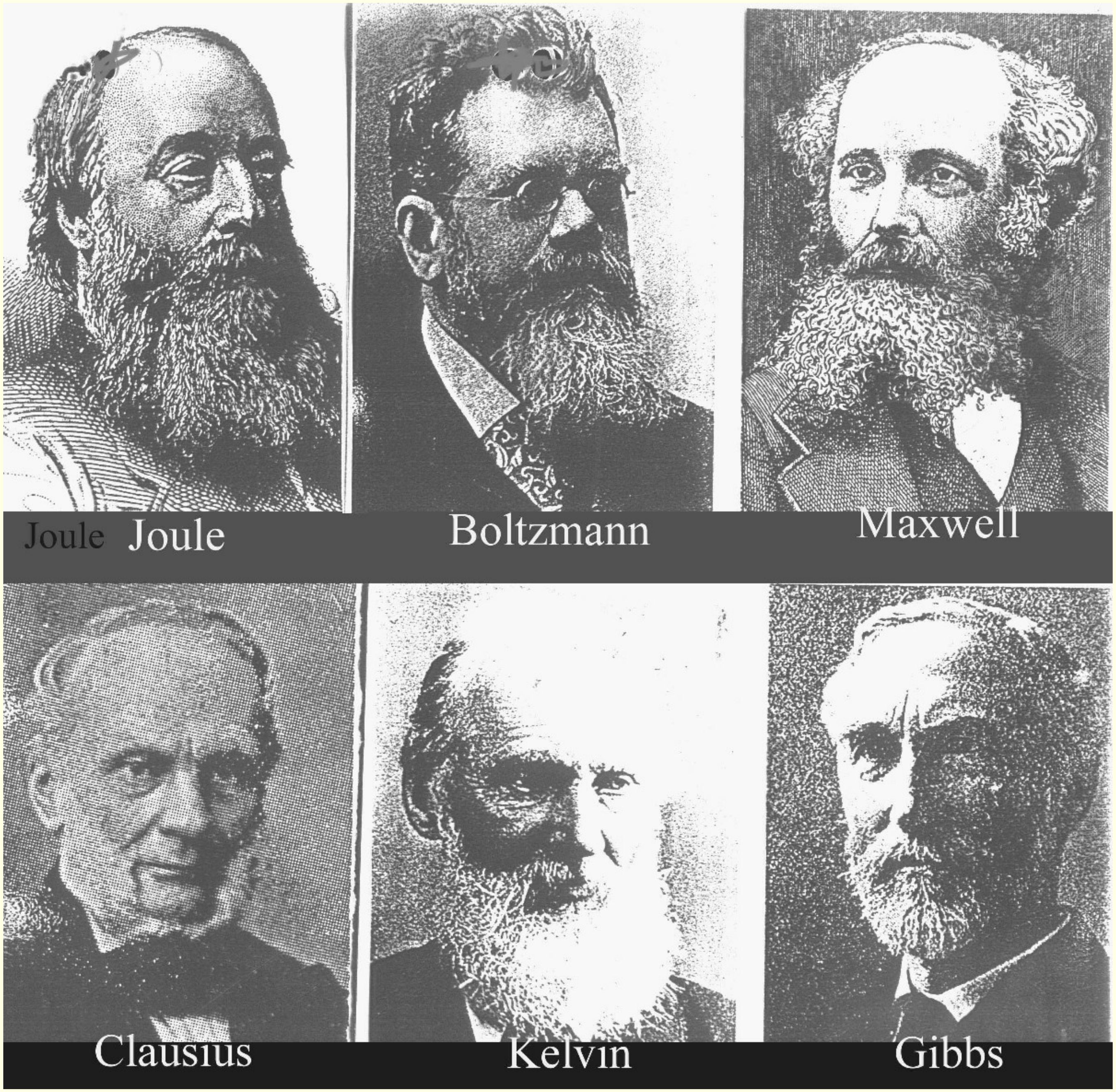
Clay Mathematics Institute

Navier-Stokes Equation

Waves follow our boat as we meander across the lake, and turbulent air currents follow our flight in a modern jet. Mathematicians and physicists believe that an explanation for and the prediction of both the breeze and the turbulence can be found through an understanding of solutions to the Navier-Stokes equations. Although these equations were written down in the 19th Century, our understanding of them remains minimal. The challenge is to make substantial progress toward a mathematical theory which will unlock the secrets hidden in the Navier-Stokes equations.

- The Millennium Problems
- Official Problem Description — Charles Fefferman
- Lecture by Luis Cafarelli (video)

The Inventors of Thermodynamics



The Primitive Equations

$$\frac{du}{dt} - \left(f + \frac{u\tan\phi}{a}\right)v + \frac{1}{\rho}\frac{\partial p}{\partial x} + F_x = 0$$

$$\frac{dv}{dt} + \left(f + \frac{u\tan\phi}{a}\right)u + \frac{1}{\rho}\frac{\partial p}{\partial y} + F_y = 0$$

$$p = R\rho T$$

$$\frac{\partial p}{\partial z} + g\rho = 0$$

$$\frac{dT}{dt} + (\gamma - 1)T\nabla\cdot\mathbf{V} = \frac{Q}{c_p}$$

$$\frac{\partial \rho}{\partial t} + \nabla\cdot\rho\mathbf{V} = 0$$

$$\frac{\partial \rho_w}{\partial t} + \nabla\cdot\rho_w\mathbf{V} = [\mathbf{Sources} - \mathbf{Sinks}]$$

Seven equations; seven variables $(u, v, w, p, T, \rho, \rho_w)$**.**

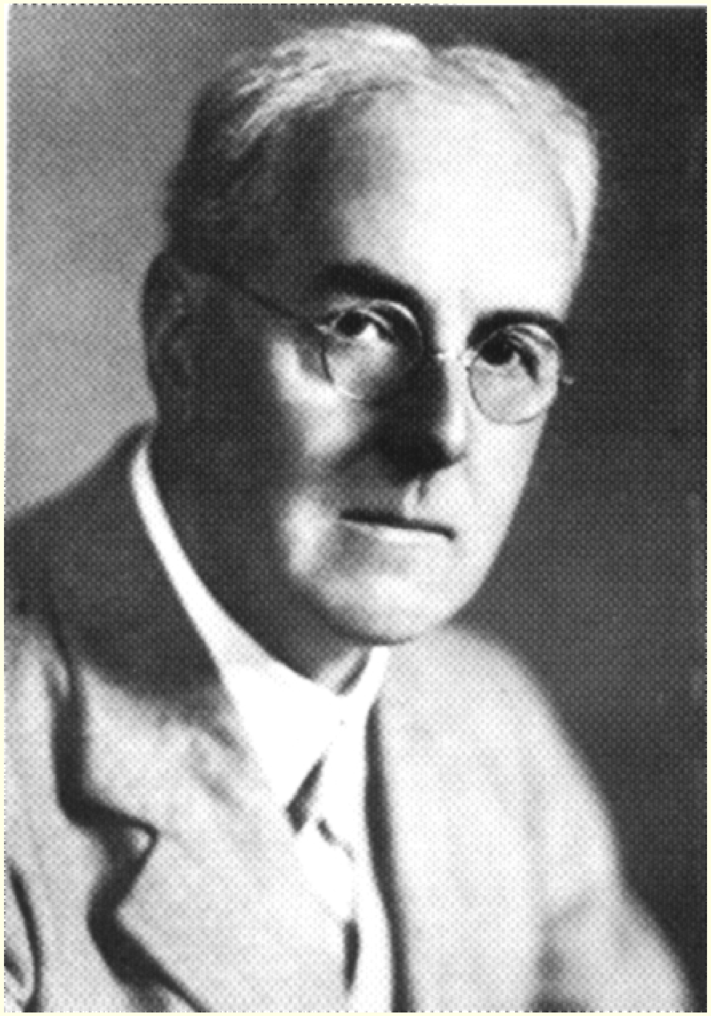
Scientific Weather Forecasting in a Nut-Shell

- **The atmosphere is a physical system**
- **Its behaviour is governed by the laws of physics**
- **These laws are expressed quantitatively in the form of mathematical equations**
- **Using observations, we can specify the atmospheric state at a given initial time: "Today's Weather"**
- **Using the equations, we can calculate how this state will change over time: "Tomorrow's Weather"**

- **The equations are very complicated (non-linear) and a powerful computer is required to do the calculations**
- **The accuracy decreases as the range increases; there is an inherent limit of predictibility.**

Richardson's Forecast

Lewis Fry Richardson, 1881–1953.



During WWI, Richardson computed by hand the pressure change at a single point.

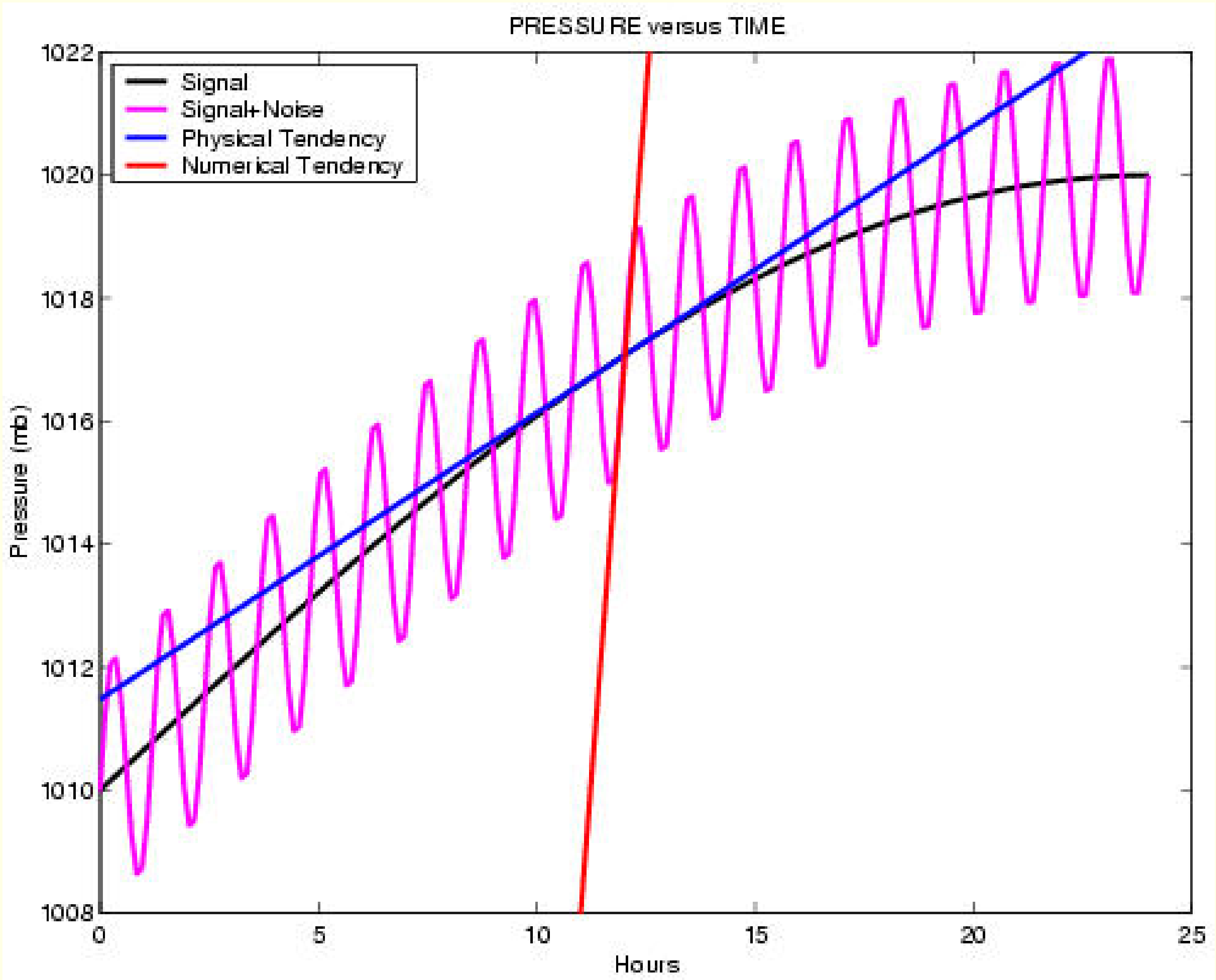
It took him two years !

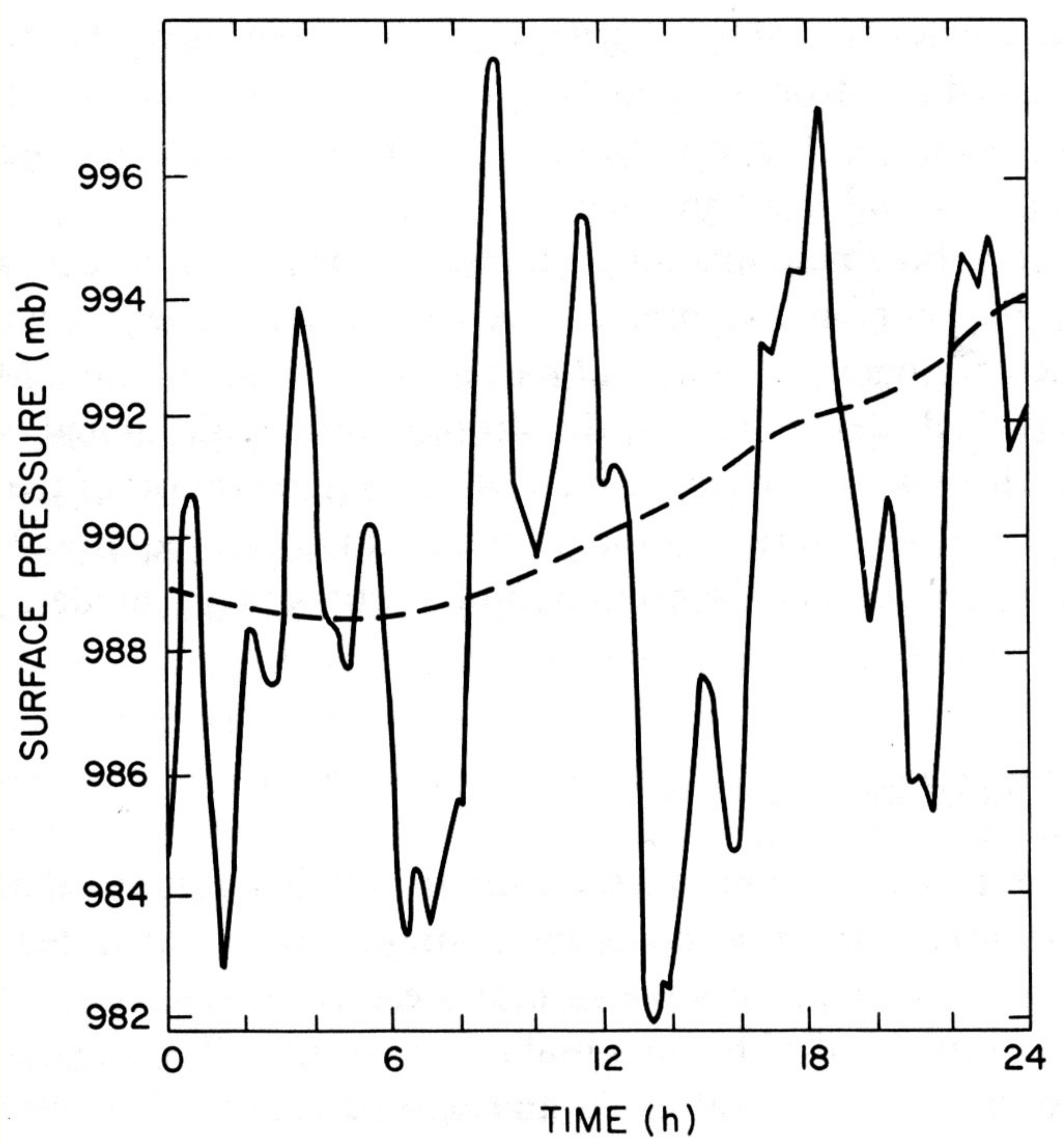
His 'forecast' was a catastrophic failure:

$$\Delta p = \mathbf{145\ hPa\ in\ 6\ hrs}$$

But Richardson's method was scientifically sound.

Tendency of a Noisy Signal





Evolution of surface pressure before and after NNMI.

(Williamson and Temperton, 1981)

Initialization of Richardson's Forecast

Richardson's Forecast has been repeated on a computer.

The atmospheric observations for 20 May, 1910, were *recovered from original sources*.

- ***ORIGINAL:*** $$\frac{dp_s}{dt} = +\mathbf{145}\,\mathrm{hPa}/6\,\mathrm{h}$$
- ***INITIALIZED:*** $$\frac{dp_s}{dt} = -\mathbf{0.9}\,\mathrm{hPa}/6\,\mathrm{h}$$

Observations: The barometer was steady!

Richardson's Forecast Factory

©François Schuiten

64,000 Computers: The first Massively Parallel Processor

The Finite Difference Scheme

Let Q be governed by an equation

$$\frac{dQ}{dt} = F(Q)\,.$$

The time interval under consideration is sliced into a finite number of discrete time steps $\{0, \Delta t, 2\Delta t, \dots\ , n\Delta t, \dots\ \}$.

The time derivitive is approximated by a finite difference:

$$\frac{dQ}{dt} \approx \frac{Q(t+\Delta t) - Q(t-\Delta t)}{2\Delta t}\,.$$

Thus, a problem in analysis becomes a problem in algebra.

Reversing History

Differential calculus depends upon justifying the limiting process $\Delta t \to 0$.

In approximating a differential equation, we reverse the procedure, and replace derivatives by ratios of increments.

We thus " . . . return to the manner in which they did things before the calculus was invented . . ." (Richardson)

Stepping Forward

The time derivative in

$$\frac{dQ}{dt} = F(Q)\,.$$

is now approximated by a centered difference

$$\frac{Q^{n+1} - Q^{n-1}}{2\Delta t} = F^n\,,$$

Then

$$Q^{n+1} = Q^{n-1} + 2\Delta t\, F^n\,.$$

This process of stepping forward is repeated a large number of times, until the desired forecast range is reached.

We can discretize space in a similar way, but . . .

The Spectral Method

The ECMWF Integrated Forecast System (IFS) uses a spectral representation of the meteorological fields.

Each field is expanded in spherical harmonics, truncated at a fixed total wavenumber N:

$$Q(\lambda_i, \phi_j, t) = \sum_{n=0}^{N} \sum_{m=-n}^{n} Q_n^m(t) Y_n^m(\lambda_i, \phi_j)$$

The functions $Y_n^m(\lambda, \phi)$ are eigensolutions of the Laplacian:

$$\nabla^2 Y_n^m = -n(n+1) Y_n^m \,.$$

The coefficients $Q_n^m(t)$ depend only on time.

When the model equations are transformed to spectral space, they become a set of ordinary differential equations for the spectral coefficients Q_n^m.

ENIAC Forecast

The Meteorology Project

Project estblished by John von Neumann in 1946.

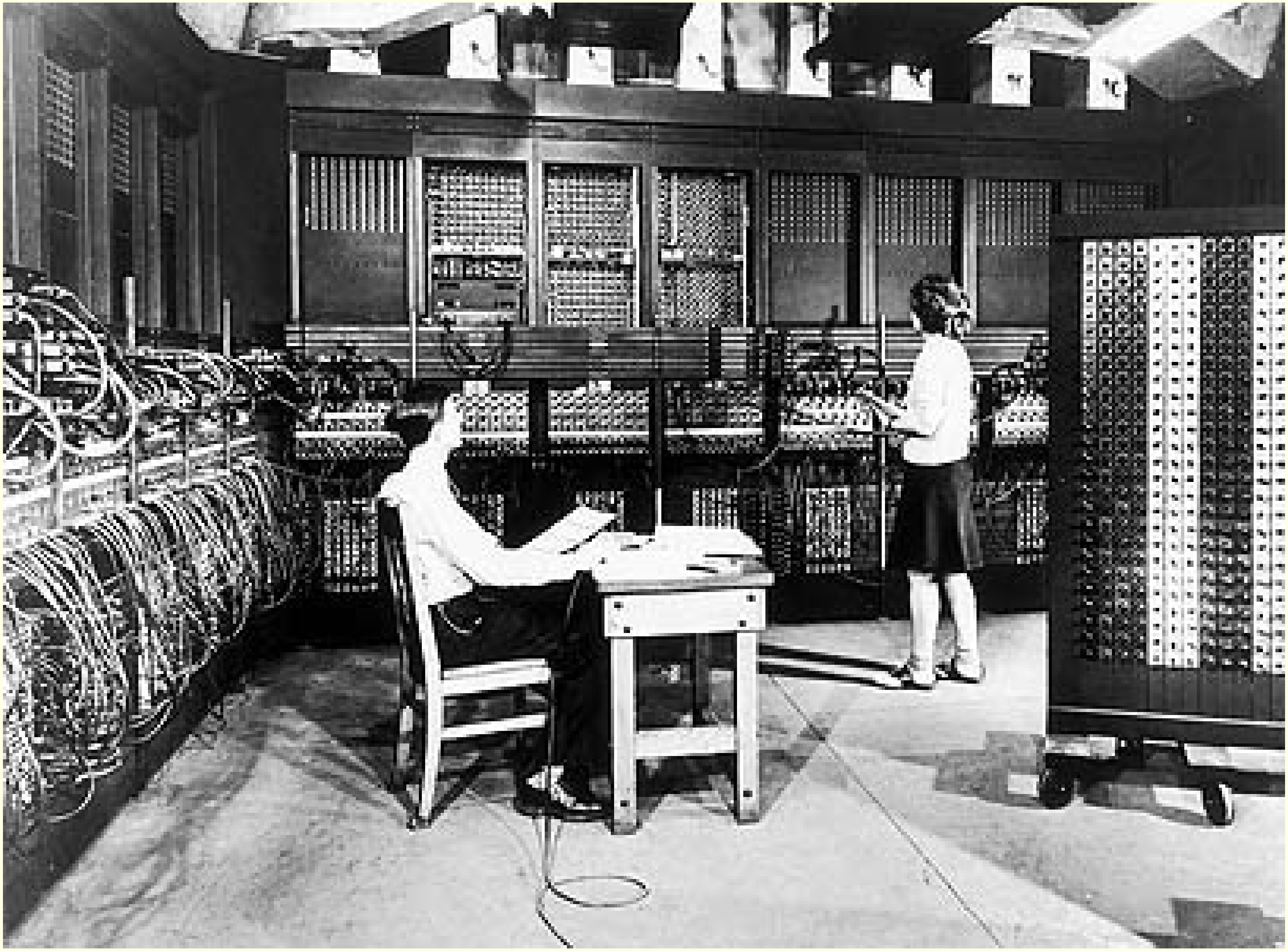
Objective of the project:

To study the problem of predicting the weather using a digital electronic computer.

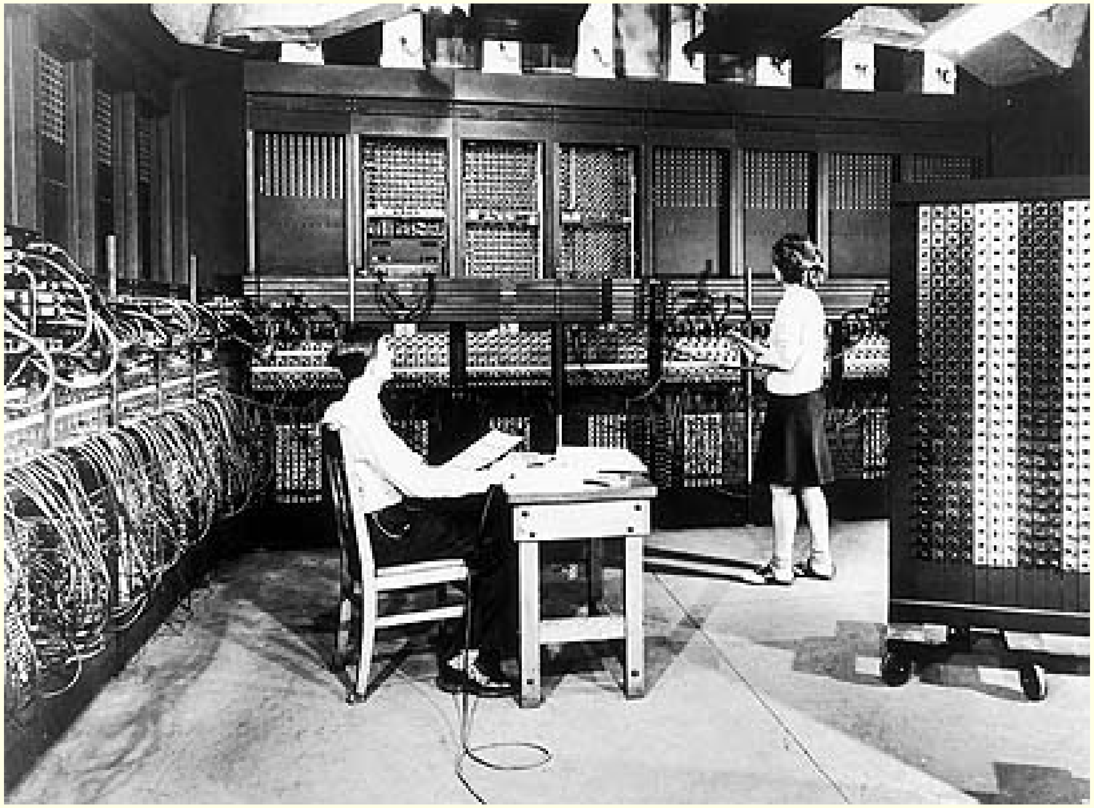
A Proposal for Funding listed three "possibilities":

- ***New methods of weather prediction***
- ***Rational basis for planning observations***
- ***Step towards influencing the weather!***

The ENIAC



The ENIAC



The **ENIAC** was the first multi-purpose programmable electronic digital computer.
It had:

- 18,000 vacuum tubes
- 70,000 resistors
- 10,000 capacitors
- 6,000 switches
- Power: 140 kWatts

Charney, et al., *Tellus*, 1950.

$$\begin{bmatrix}\textbf{Absolute}\\\textbf{Vorticity}\end{bmatrix} = \begin{bmatrix}\textbf{Relative}\\\textbf{Vorticity}\end{bmatrix} + \begin{bmatrix}\textbf{Planetary}\\\textbf{Vorticity}\end{bmatrix} \qquad \eta = \zeta + f\,.$$

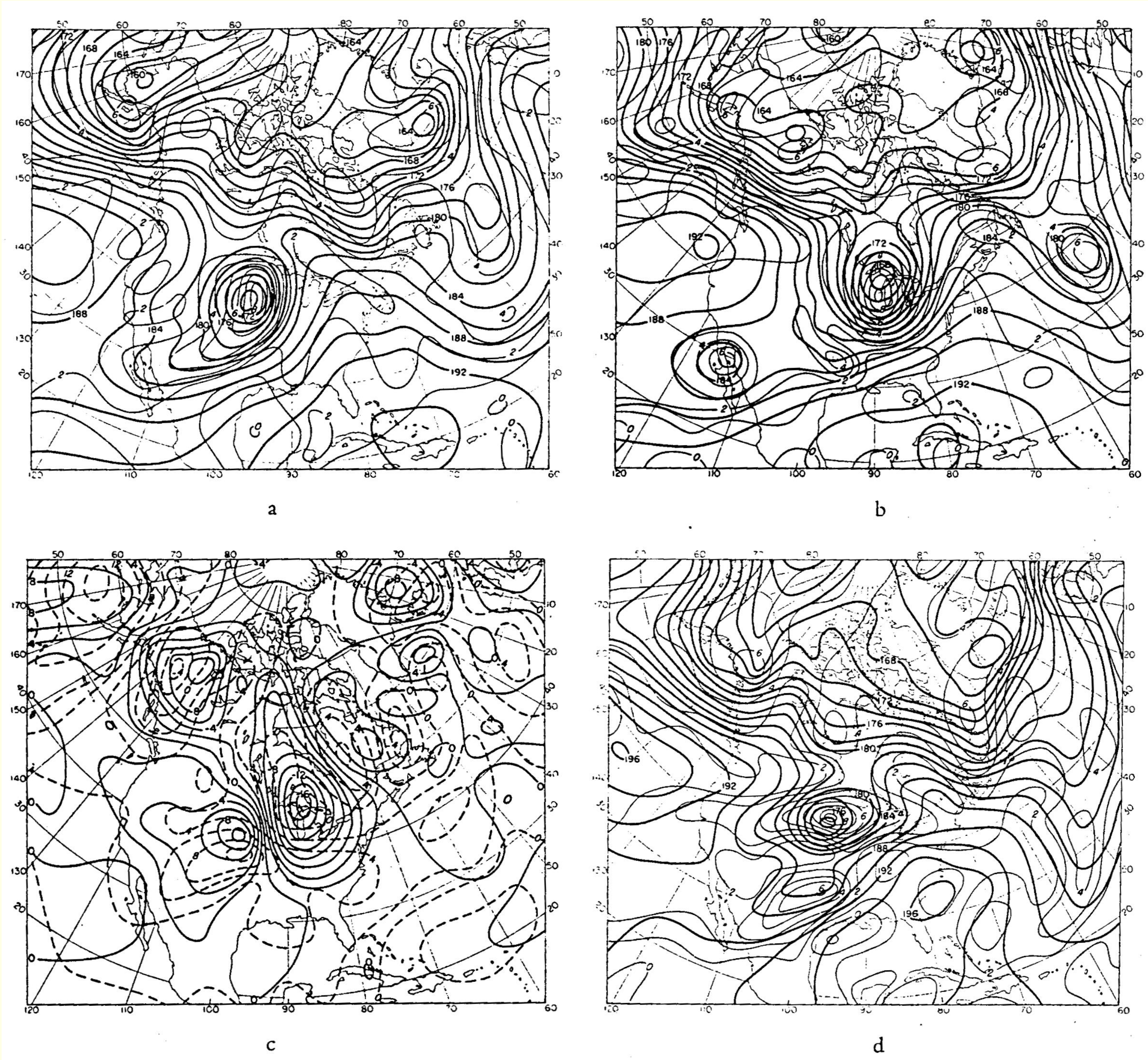
- **The atmosphere is treated as a single layer.**
- **The flow is assumed to be nondivergent.**
- **Absolute vorticity is conserved.**

$$\frac{d(\zeta + f)}{dt} = 0.$$

This equation looks deceptively simple. But it is nonlinear:

$$\frac{\partial}{\partial t}[\nabla^2\psi] + \left\{\frac{\partial\psi}{\partial x}\frac{\partial\nabla^2\psi}{\partial y} - \frac{\partial\psi}{\partial y}\frac{\partial\nabla^2\psi}{\partial x}\right\} + \beta\frac{\partial\psi}{\partial x} = 0\,,$$

ENIAC Forecast for Jan 5, 1949



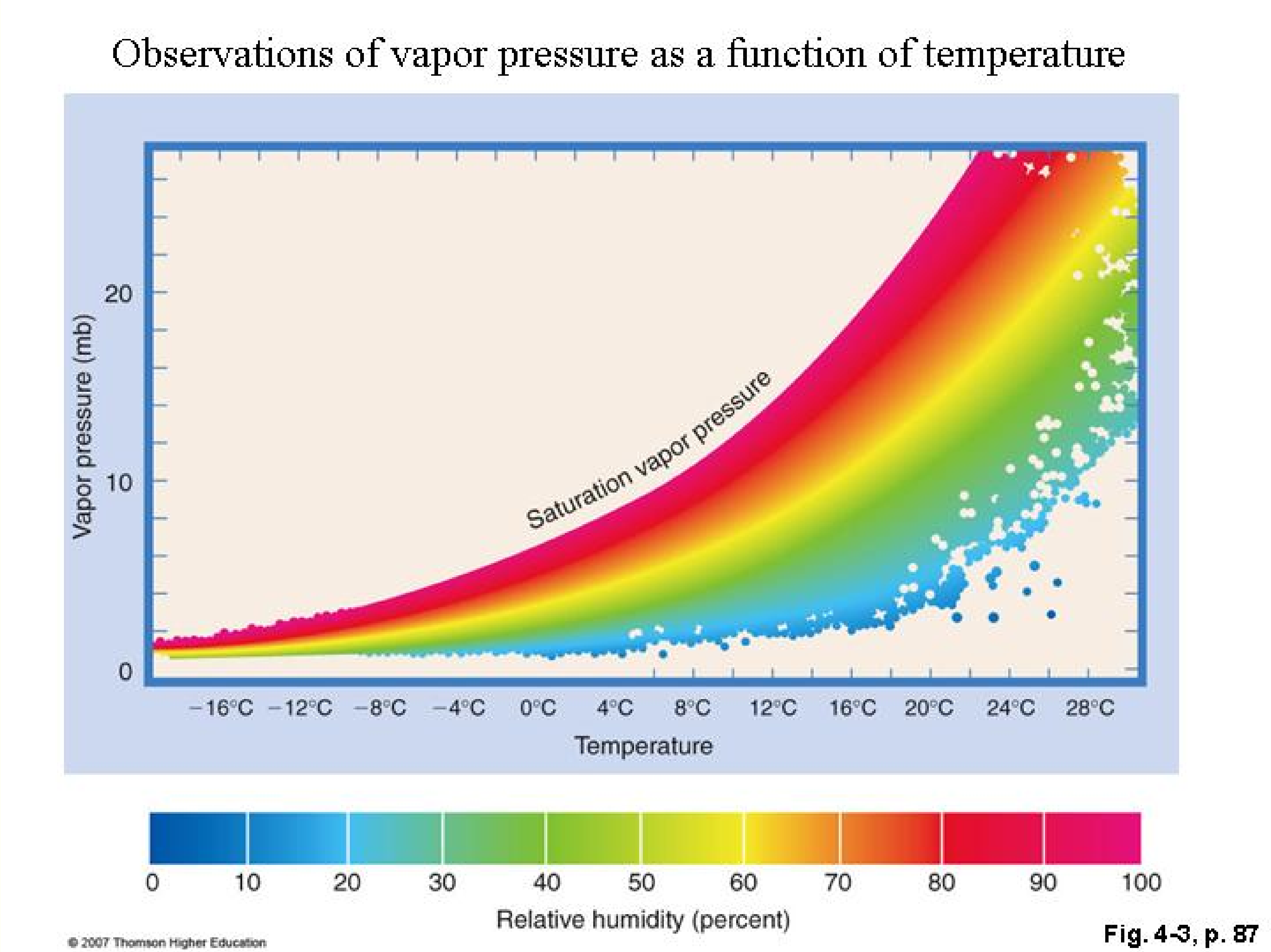
NWP Operations

The Joint Numerical Weather Prediction Unit was established on July 1, 1954:

- ***Air Weather Service of US Air Force***
- ***The US Weather Bureau***
- ***The Naval Weather Service.***

Operational numerical weather forecasting began in May, 1955, using a three-level quasi-geostrophic model.

Interlude



Temperature, Humidity and Climate Change

Data Assimilation

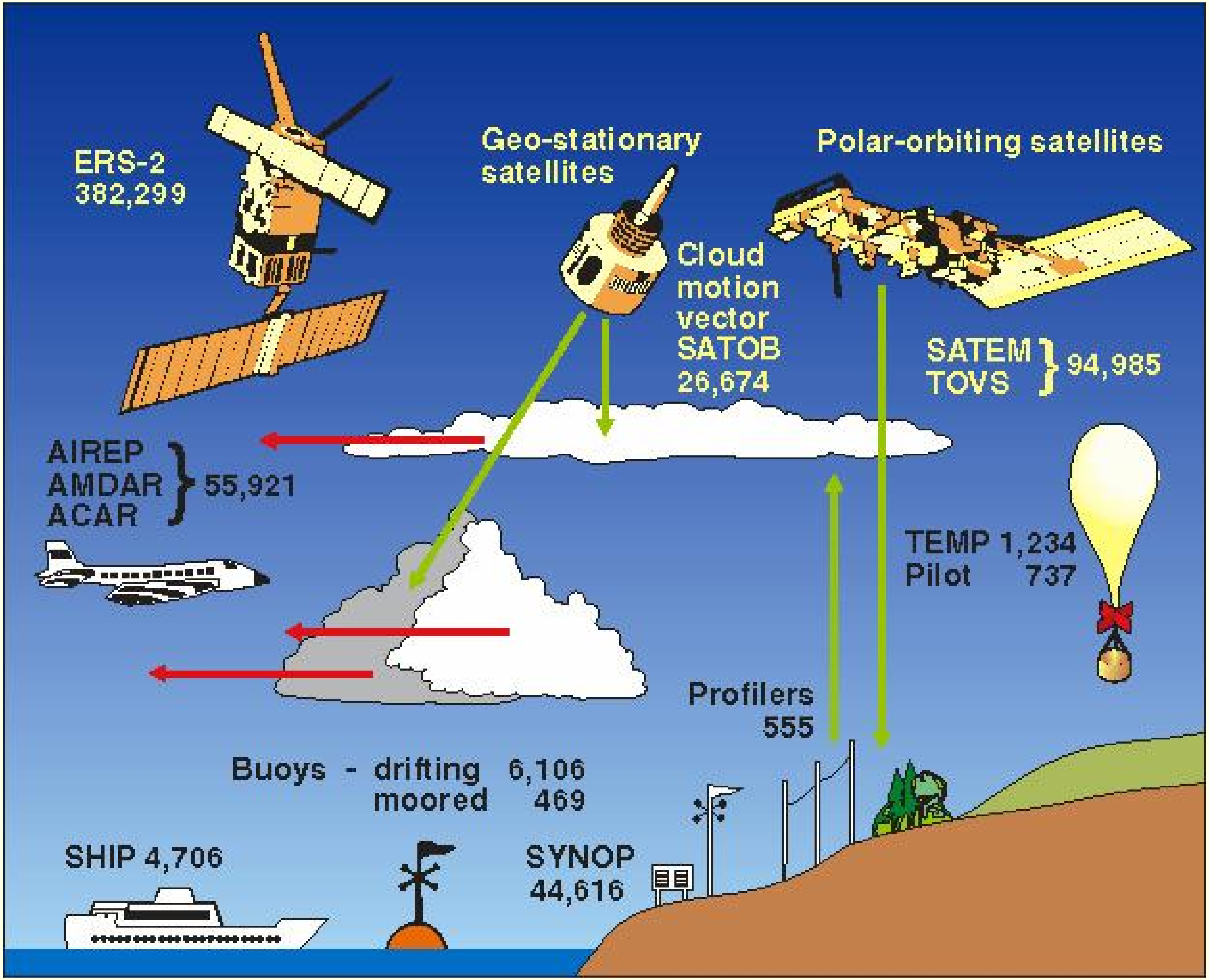
Data Assimilation

NWP: An initial/boundary value problem

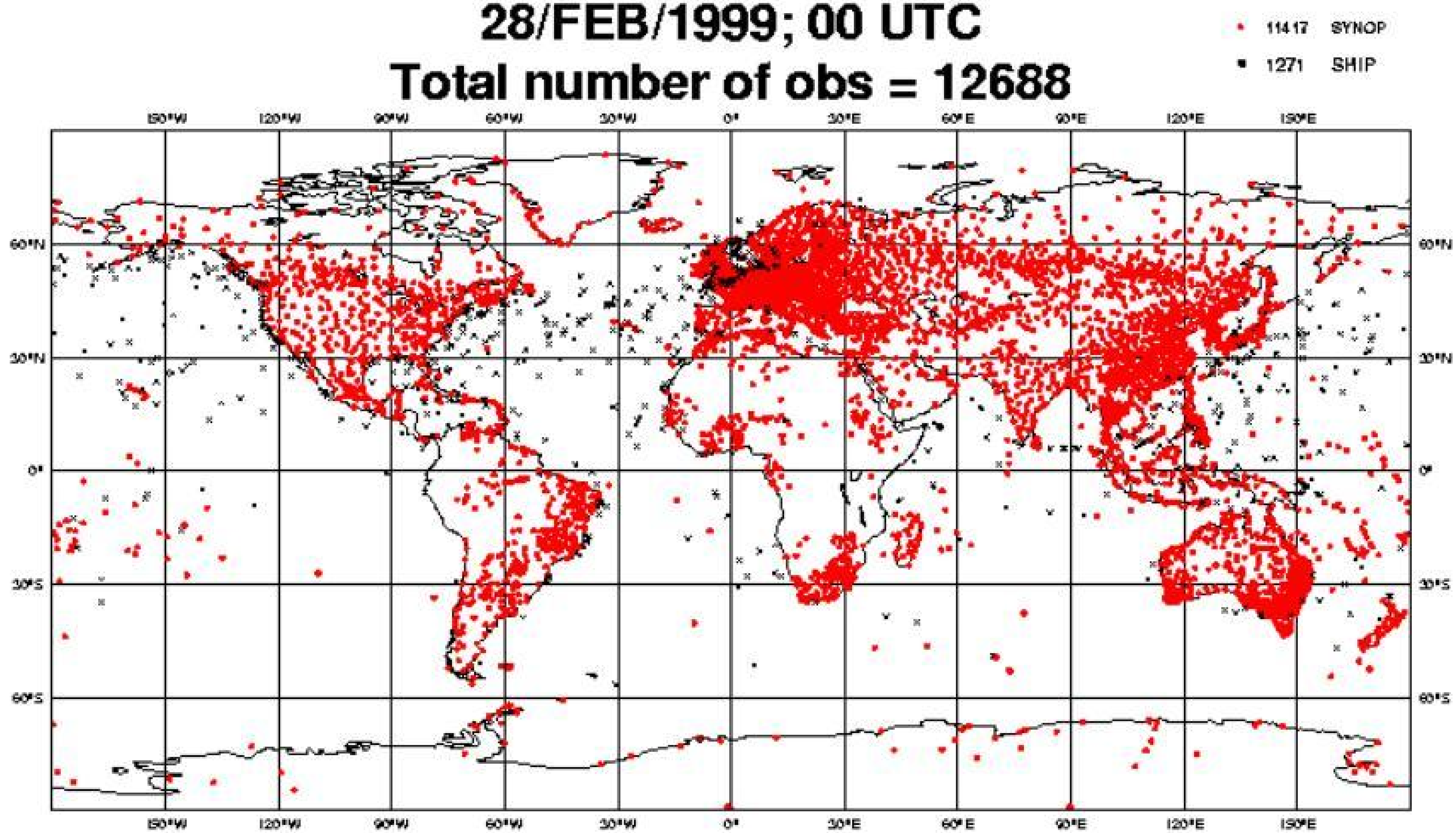
- Given
  - an estimate of the present state of the atmosphere (initial conditions)
  - appropriate surface and lateral boundary conditions

  the model forecasts the evolution of the atmosphere.

- Operational NWP centers produce initial conditions from a statistical combination of observations and short-range forecasts. This is called data assimilation.


ERS-2
382,299
Geo-stationary
satellites
Polar-orbiting satellites
Cloud
motion
vector
SATOB
26,674
SATEM
TOVS
94,985
AIREP
AMDAR
ACAR
55,921
TEMP 1,234
Pilot 737
Profilers
555
Buoys - drifting 6,106
moored 469
SHIP 4,706
SYNOP
44,616

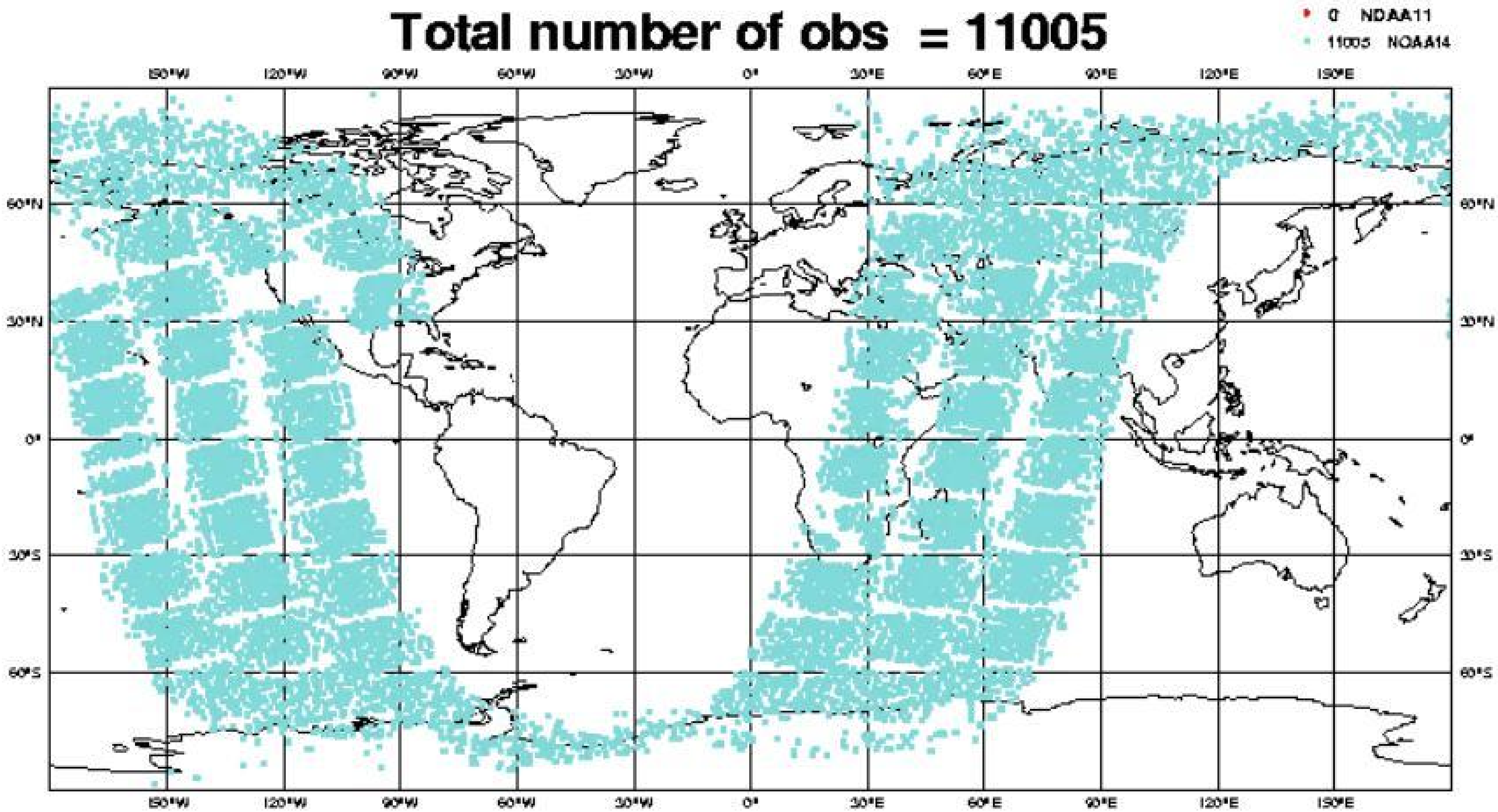
ECMWF Data Coverage - SYNOP/SHIP
28/FEB/1999; 00 UTC
Total number of obs = 12688



ECMWF Data Coverage - TOVS (120km)

28/FEB/1999; 00 UTC

Total number of obs = 11005



Optimal Interpolation

The analysis problem is to find an optimum atmospheric state, $\mathbf{x}_a$, given

- **A background field $\mathbf{x}_b$ (on a regular grid)**
- **A set of (irregularly spaced) p observations $\mathbf{y}_o$**

The analysis is cast as background *plus* increment:

$$\mathbf{x}_a = \mathbf{x}_b + \mathbf{W}[\mathbf{y}_o - H(\mathbf{x}_b)]$$

The analysis and the background are vectors of length n.

The weights are given by a matrix $\mathbf{W}$ of size $(n \times p)$.

The Full Set of OI Equations

The result of the (least squares) optimization is:

$$\mathbf{x}_a = \mathbf{x}_b + \mathbf{W}[\mathbf{y}_o - H(\mathbf{x}_b)]$$

$$\mathbf{W} = \mathbf{B}\mathbf{H}^T(\mathbf{R} + \mathbf{H}\mathbf{B}\mathbf{H}^T)^{-1}$$

$$\mathbf{P}_a = (\mathbf{I} - \mathbf{W}\mathbf{H})\mathbf{B}$$

All the covariance matrices are modelled using simplifying assumptions.

Solution is a formidable computational task:
The matrices are huge. Many shortcuts are needed.

Variational Assimilation

Another approach to objective analysis is the variational assimilation technique.

Problem:

Find the analysis x that minimizes a *cost function*:

$$J(\mathbf{x}) = \frac{1}{2}\left\{(\mathbf{x}-\mathbf{x}_b)^T\mathbf{B}^{-1}(\mathbf{x}-\mathbf{x}_b) + [\mathbf{y}_o - H(\mathbf{x})]^T\mathbf{R}^{-1}[\mathbf{y}_o - H(\mathbf{x})]\right\}$$

the distance between x and the background $\mathbf{x}_b$,
***plus* the distance to the observations $\mathbf{y}_{o,}$:**

Variational assimilation has been shown to yield significant improvements in forecast accuracy.

The gradient of J with respect to $\mathbf{x}$ is

$$\nabla J(\mathbf{x}) = [\mathbf{B}^{-1} + \mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}](\mathbf{x} - \mathbf{x}_b) - \mathbf{H}^T\mathbf{R}^{-1}\{\mathbf{y}_o - H(\mathbf{x}_b)\}$$

To find a minimum of J, we set

$$\nabla J(\mathbf{x}) = 0\,.$$

The result is:

$$\mathbf{x} = \mathbf{x}_b + \left[\mathbf{B}^{-1} + \mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\right]^{-1}\mathbf{H}^T\mathbf{R}^{-1}\{\mathbf{y}_o - H(\mathbf{x}_b)\}$$

This is the (formal) solution of the 3-dimensional variational (3D-Var) analysis problem.

The matrices are huge: perhaps $10^7 \times 10^7$.

Minimization

In practical 3D-Var, we do not invert a huge matrix.

We find the minimum of $J(\mathbf{x})$ by computing the cost function and using an optimization technique.

The idea is to "proceed downhill" as fast as possible:

- **Steepest Descent algorithm,**
- **Newton's method,**
- **Conjugate Gradient algorithm.**

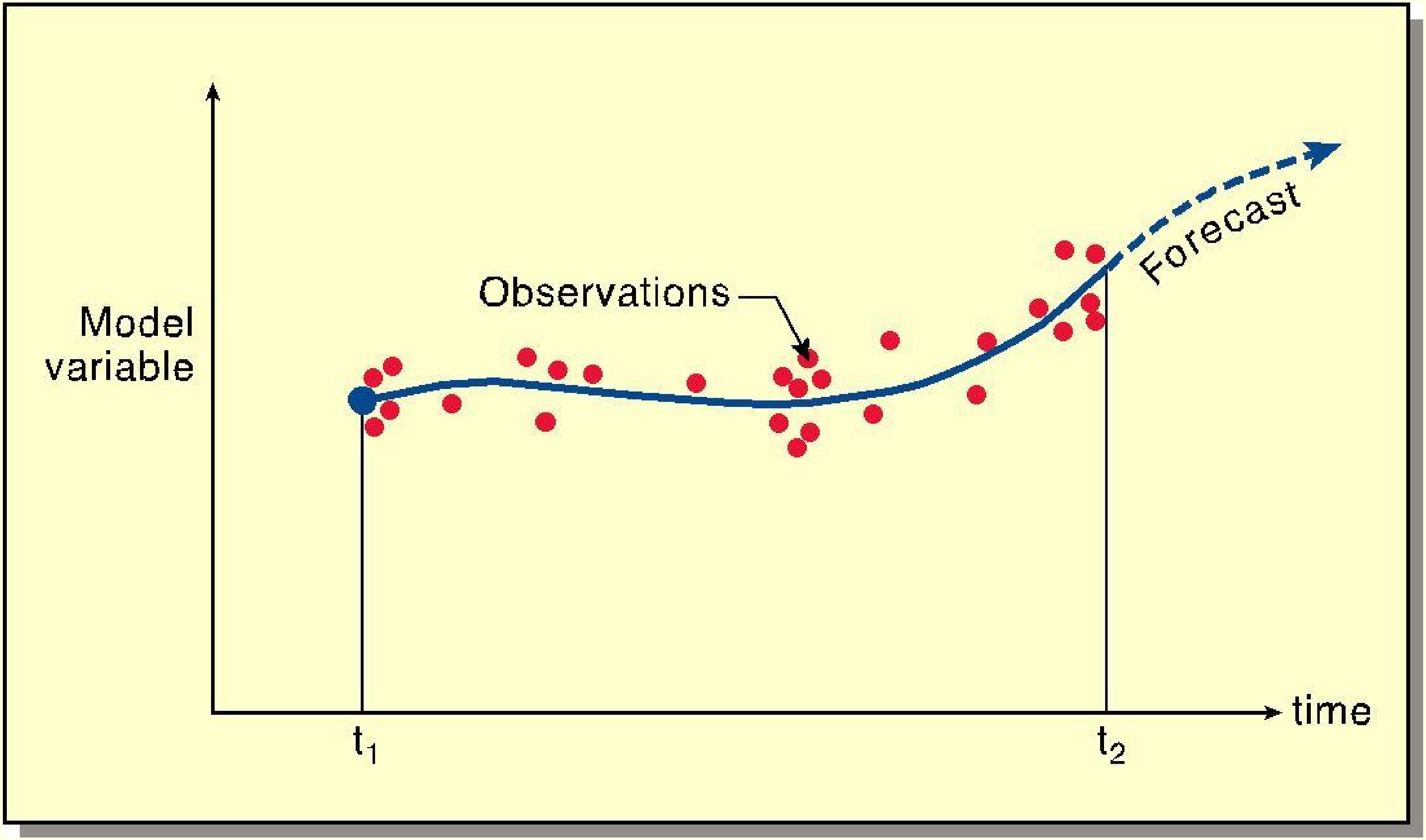
4D-Variational Assimilation

Four-dimensional variational assimilation (4D-Var) is an extension of 3D-Var to allow for observations distributed within a time interval (t_0, t_n).

The cost function includes terms for the distance to observations at the time of the observation.

$$J[\mathbf{x}(t_0)] = \frac{1}{2}[\mathbf{x}(\mathrm{t}_0) - \mathbf{x}^b(t_0)]^T \mathbf{B}_0^{-1} [\mathbf{x}(\mathrm{t}_0) - \mathbf{x}^b(t_0)]$$
$$+\frac{1}{2}\sum_{i=0}^{N} \left[H(\mathbf{x}_i) - \mathbf{y}_i^o\right]^T \mathbf{R}_i^{-1} \left[H(\mathbf{x}_i) - \mathbf{y}_i^o\right]$$

The control variable is the *initial state* $\mathbf{x}(t_0)$.



Schematic diagram of four dimensional variational assimilation.

Tangent Linear Model

The solution at time t_{i+1} is computed from the solution at time t_i by a (nonlinear) model:

$$\mathbf{x}_{i+1} = M_i[\mathbf{x}_i]\,.$$

If we perturb the initial conditions, the solution is

$$\mathbf{x}_{i+1} + \delta\mathbf{x}_{i+1} = M_i\left[\mathbf{x}_i + \delta\mathbf{x}_i\right]$$

The linear tangent model is the (Jacobian) matrix:

$$[\mathbf{L}_i]_{j,k} = \frac{\partial[M(\mathbf{x}_i)]_j}{\partial(x_i)_k}$$

Then, to first order,

$$\delta\mathbf{x}_{i+1} = \mathbf{L}_i\,\delta\mathbf{x}_i\,.$$

The Adjoint Model

The transpose of the linear tangent model is called the adjoint model.

The Gradient of the cost function is:

$$\frac{\partial J}{\partial \mathbf{x}_0} = -\sum_{i=0}^{N} \left[\mathbf{L}_0^T \mathbf{L}_1^T \cdots\ \mathbf{L}_{i-1}^T\right] \mathbf{H}_i^T \mathbf{R}_i^{-1} \mathbf{d}_i$$

Every iteration of the 4D-Var minimization requires the computation of the gradient:

- **Compute the observation increments $\mathbf{d}_i$ during a forward integration**
- **Multiply them by $\mathbf{H}_i^T \mathbf{R}_i^{-1}$**
- **Integrate these weighted increments backward to the initial time using the adjoint model.**

Atmospheric Normal Modes

Oscillations of the Atmosphere

We treating the atmosphere as a thin single layer:

$$\begin{aligned}
\frac{du}{dt} - fv - \frac{uv\tan\phi}{a} + g\frac{\partial h}{\partial x} &= 0 \\
\frac{dv}{dt} + fu + \frac{u^2\tan\phi}{a} + g\frac{\partial h}{\partial y} &= 0 \\
\frac{dh}{dt} + h\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} - \frac{v\tan\phi}{a}\right) &= 0
\end{aligned}$$

These are the shallow water quations on the sphere.

We linearize about a motionless state with depth H:

$$h(\lambda, \phi, t) = Y(\phi)\exp[i(m\lambda - \sigma t)]$$

After some algebra we get an equation for $Y(\phi)$:

$$\frac{d}{d\mu}\left[\left(\frac{1-\mu^2}{\sigma^2-\mu^2}\right)\frac{dY}{d\mu}\right] + \left\{\frac{1}{\sigma^2-\mu^2}\left[\frac{m}{\sigma}\frac{\sigma^2+\mu^2}{\sigma^2-\mu^2} - \frac{m^2}{1-\mu^2}\right] + \epsilon\right\}Y = 0\,.$$

where $\mu = \sin\phi$ and $\epsilon = (2\Omega a)^2/gh$.

Laplace Tidal Equation

Again, the meridional structure is given by

$$\frac{d}{d\mu}\left[\left(\frac{1-\mu^2}{\sigma^2-\mu^2}\right)\frac{dY}{d\mu}\right]+\left\{\frac{1}{\sigma^2-\mu^2}\left[\frac{m}{\sigma}\frac{\sigma^2+\mu^2}{\sigma^2-\mu^2}-\frac{m^2}{1-\mu^2}\right]+\epsilon\right\}Y=0\,.$$

where $\mu = \sin\phi$ **and** $\epsilon = (2\Omega a)^2/gh$**.**

The normal modes are determined by the eigensolutions of this second order o.d.e., the Laplace Tidal Equation.

Boundary conditions require Y **to be regular at the poles.**

The Laplace Tidal Equation is not in Sturm-Lioville form.

Mathematical Difficulties

The standard form of the Sturm-Liouville equation is

$$\frac{d}{d\mu}\left(p(\mu)\frac{dY}{d\mu}\right) + [q(\mu) + \lambda r(\mu)]Y = 0$$

where $p(\mu)$ is regular and has no zeros within the domain.

For the Sturm-Liouville Equation:

1. The equation is self-adjoint and the eigenvalues λ are real.
2. The eigenfunctions for different λ are orthogonal.
3. The eigenfunctions form a complete set.
4. There is a denumerable infinity of non-negative eigenvalues with a single limit point at $+\infty$.
5. The zeros of the eigenfunctions behave according to the Sturmian oscillation theorems.

For the LTE, $p(\mu) = (1 - \mu^2)/(\sigma^2 - \mu^2)$ blows up at the 'critical latutudes' where $\mu = \pm\sigma$, and the equation is singular.

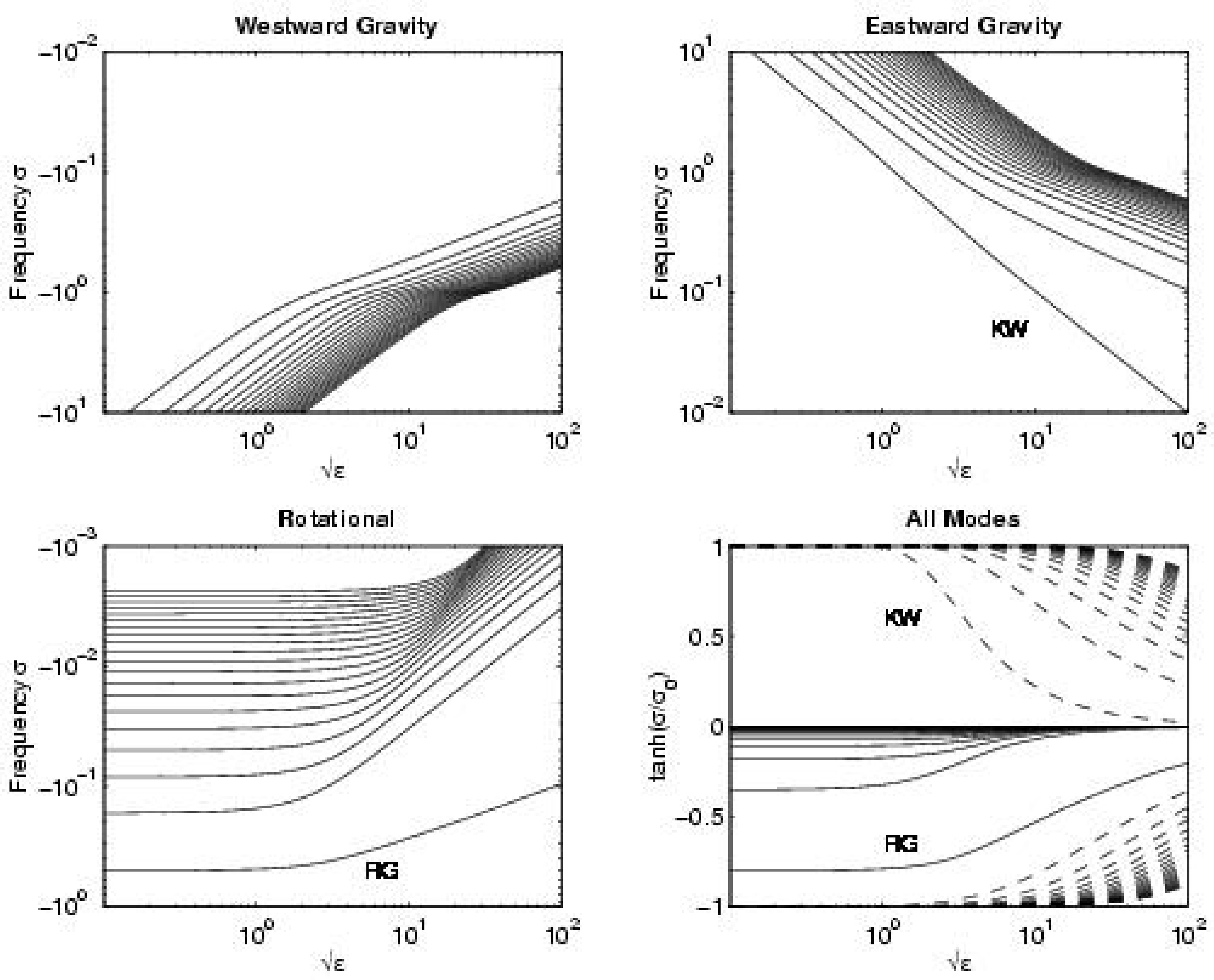
Since the LTE cannot be written in standard Sturm-Liouville form, the five properties may not hold.

It has been shown that the eigenvalues ϵ_n of the LTE are real and the eigenfunctions form a complete, orthogonal set.

Fourth and fifth properties do not hold.

For $|\sigma| < 1$ there is a double infinity of eigenvalues, with limit points at both $+\infty$ and $-\infty$.

The zeros of the eigenfunctions do not behave in a simple manner like for a regular Sturm-Liouville problem.



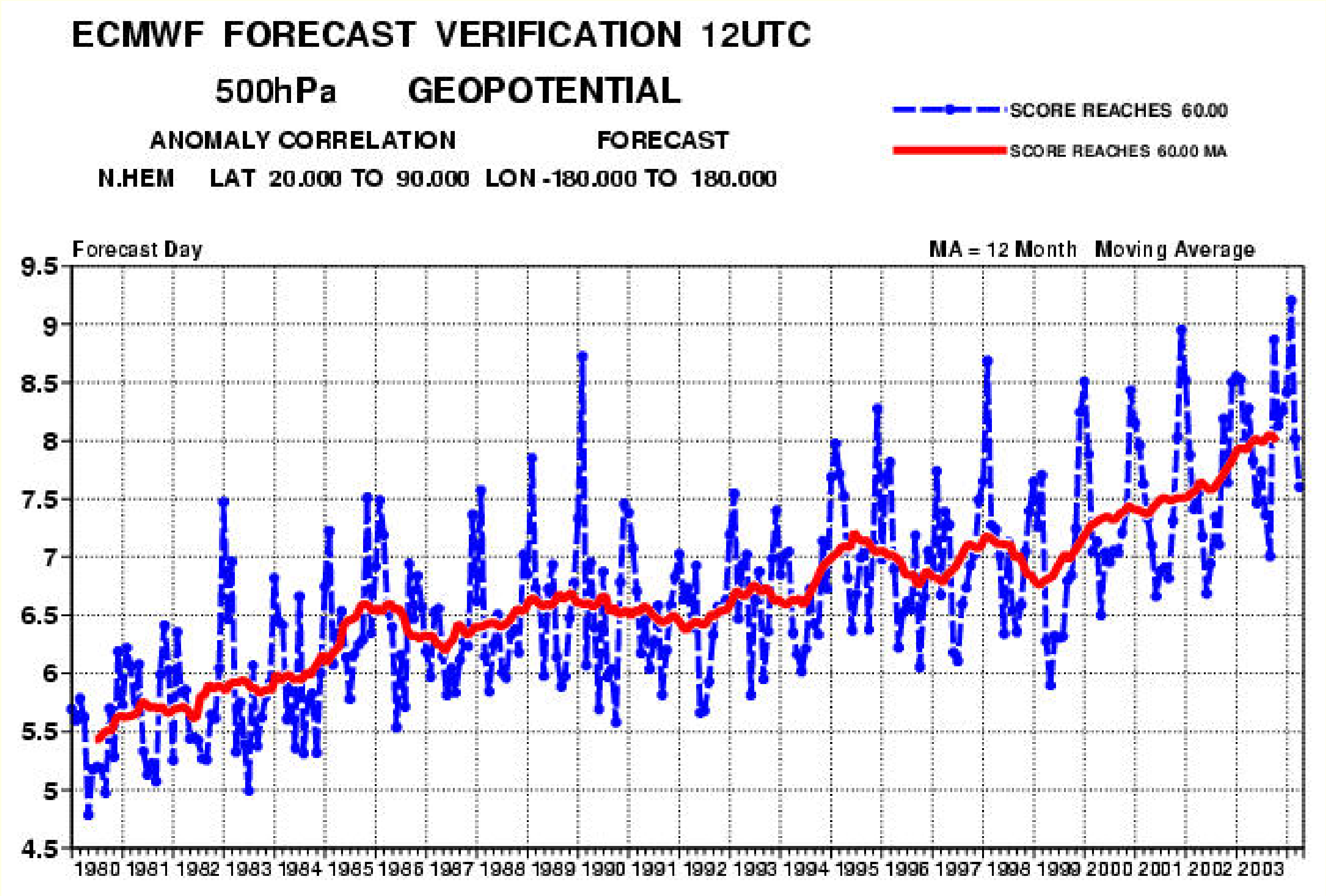
Eigenfrequencies σ of the LTE.

Atmospheric Predictability

and

Ensemble Forecasting

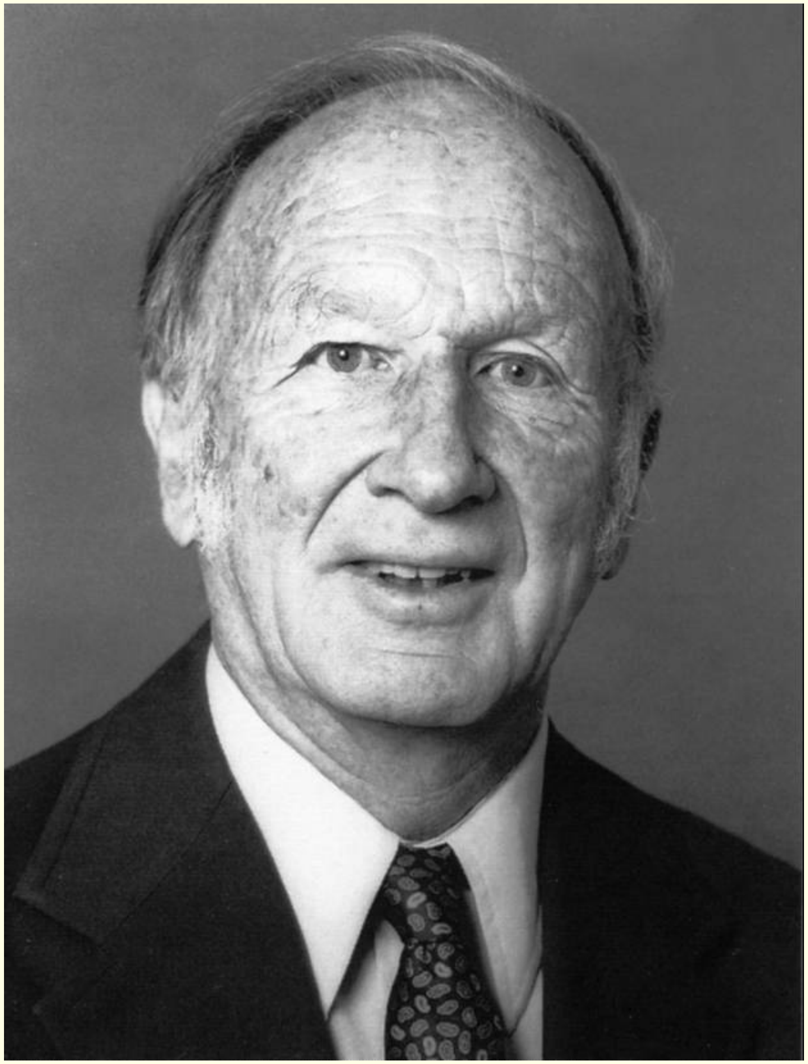
Progress in numerical weather prediction over the past fifty years has been quite dramatic.



Forecast skill continues to increase . . .
by one day per decade.

However, there is a limit . . .

Chaos in Atmospheric Flow



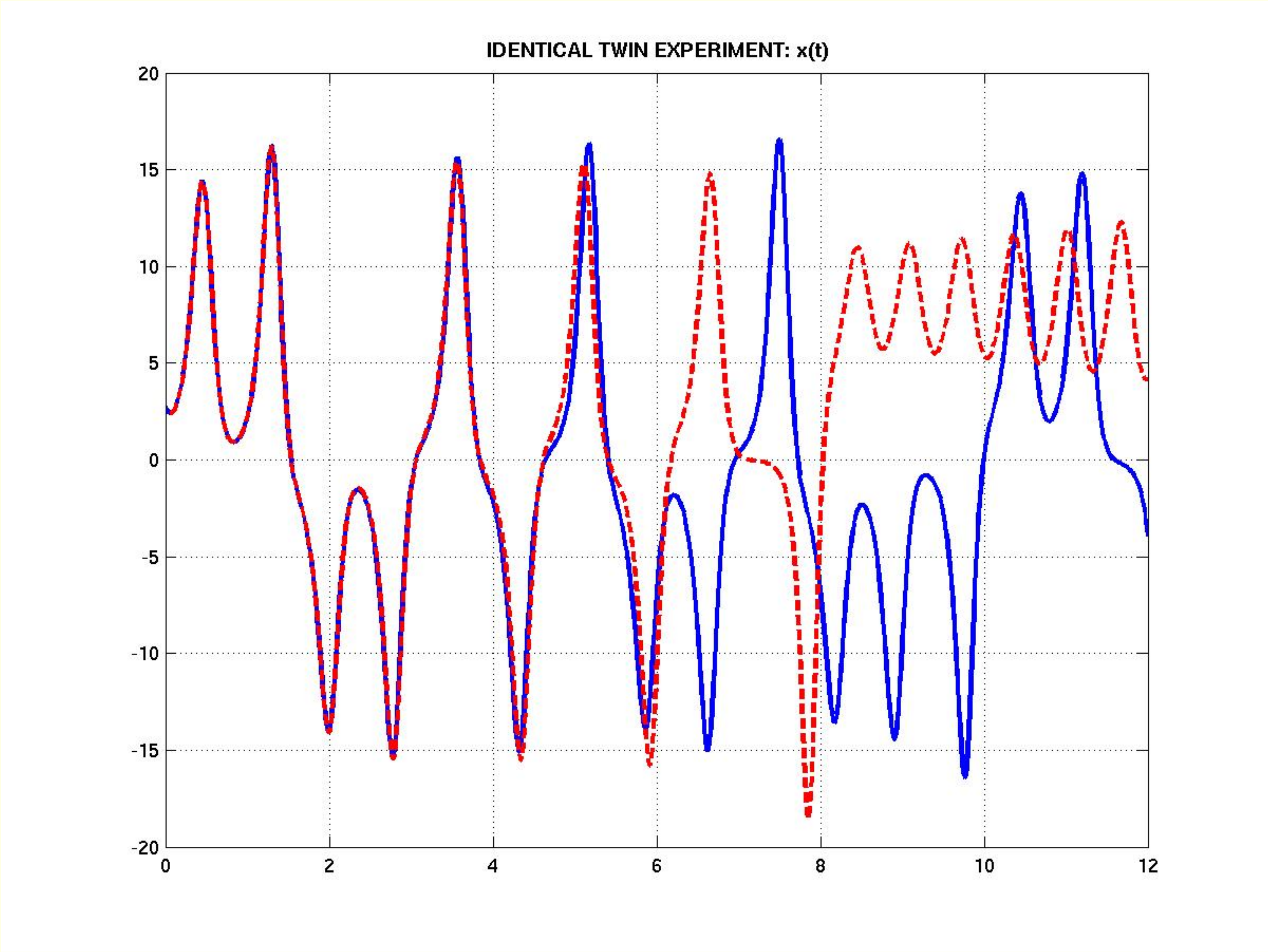
Edward Lorenz (b. 1917)

In a paper published in 1963, entitled *Deterministic Nonperiodic Flow*, Edward Lorenz showed that the solutions of the system

$$\begin{aligned} \dot{x} &= -\sigma x + \sigma y \\ \dot{y} &= -xz + rx \\ \dot{z} &= +xy - bz \end{aligned}$$

are *highly sensitive to the initial conditions.*

Identical Twin Experiment



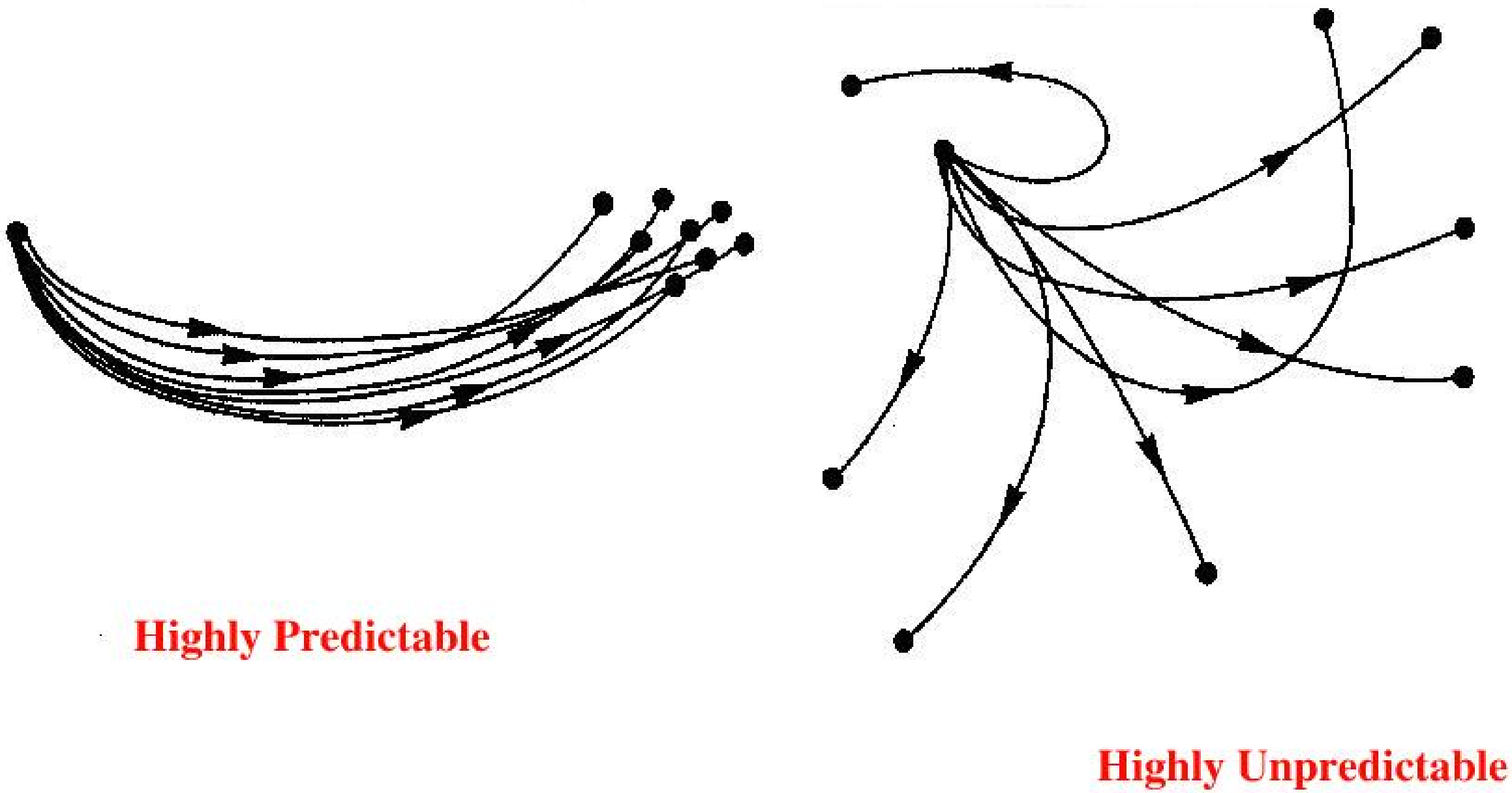
Ensemble Forecasting

In recognition of the chaotic nature of the atmosphere, focus has now shifted to predicting the probability of alternative weather events rather than a single outcome.

European Centre for Medium range Forecasts. Reading Headquarters.

The mechanism is the *Ensemble Prediction System* (EPS) and the world leader in this area is the European Centre for Medium-range Weather Forecasts (ECMWF).

Variation in Predictability



Ensemble remains compact **Ensemble spreads out**

Singular Vectors

The linear tangent model $\mathbf{L}_i$ transforms a perturbation at time t_i to a perturbation at time t_{i+1}:

$$\delta\mathbf{x}(t_{i+1}) = \mathbf{L}_i \delta\mathbf{x}(t_i)$$

Perturbation growth is measured by the norm:

$$||\delta\mathbf{x}(t_{i+1})||^2 = ||\mathbf{L}_i \delta\mathbf{x}(t_i)||^2 = \langle \mathbf{L}\delta\mathbf{x}, \mathbf{L}\delta\mathbf{x} \rangle = \langle \delta\mathbf{x}, \mathbf{L}^{\mathrm{T}}\mathbf{L}\delta\mathbf{x} \rangle \,.$$

This depends on the eigenvalues of $\mathbf{L}^{\mathrm{T}}\mathbf{L}$, the singular values.

The singular vector corresponding to the maximum singular value gives the component that grows fastest.

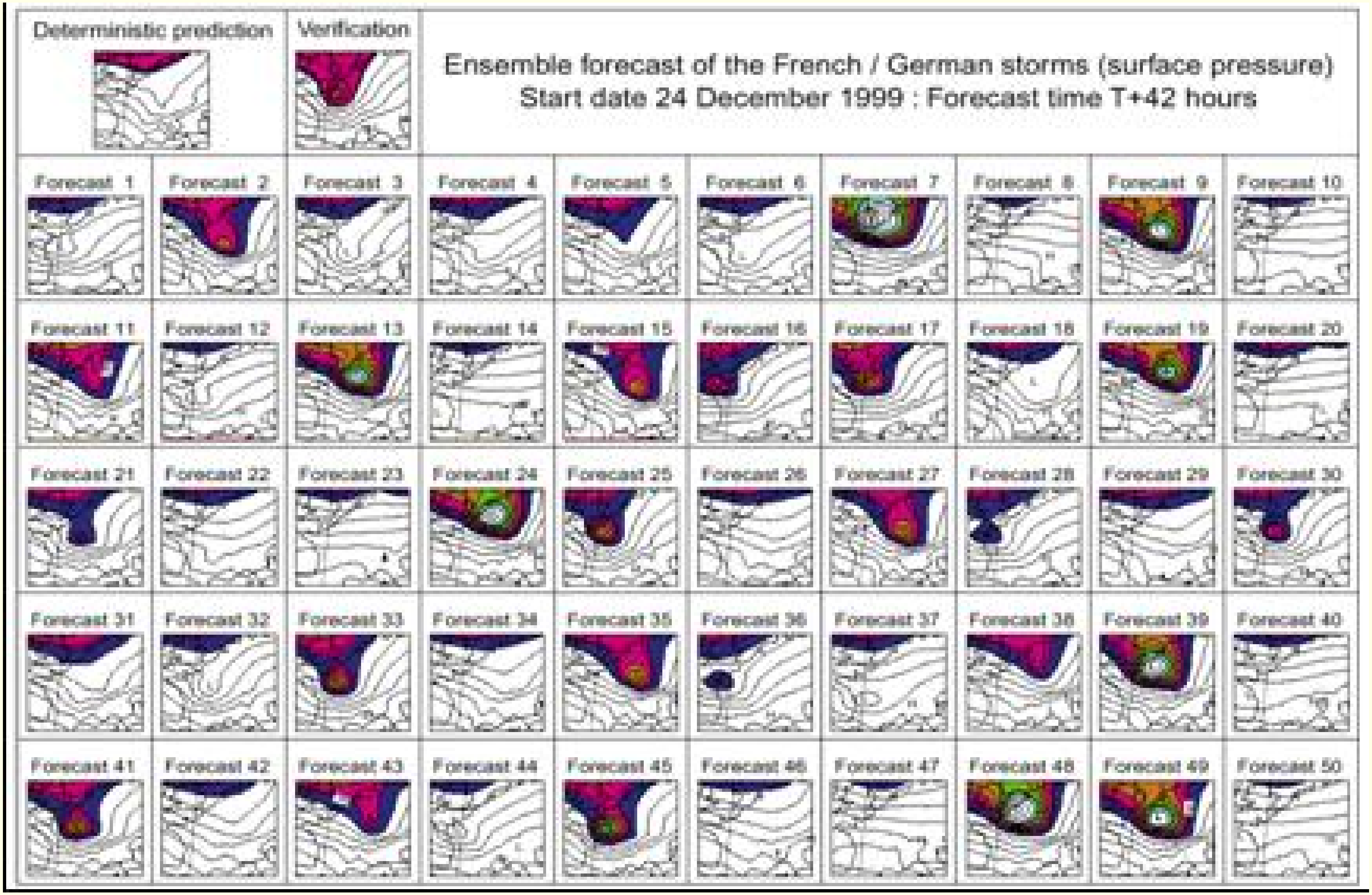
EPS: Ensemble Prediction System

We calculate the 25 largest singular values, and the corresponding 25 singular vectors.

Fifty perturbed initial states are constructed by adding and subtracting from the analysis.

This gives us fifty-one initial states.

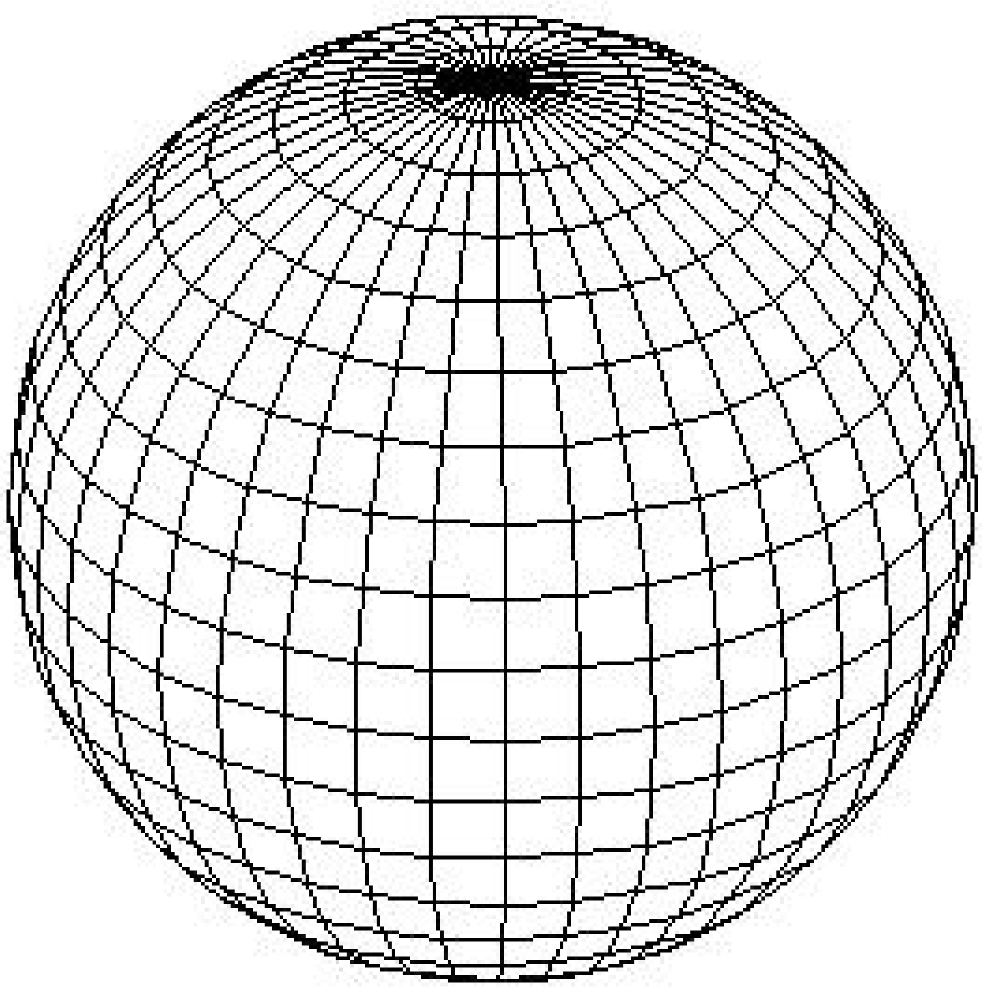
Fifty-one forecasts are done, starting from these.



Ensemble of fifty-one 42-hour forecasts.

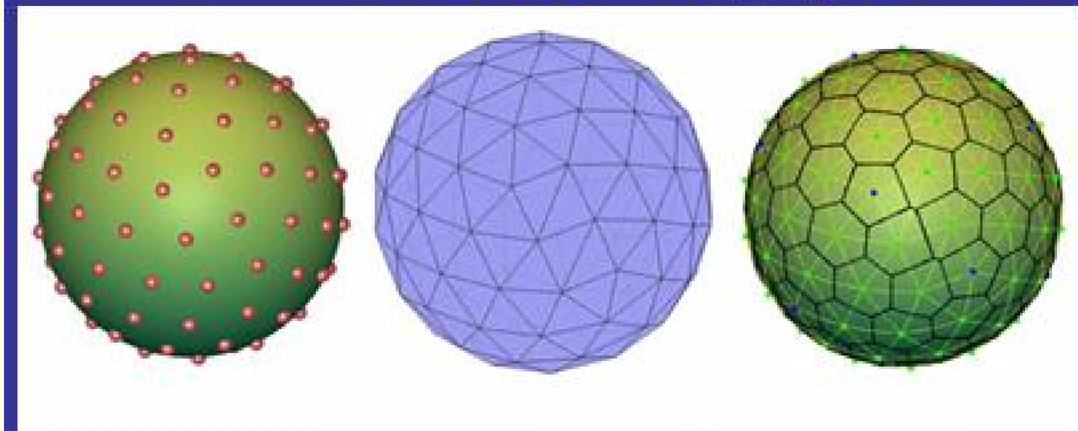
Valid time: 0600 UTC, 26th December, 1999

Discretizing the Sphere



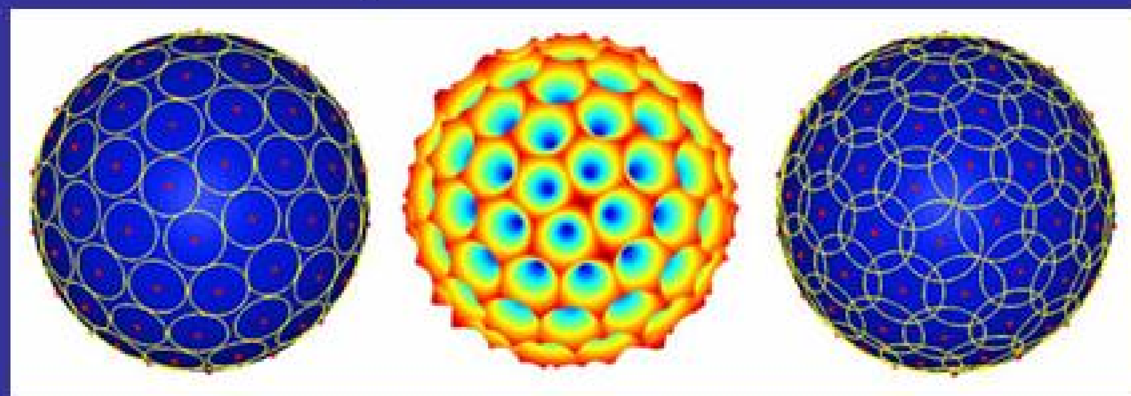
Regular Latitude-Longitude Grid

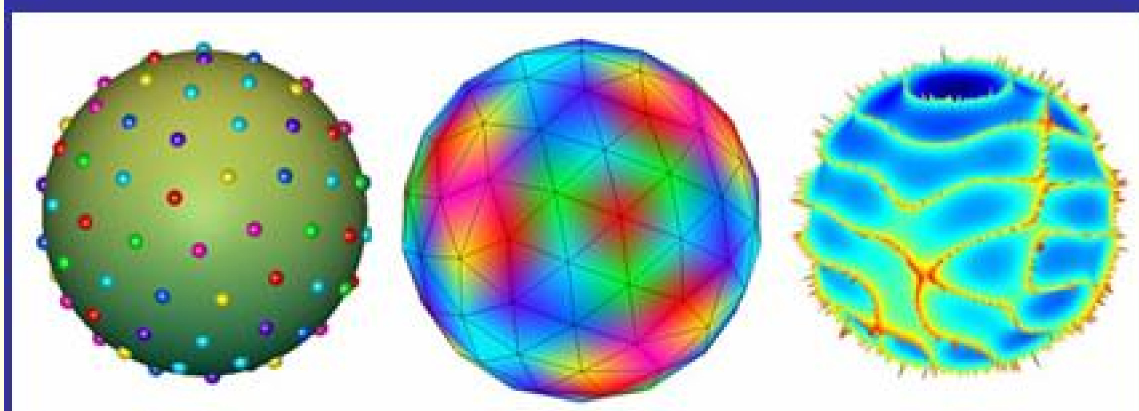
Distributing points on the sphere



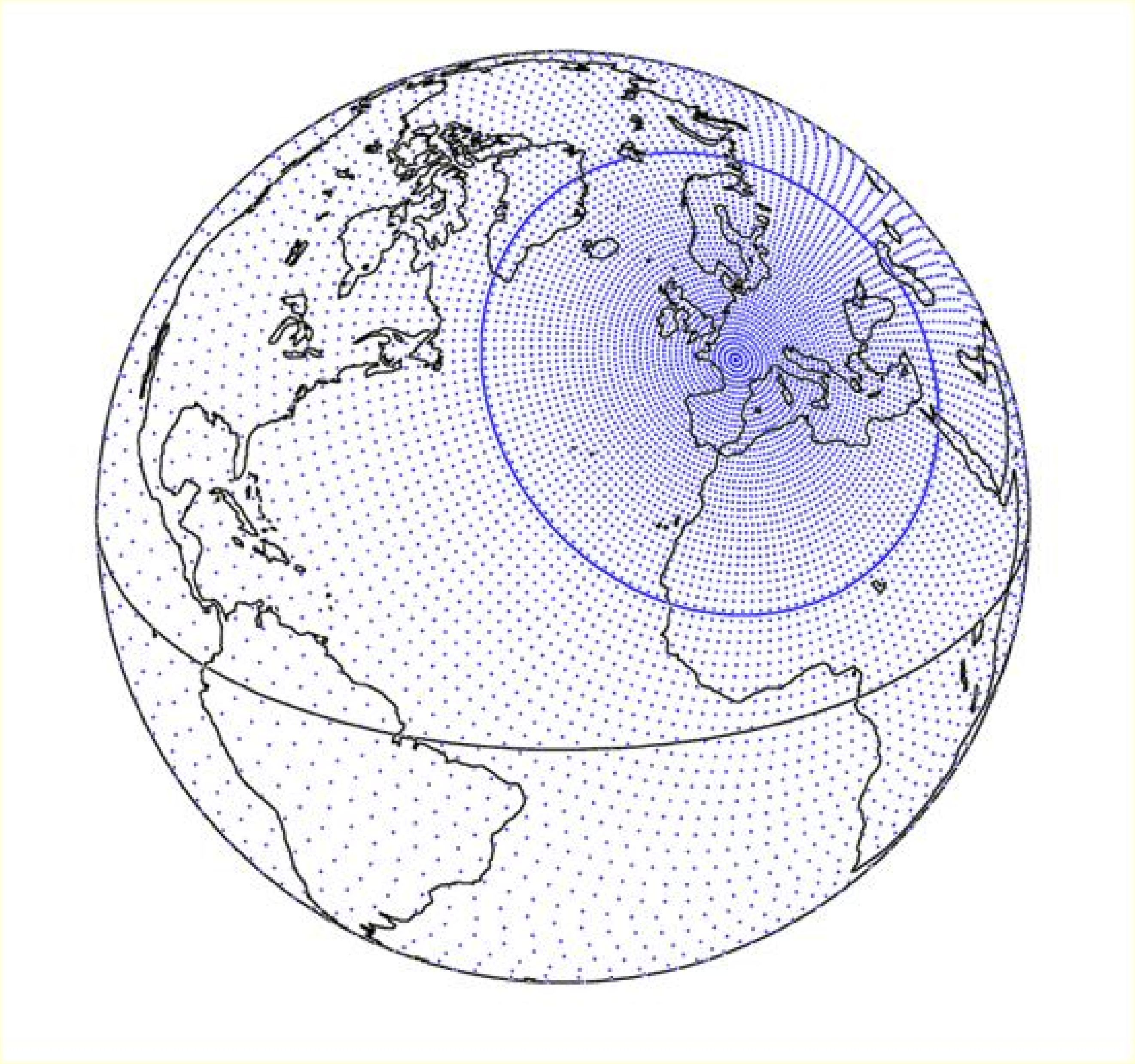
Convex hull, Voronoi cells and Delaunay triangulation

Covering and packing with spherical caps



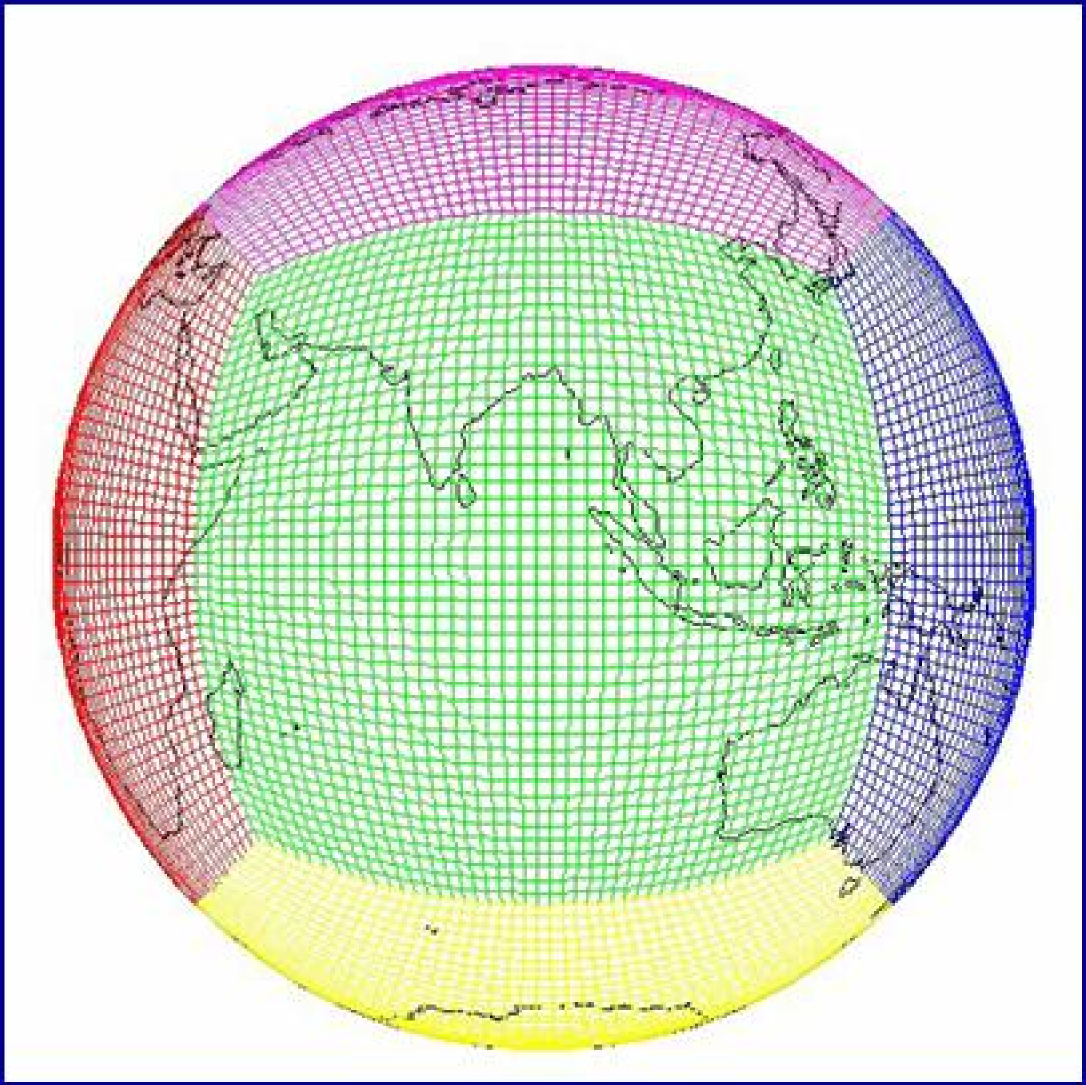


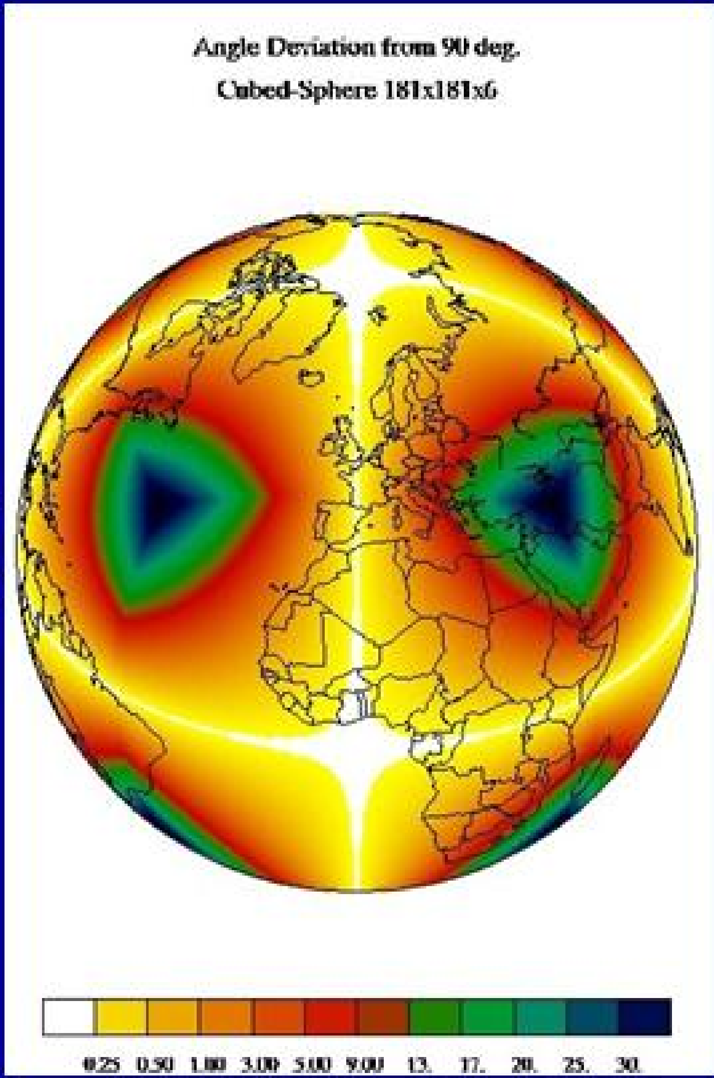
Interpolatory cubature, cubature weights and determinants

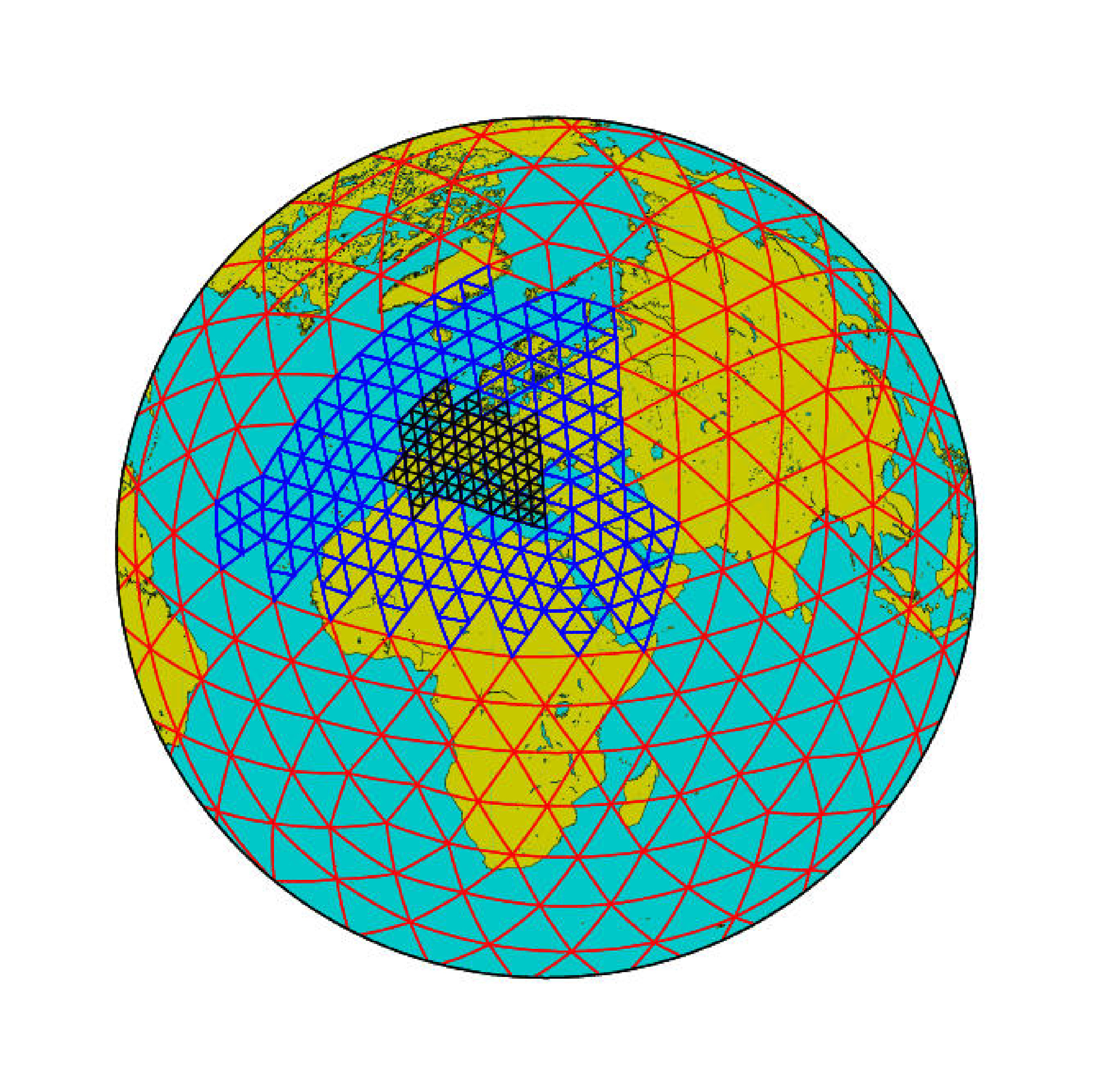


Conformal Stretched Grid

The Cubed Sphere

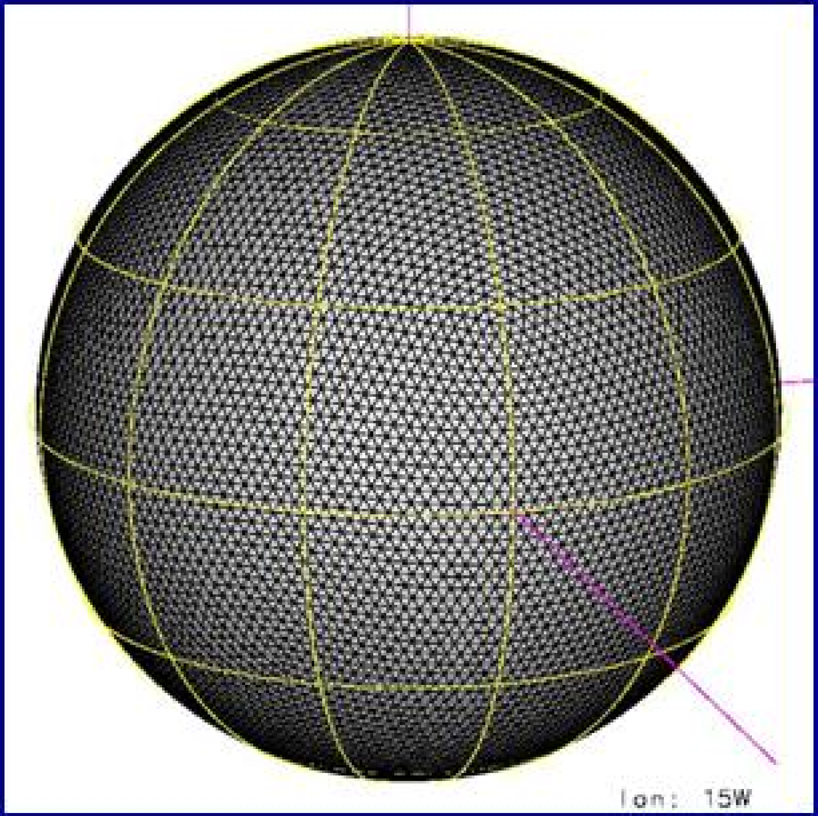


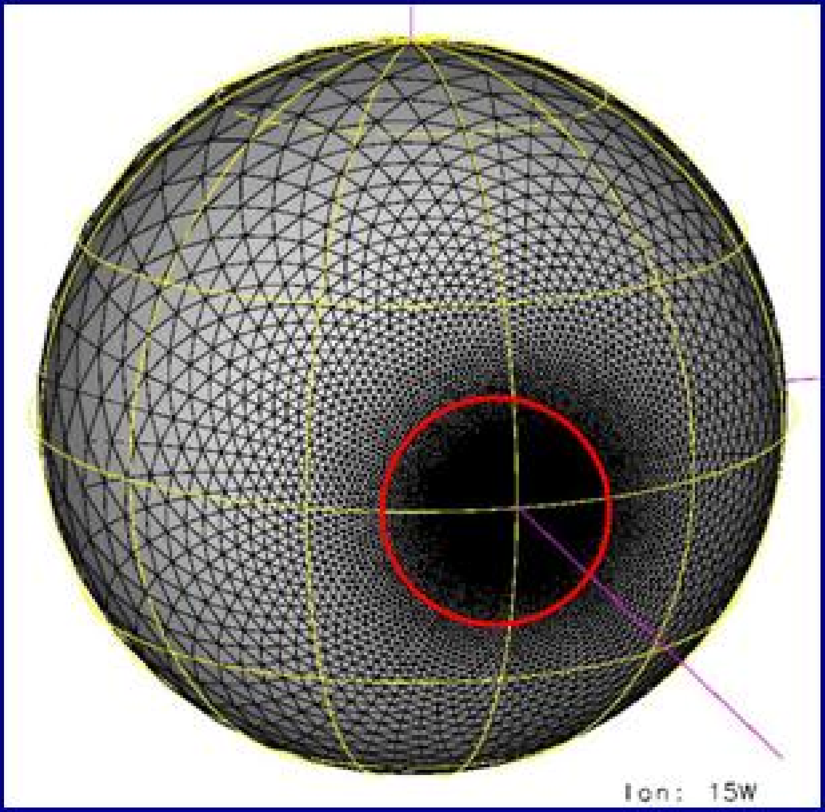




Triangulated Icosahedral Grid

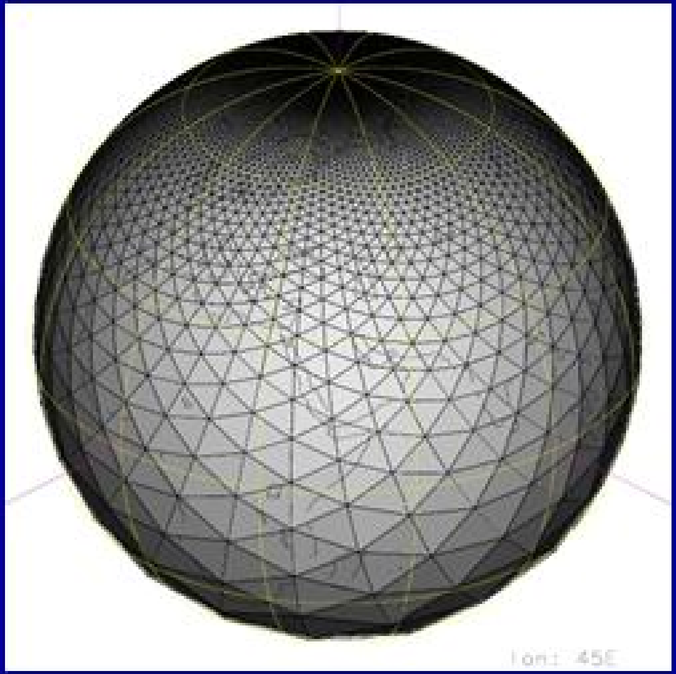
Stretched Icosahedral Grid

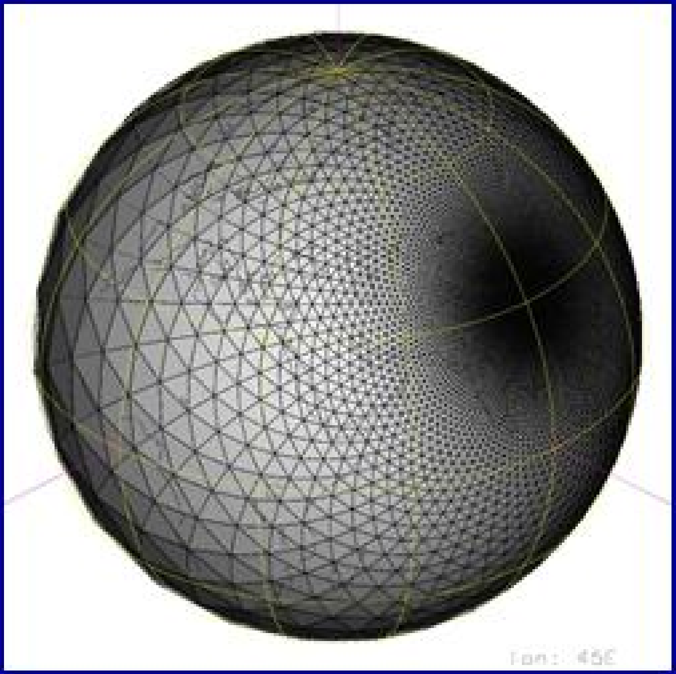




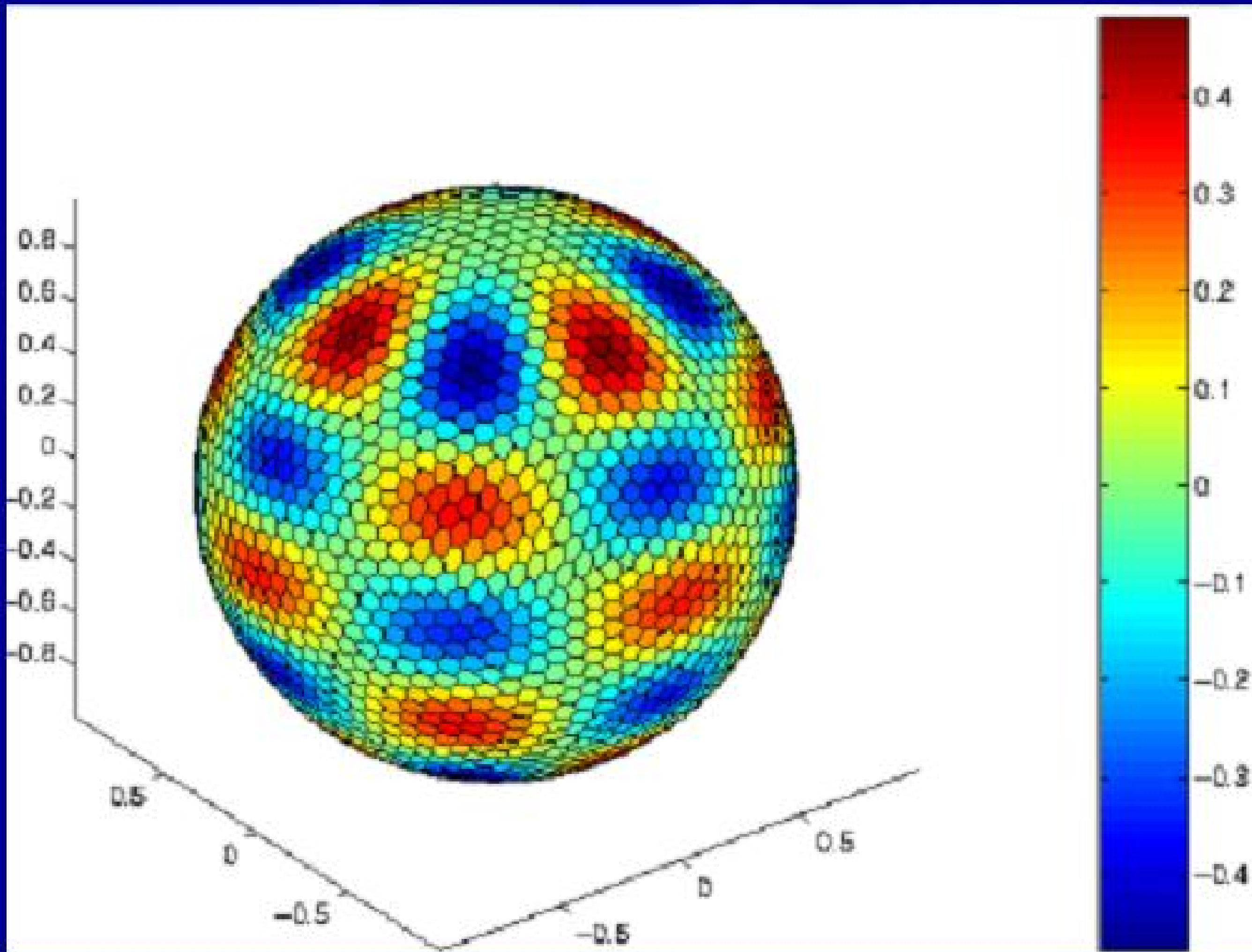
To make a stretched grid

- Gather the grid points in the north pole region (left figure)
- Rotate the grid system to the interested region (right figure)

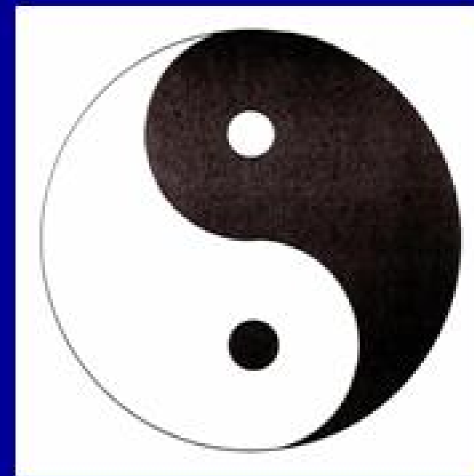


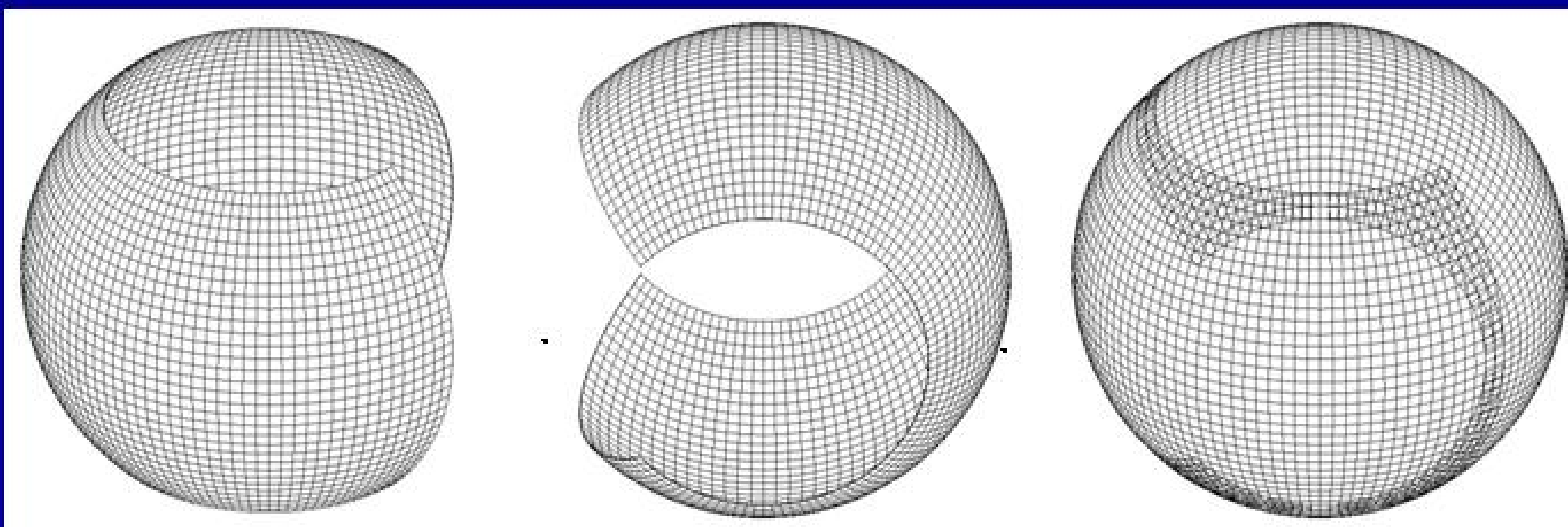


Penta-Hexagonal Grid



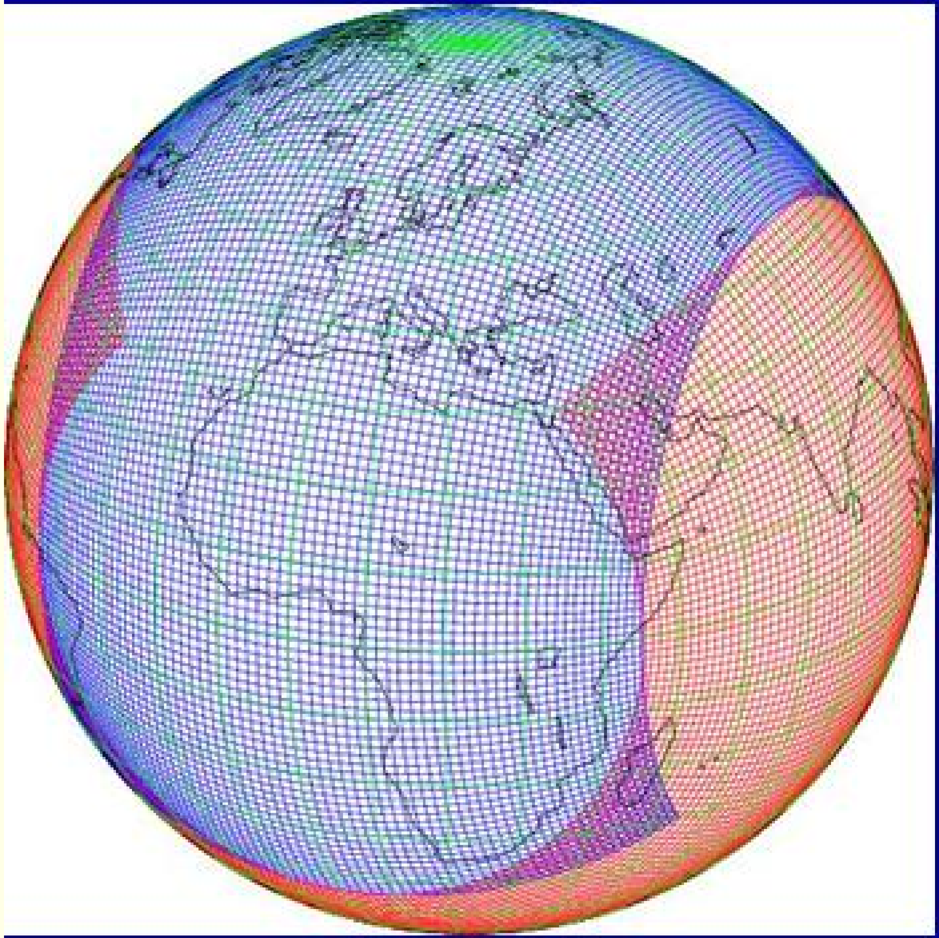
Yin-Yang grid



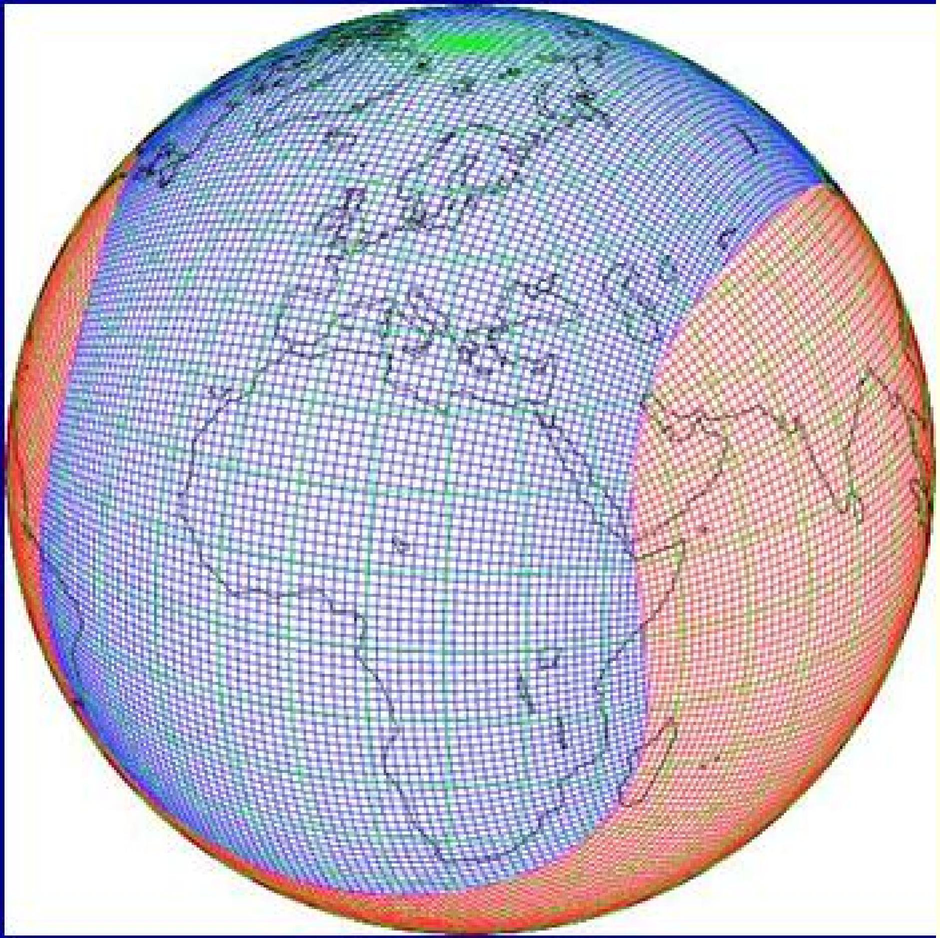


Yang (N) zone Yin (E) zone Yin-Yang composition

Rectangles, minimal overlap



Overlaps trimmed to median



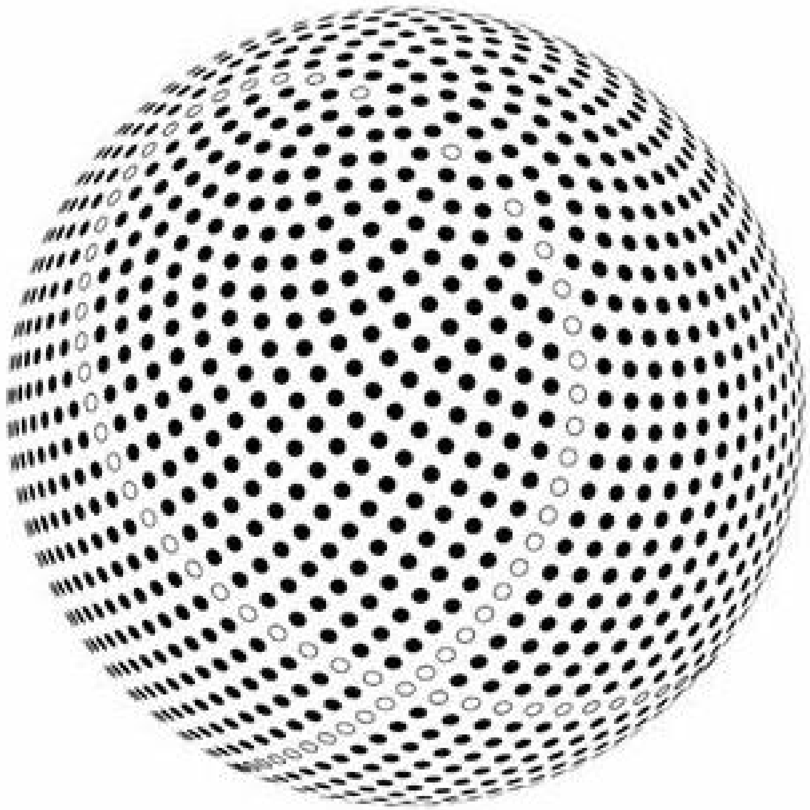


Figure 2. A spherical Fibonacci grid, at resolution $N = 1000$ (2001 grid points). As in Fig. 1, the spiral structure is highlighted by marking every 34th and 55th grid point.

Fibonacci Grid

Inspired by Sun-flowers and Pineapples

The ultimate grid remains elusive.

“Ultimate” depends on the application.

The End

Typesetting Software: TEX, *Textures*, LATEX, `hyperref`, `texpower`, Adobe Acrobat 4.05
Graphics Software: Adobe Illustrator 9.0.2
LATEX Slide Macro Packages: Wendy McKay, Ross Moore