Replication of Léon Foucault’s Pendulum Experiment in Dublin

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Outline

Introduction

Galbraith and Haughton

Pendulum Experiments in Dublin

The Main Results

The Sine Factor

Outline

Pendulum Experiment in Dublin

ABSTRACT.

Léon Foucault's pendulum experiment in 1851 generated widespread interest. The experiment was repeated in numerous locations in Europe and America. The more careful of these demonstrations confirmed the effect of the Earth's rotation on the precession of the swing-plane of the pendulum. A set of pendulum experiments were carried out by Joseph Galbraith and Samuel Haughton in Dublin and a comprehensive mathematical analysis of them was published.

Setting the Scene: Is the Earth Spinning?

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In 1851, a way was found to demonstrate by mechanical means that the Earth is spinning.

We examine the remarkable pendulum experiment of Léon Foucault, and some of its consequences.

Watch for a While: Swing-plane Rotates.

Figure : Foucault pendulum in the Panthéon, Paris.

"You are Invited to See the Earth Turn . . . "

In March 1851, a large crowd gathered in the Panthéon in Paris to witness the Earth spinning.

With simple apparatus comprising a heavy ball swinging on a wire, Léon Foucault showed how the Earth rotates on its axis.

His pendulum demonstration caused a sensation, and Foucault achieved instant and lasting fame.

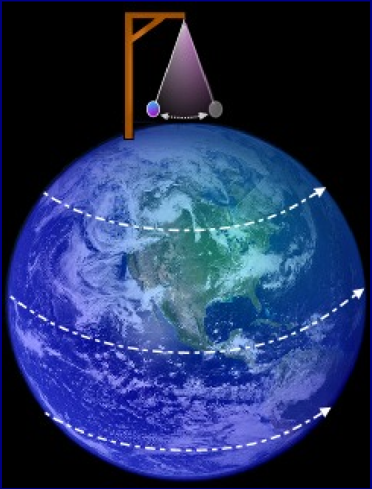


Figure : Simplistic explanation of pendulum precession.

A Few Details

The announcement of the experiment read:

“You are invited to see the Earth turn . . . ”.

- ***Pendulum length*: 67 metres. *Mass of bob*: 28 kg.**
- ***Swing-arc*: 6 metres. *Swing-angle*: about 5°.**
- **Position of bob shown on large circular scale.**
- **Daily demonstrations attracted large crowds.**

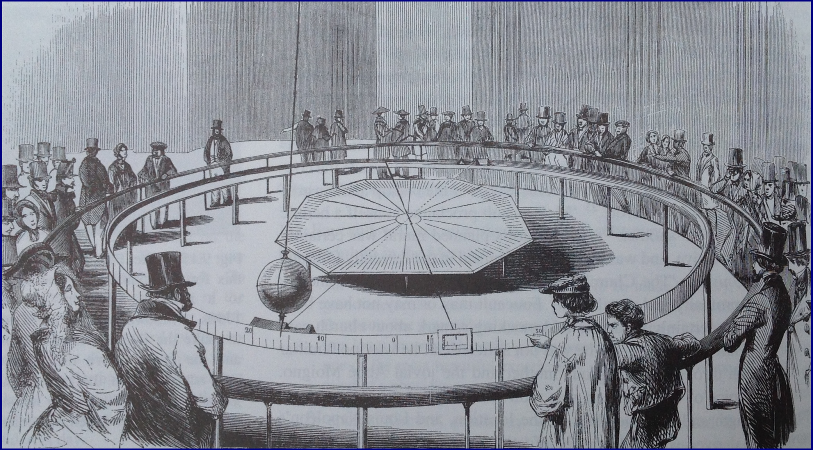


Figure : Engraving in *L'Illustration* of Foucault's pendulum.

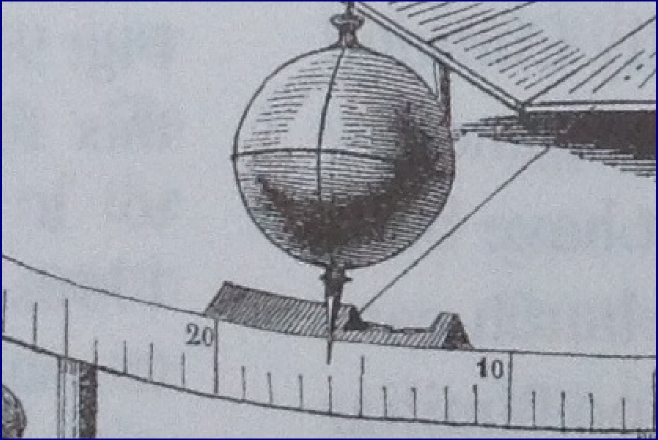


Figure : Detail of Foucault's pendulum.

Widespread International Interest

In the aftermath of Foucault's demonstration, pendulum mania raged across Europe and USA.

The experiment was repeated hundreds of times.

By June 1851 it had been repeated in many cities in Europe and America, including Dublin.

Many of these attempts were done without due care.

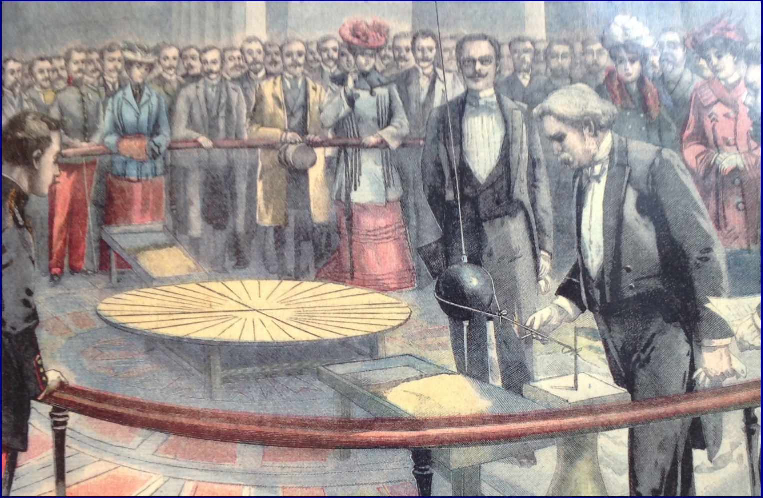
Press Reaction in London

The *London Literary Gazette* reported on several cases in which ‘to the horror of the spectators, the Earth has been shown to turn the wrong way’.

These errors were probably due to

- Lack of care in setting up the experiment.
- Incorrect starting conditions.
- Elliptical bob trajectories.
- Stray air currents.

Illustration from *Flammarion*



Correct way to start the pendulum swinging.

In September 1851, the *American Journal of Science* surveyed several pendulum demonstrations in Europe and America.

These included the experiments carried out in Dublin.

Joseph Galbraith and Samuel Haughton of Dublin University replicated the experiments shortly after Foucault had reported his findings.

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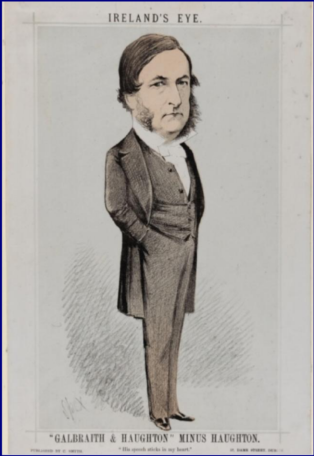
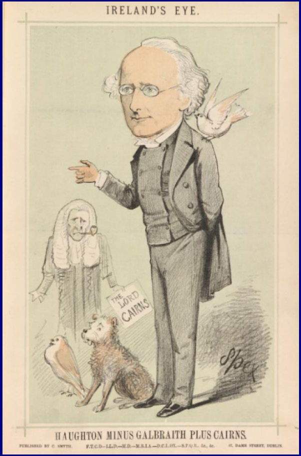
Galbraith & Haughton

Joseph Galbraith (1818–1890)
Samuel Haughton (1821–1897)

- Close contemporaries and life-long collaborators.
- Both highly talented applied mathematicians.
- Both took holy orders in the Church of Ireland.
- Both Fellows of Trinity College Dublin.
- Both were members of the Royal Irish Academy.
- Both were Freemasons in the same Lodge.
- Both sympathasised with Irish nationalism.
- Wrote many mathematical textbooks together.





Cartoons from *Ireland's Eye*, October 1874.
Left: 'Galbraith & Haughton minus Haughton'.
Right: 'Haughton minus Galbraith plus Cairns'.

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Location: Engine-factory of the D&KR.

Experiments were done in April and May 1851.

Objective of the Experiments:
"To determine the azimuthal motion of the plane of vibration of a freely suspended pendulum".

The experiments were at the engine-factory of the Dublin & Kingstown Railway, where Samuel's cousin Wilfred Haughton was chief engineer.

In 1839 the D&KR had acquired the premises of the Dock Distillery at Grand Canal Street. (Engine Factory indicated on map overleaf)

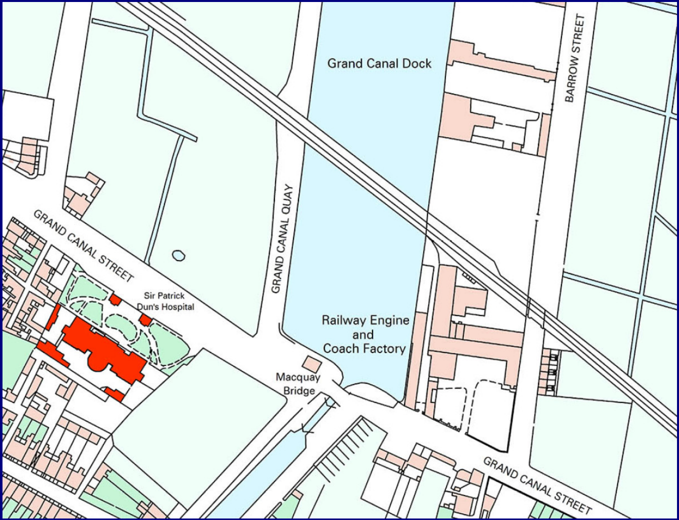
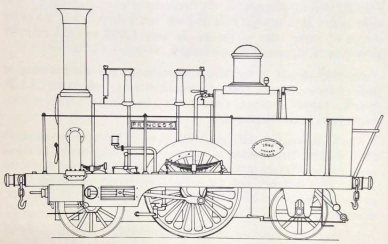


Figure : Detail from map of Dublin, 1843–7, showing the location of the locomotive factory at Grand Canal Dock.

First Locomotive Built in Dublin

The first locomotive built there was named Princess, after Victoria, the first daughter of Queen Victoria.



Total cost of production of the engine: about £1000.

Princess, completed in 1841, continued to give service until it was finally scrapped in 1883.

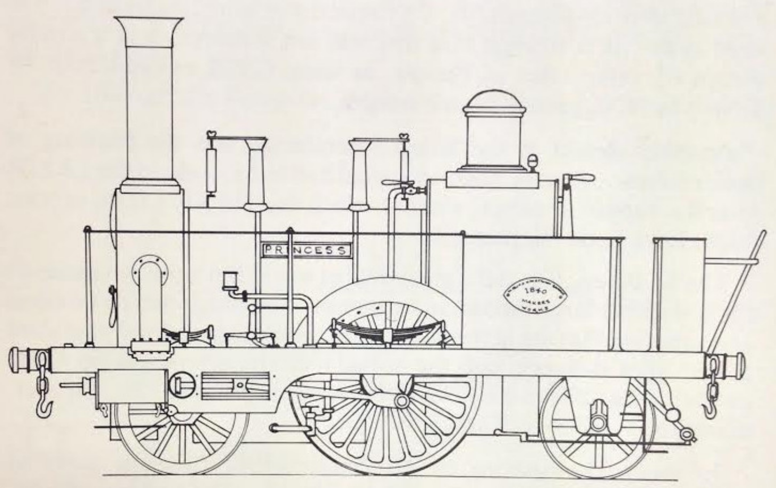
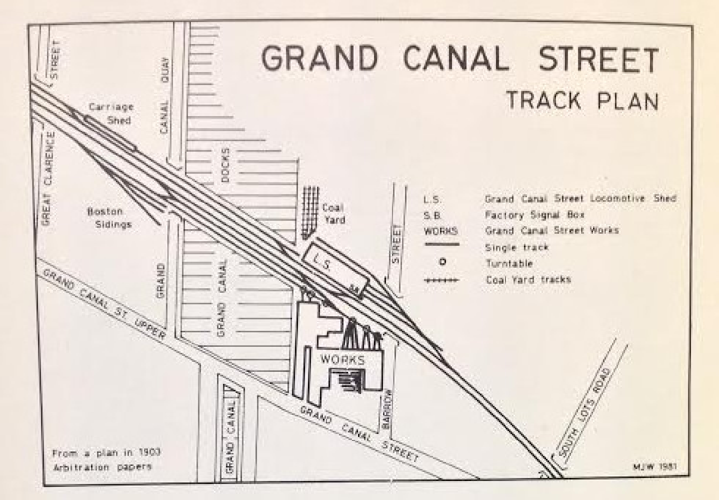


Figure : First locomotive built in Dublin, 1840/41.

Track plan of D&KR, Grand Canal Street



[From a plan in 1900 Arbitration papers]

Details of the Dublin Set-up

A Foucault pendulum should be as long as possible, with a small swing angle and a heavy bob.

Engine houses have lofty roofs to allow for the dispersal of smoke and steam.

As the D&KR was on an embankment, the engine rooms were on the first floor of the building.

There was ample head-room and a pendulum of over 35 feet in length could be accommodated.

Grand Canal Street Works, 1902

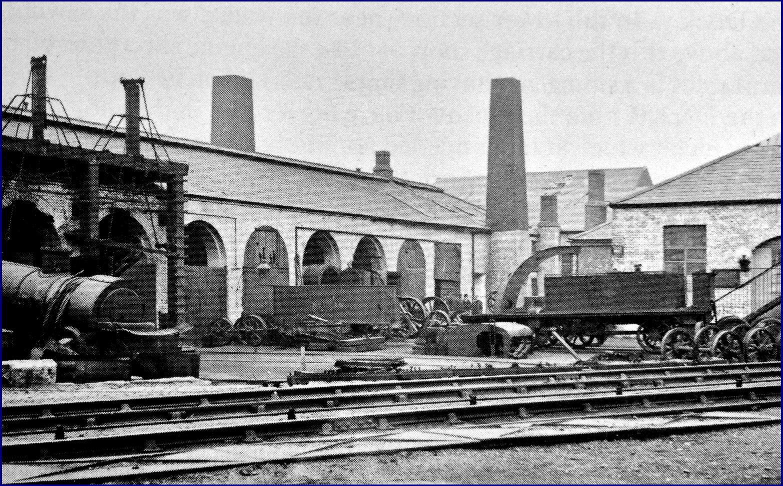


Figure : Engine factory of D&KR at Grand Canal Street.

The construction bays were accessed by turntables from a siding close to where the new Grand Canal Dock DART station now stands.

Spherical bob of iron, weighing 30 pounds, with a downward spike attached to indicate the position.

Initially set swinging in a north-south plane. Below the bob, a graduated circle indicated the azimuth.

Various modes of suspending the pendulum.

Six experiments were carried out, generally lasting between fifteen and twenty hours, with the azimuth being recorded every twenty minutes.

In the final experiment a full rotation of 360° was achieved after 28 hours 26 minutes.

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Observed Precession Rate

Foucault had derived the theoretical result:

$$\omega = \Omega \sin\theta$$

For Dublin, this implies precession through a full circle in about 30 hours.

The theoretical precession rate at the latitude of Dublin is $\omega = 12.07^\circ/$hr.

The mean rate for the experiments was $11.9^\circ/$hr.

Effects of Ellipticity

Galbraith and Haughton also analysed the effects of ellipticity of the trajectory.

They derived a mathematical expression for the precession due to this effect.

For a pendulum of length ℓ tracing an area A, the precession for each orbit is

$$\Delta_{\mathrm{ELL}} = \frac{3A}{4\ell^2}$$

This amounts to about half a degree per hour, small compared to the change due to the Earth's rotation.

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The observed change in the swing plane of the pendulum is often stated to be due to the Earth turning beneath it. This is an over-simplification.

The turning rate for a pendulum at the North Pole is one revolution per day.

At other locations the turn is proportional to the sine of the latitude ($\sin\theta$).

Thus, after one day, the swing plane does not return to its original position.

At 53.5°N, the period is $24\,\mathrm{h}/\sin 53.5^\circ \approx$ 29.86 hours.

This *anholonomy* has been a source of confusion ever since Foucault's demonstration.

A simple way of explaining the turning of the swing plane as a beat phonomenon was given by Opat.

In fact, a similar explanation was published 140 years earlier by the Irish mathematician Matthew O'Brien.

The quasi-linear motion of the bob is considered as a combination of two counter-rotating conical motions with equal amplitudes but different frequencies.

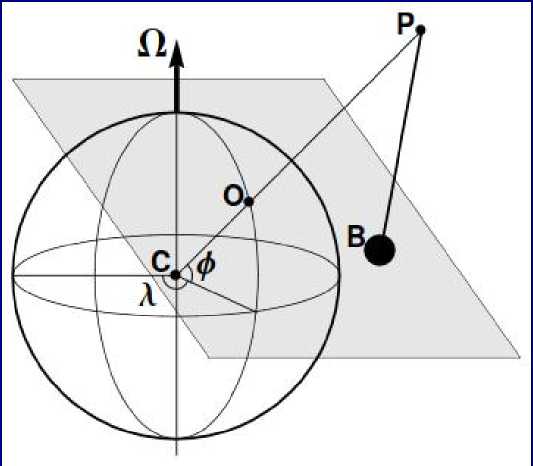

Figure : Geometry of the Foucault pendulum. The plane tangent to the Earth at the subpoint is shown in grey.

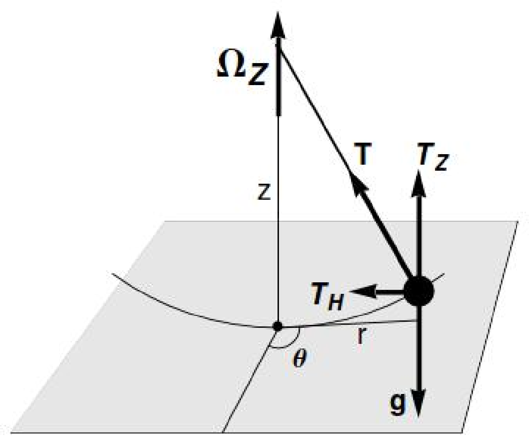

Figure : Foucault pendulum in tangent plane geometry.

For small amplitude the bob remains close to a plane.

Forces acting on the bob: Gravity g and tension T.

Additional force, the Coriolis force, in rotating frame.

Horizontal Equation of Motion:

$$\text{mass} \times \begin{pmatrix} \text{Centripetal} \\ \text{Acceleration} \end{pmatrix} = \begin{pmatrix} \text{Tension} \\ \text{Force} \end{pmatrix} + \begin{pmatrix} \text{Coriolis} \\ \text{Force} \end{pmatrix} .$$

For anticlockwise motion, the Coriolis force *opposes* the tension while for clockwise motion it *reinforces* it.

The frequency is slightly smaller for anticlockwise than for clockwise motion.

$\therefore$ slow clockwise precession of the swing plane.

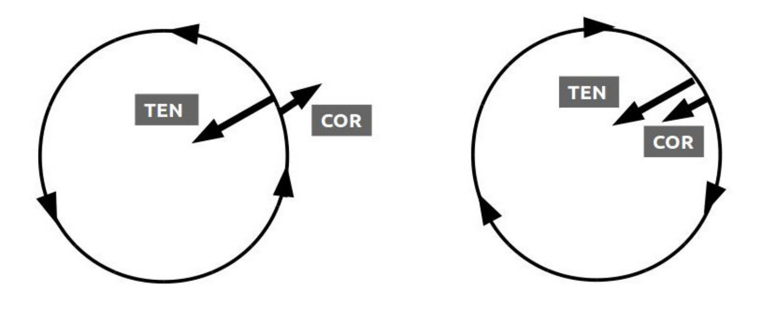

Figure : Forces acting in the case of conical motion.

Irish Mathematician Matthew O'Brien

Matthew O'Brien (1814–1855) an inventor of Vector Analysis.

He explained the precession of the pendulum in terms of combined conical motions.

Another View

According to Horace Lamb:

"The simple view of the matter is that the Earth is rotating about the vertical at the rate $\Omega \sin \phi$, in the positive direction, beneath the pendulum".

$\star \quad \star \quad \star$

For a detailed mathematical analysis of the precession of the azimuthal angle see Lynch (2016, Appendix B).

A complete account, with extracts from the Diary of Joseph Galbraith, is available online:

In Retrospect

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https://maths.ucd.ie/~plynch/Publications

International Reaction

According to the *Philosophical Magazine*, 'Messrs. Galbraith and Haughton ... have pursued their research with all imaginable precautions'.

Their impressive results confirm this assessment.

Thank you