Balanced Flow on the Spinning Globe

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EMS Silver Medal Awardees

- 2008: Karin Labitzke, Germany
- René Morin, France
- 2009: Lennart Bengtsson, Sweden
- 2010: David Burridge, UK
- 2011: Jean François Geleyn, France
- 2012: Tim Palmer, UK
- 2013: Hartmut Graßl, Germany
Outline

Introduction

Atmospheric Balance

Coriolis Effect

Richardson’s Forecast
Outline

Introduction

Atmospheric Balance

Coriolis Effect

Richardson’s Forecast
Galileo Galilei (1564–1642)

Formulated law of falling bodies ... verified by measurements.

Constructed a telescope, and found
  • lunar craters
  • four moons of Jupiter

Galileo invented the thermometer

Evangelista Torricelli invented the barometer

Thus began quantitative meteorology.
Galileo Galilei and Leaning Tower of Pisa
We have Viviani’s word that Galileo dropped various weights from the Leaning Tower . . .

“... to the dismay of the philosophers, different weights fell at the same speed . . .”

Galileo on the Universe

The Assayer (*IL SAGGIATORE*) was published in Rome in 1623.

[The universe] ... is written in the language of mathematics ... without which it is ... impossible to understand a single word of it.
As easy as A, B, C

Three-term equation:

$$A + B + C = 0$$
As easy as A, B, C

Three-term equation:

\[ A + B + C = 0 \]

Suppose one term is small relative to the others.

There are three possibilities:

- **A SMALL** \(\Rightarrow\) \(B + C \approx 0\)
- **B SMALL** \(\Rightarrow\) \(A + C \approx 0\)
- **C SMALL** \(\Rightarrow\) \(A + B \approx 0\)
Outline

Introduction

Atmospheric Balance

Coriolis Effect

Richardson’s Forecast
A Most Surprising Property of Atmospheric & Oceanic Motion

The motion of the atmosphere and ocean systems is remarkably persistent.

Why doesn’t air rush in to fill low pressure areas?
The motion of the atmosphere and ocean systems is remarkably persistent.

Why doesn’t air rush in to fill low pressure areas?

The crucial factor is the rotation of the Earth.
“If a stone is thrown into an infinite resting ocean, the gravitational oscillations engendered will radiate their energy to infinity, leaving the ocean . . . undisturbed;
“If a stone is thrown into an infinite resting ocean, the gravitational oscillations engendered will radiate their energy to infinity, leaving the ocean . . . undisturbed;

“If a stone is thrown into an infinite rotating ocean, some of the energy . . . will be converted into rotational motions . . . and these will persist . . . .”

The Twentieth Century Reanalysis Project, supported by the Earth System Research Laboratory Physical Sciences Division from NOAA and the University of Colorado CIRES/Climate Diagnostics Center, is an effort to produce a global reanalysis dataset spanning the entire twentieth century. It assimilating only surface observations of synoptic pressure, monthly sea surface temperature and sea ice distribution (Version II includes the years 1871 to 2008). Products include 6-hourly ensemble mean and spread analysis fields on a 2x2 degree global analysis grid, and 3 and 6-hourly ensemble mean and spread forecast (first guess) fields on a global Gaussian T-62 grid. Fields are accessible in yearly time series files (1 file/parameter). Ensemble grids, spectral coefficients, and other information will available by offline request in the future.
A global reanalysis dataset spanning the entire twentieth century . . .

Assimilating only surface pressure observations . . .

. . . the analysis covers the entire troposphere.

Resolution:
T62 (300km), 28 Levels. 56-Member Ensemble.
Mean Zonal Wind Analysis

20CR

ERA40
20th Century Reanalysis: Conclusion

How do they do that?

How do they reconstruct the troposphere from surface observations?
How do they do that?

How do they reconstruct the troposphere from surface observations?

Reconstruction of the complete three-dimensional structure of the troposphere is possible ... ... because the atmosphere is in a state of balance.
How do they do that?

How do they reconstruct the troposphere from surface observations?

Reconstruction of the complete three-dimensional structure of the troposphere is possible . . .

. . . because the atmosphere is in a state of balance.

ERA-CLIM2: 20th Century Reanalysis coming soon.
Examples of Balance in the Atmosphere

- Hydrostatic balance
- Geostrophic balance
- Quasi-nondivergence
- Quasi-incompressibility
- Ocean atmosphere balance
- Energy balance
- Ice sheet balance
- Etc., etc., etc.
The Thin Atmosphere
What keeps the air aloft?
Something must be balancing gravity.
What is it?
Hydrostatic Balance

*What keeps the air aloft?*

Something must be balancing gravity. What is it?

For a parcel of air:

- The air below is pushing it upwards.
- The air above is pushing it down.
- The push upwards is greater.
- The difference balances the pull of gravity.
Examine the terms in the vertical equation

\[ \frac{dw}{dt} = \frac{u^2 + v^2}{r} + 2\Omega u \cos \phi \frac{1}{\rho} \frac{\partial p}{\partial z} - g + F_Z. \]

Vertical pressure gradient force and gravity dominate.
Examine the terms in the vertical equation

\[
\frac{dw}{dt} = \frac{u^2 + v^2}{r} + 2\Omega u \cos \phi - \frac{1}{\rho} \frac{\partial p}{\partial z} - g + F_Z.
\]

Vertical pressure gradient force and gravity dominate.

Keeping just the two large terms, we have:

\[
\frac{\partial p}{\partial z} = -g \rho
\]
Hydrostatic Balance

- The vertical wind is generally very small.
- There is balance between the vertical pressure gradient force and gravity.
- This balance is called *hydrostatic balance*.

\[ \frac{\partial p}{\partial z} + g\rho = 0 \]
Examples of Balance in the Atmosphere

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- Etc., etc., etc.
Geostrophic Balance

\[ \gamma \xi \theta = \text{geo strophe} = \text{Earth Turning} \]

The term was coined by Sir Napier Shaw, Director of the Met Office.
Buys Ballot

Christophorus Henricus Diedericus Buys Ballot (1817–1890)

Dutch meteorologist and chemist and mineralogist and geologist and mathematician.
Buys Ballot’s Law

In the Northern Hemisphere, if you stand with your back to the wind, the Low Pressure is to your left.
Buys Ballot’s Law

In the Northern Hemisphere, if you stand with your back to the wind, the Low Pressure is to your left.

The GPS Version:

If you stand with your back to the wind, and the low pressure is to your left, then you must be in the Northern Hemisphere.
Buys Ballot’s Law

In the Northern Hemisphere, if you stand with your back to the wind, the Low Pressure is to your left.

The GPS Version:

If you stand with your back to the wind, and the low pressure is to your left, then you must be in the Northern Hemisphere.

RaymondHide
William Ferrel
(1817–1891)
American meteorologist.
Ferrel’s 1856 Paper

An essay on the winds and the currents of the oceans.

[Nashville Journal of Medicine and Surgery, 1856.]
Ferrel’s 1856 Paper

An essay on the winds and the currents of the oceans.

[Nashville Journal of Medicine and Surgery, 1856.]

“In consequence of the atmosphere’s revolving ... each particle is impressed with a centrifugal force.

“But if the rotatory motion of the atmosphere is greater than that of the Earth, this force is increased.

“and if ... [less] ... it is diminished.

“This difference gives rise to a disturbing force ... which materially influences the motion.”
Force Balance for Low and High Pressure

Gradient balance around low and high pressure.

Image from ATPM Manual, Oxford Aviation Training

Gradient Wind Speed Around a Depression (Northern Hemisphere)
Horizontal Equations of Motion

\[ \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} + 2\mathbf{\Omega} \times \mathbf{V} + \frac{1}{\rho} \nabla p = 0 \]
Horizontal Equations of Motion

\[ \frac{\partial V}{\partial t} + V \cdot \nabla V + 2\Omega \times V + \frac{1}{\rho} \nabla p = 0 \]

For steady motion we get a three-way balance:

\[ V \cdot \nabla V + 2\Omega \times V + \left(\frac{1}{\rho}\right) \nabla p = 0 \]

(CFF) (COR) (PGF)
Horizontal Equations of Motion

\[ \frac{\partial V}{\partial t} + V \cdot \nabla V + 2\Omega \times V + \frac{1}{\rho} \nabla p = 0 \]

For steady motion we get a three-way balance:

\[ V \cdot \nabla V + 2\Omega \times V + \left( \frac{1}{\rho} \right) \nabla p = 0 \]

This is as easy as ABC:

\[ A + B + C = 0 \]
Three-way Balance

Also known as **Gradient Balance**:

\[ \mathbf{V} \cdot \nabla \mathbf{V} + 2\Omega \times \mathbf{V} + \frac{1}{\rho} \nabla p = 0 \]

- **CFF**
- **COR**
- **PGF**

Three for the price of one!

Intro Balance Coriolis LFR
Three-way Balance

Also known as Gradient Balance:

\[ \mathbf{V} \cdot \nabla \mathbf{V} + 2\Omega \times \mathbf{V} + \frac{1}{\rho} \nabla p = 0 \]

- **CFF** small \(\Rightarrow\) Geostrophic Balance
- **COR** small \(\Rightarrow\) Cyclostrophic Balance
- **PGF** small \(\Rightarrow\) Inertial Balance

Three for the price of one!
Balance between the Coriolis force and the pressure gradient force:

\[ \mathbf{f k} \times \mathbf{V} + \frac{1}{\rho} \nabla p = 0 \]

We can determine the wind from the pressure!
Geostrophic Balance

Balance between the Coriolis force and the pressure gradient force:

\[ f k \times V + \frac{1}{\rho} \nabla p = 0 \]

\[ V_{\text{GEO}} = \frac{1}{f \rho} k \times \nabla p \]
Geostrophic Balance

Balance between the Coriolis force and the pressure gradient force:

\[
\begin{align*}
\mathbf{f} \times \mathbf{V} + \left(\frac{1}{\rho}\right) \nabla p &= 0 \\
\text{COR} + \text{PGF} &= 0
\end{align*}
\]

\[
\mathbf{V}_{\text{GEO}} = \frac{1}{f \rho} \mathbf{k} \times \nabla p
\]

We can determine the wind from the pressure!
Time-scale for Atmospheric Motions

Non-rotating Earth:

\[ \frac{\partial V}{\partial t} + \frac{1}{\rho} \nabla p = 0 \quad \text{or} \quad T = \frac{\rho LV}{\Delta p} \]

For typical synoptic values this gives \( T \approx 3 \) hours.
Time-scale for Atmospheric Motions

Non-rotating Earth:

\[
\frac{\partial V}{\partial t} + \frac{1}{\rho} \nabla \rho = 0 \quad \text{or} \quad T = \frac{\rho L V}{\Delta p}
\]

For typical synoptic values this gives \( T \approx 3 \text{ hours} \).

Rotating Earth:

\[
\frac{\partial V}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} = 0 \quad \text{or} \quad T = \frac{L}{V}
\]

For typical synoptic values this gives \( T \approx 30 \text{ hours} \).
Examples of Balance in the Atmosphere

- Hydrostatic balance
- Geostrophic balance
- Quasi-nondivergence
- Quasi-incompressibility
- Ocean atmosphere balance
- Energy balance
- Ice sheet balance
- Etc., etc., etc.
Geostrophic Flow is Quasi-nondivergent

\[ \mathbf{V}_{\text{GEO}} = \frac{1}{f\rho} \mathbf{k} \times \nabla \rho \]

Ignore variations in \( f \) and \( \rho \):

\[ \mathbf{V}_{\text{GEO}} = \mathbf{k} \times \nabla \left( \frac{\rho}{f\rho} \right) = \nabla \times \left( -\frac{\rho}{f\rho} \right) \mathbf{k} \]

Divergence of a curl is zero:

\[ \nabla \cdot \mathbf{V}_{\text{GEO}} = 0 \]
The Rossby Number

\[ Ro = \frac{\text{Centrifugal Force}}{\text{Coriolis Force}} = \frac{V}{fL} \]

\[ Ro = \frac{\text{Spin of the Flow}}{\text{Spin of the Earth}} = \frac{\zeta}{f} \]

C. G. Rossby in *Time*
500 mb geopotential and wind field
500 mb Rossby Number $|V \cdot \nabla V| / |fV|$

Image from Marshall & Plumb, © Elsevier.
Balance at Different Scales

- Extra-tropical Depressions
- Tropical Cyclones
- Tornadoes
- Domestic.
Balance at Different Scales: Depressions

Extra-tropical Depression

\[ Ro \approx \frac{1}{10} \]

Geostrophic Balance Good

Gradient Balance Better.
Balance at Different Scales: Tropical Cyclones

Hurricane

$Ro \approx 10 - 100$

Geostrophic Balance Bad

Gradient Balance Better.
Tropical Cyclone Tracks

Tracks and Intensity of Tropical Cyclones, 1851-2006

Saffir-Simpson Hurricane Intensity Scale

NASA
“I always find my pen sticks to the paper and refuses to move when I try to draw an isobar across the equator.”

Balance at Different Scales: Tornadoes

Tornado

$Ro \approx 10,000$

Close to Cyclostrophic Balance

Coriolis effect influences background flow
Traffic Flow and Vorticity

Effect of vorticity pollution by motor vehicles on tornadoes.
Isaacs, J. D., J. W. Stork, D. B. Goldstein & G. L. Wick
> 98% of tornadoes are cyclonic, but . . .
> 98% of tornadoes are cyclonic, but ...

- Notable Anticyclonic Tornadoes:
  - West Bend tornado
  - Grand Island tornado
  - Woodward, Oklahoma April 10th 2012
  - Aurora Nebraska, 2009
  - Freedom, Oklahoma, June 6, 1975
  - Sunnyvale, California, May 4, 1998
  - El Reno, Oklahoma, May 31, 2013

- Anticyclonic tornadoes rotate clockwise (in NHS)
Balance at Different Scales: Domestic

Down the Plughole

Ro ≈ 100,000

Cyclostrophic Balance
Balance at Different Scales: Domestic

Down the Plughole

\[ \text{Ro} \approx 100,000 \]

Cyclostrophic Balance

Coriolis Effect Completely Irrelevant

... unless you believe Homer Simpson
Review of Dynamical Balance

When the forces acting on a parcel sum to zero, a balance is achieved.

With balance, there is **steady flow**.

- Hydrostatic Balance
- Geostrophic Balance
- Gradient Balance
- Cyclostrophic Balance
Outline

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Richardson’s Forecast
Newtonian mechanics assumes the existence of an absolute, unaccelerated frame of reference.

Newton’s laws are **covariant in all inertial frames**.

They keep the same mathematical form under Galilean transformations.

They are **not covariant in accelerating frames**; there are additional terms.
Newtonian Mechanics

The second law of motion in vector form is

\[ \frac{dp}{dt} = F \]

This equation is valid in all inertial frames.
Newtonian Mechanics

The second law of motion in vector form is

$$\frac{d\mathbf{p}}{dt} = \mathbf{F}$$

This equation is valid in all inertial frames.

However, the component form of the equation,

$$\frac{dp_i}{dt} = F_i$$

is true only for cartesian coordinates.
Newtonian Mechanics

In cartesian coordinates in two dimensions:

\[ \frac{d^2 x}{dt^2} = \frac{F_x}{m} \quad \frac{d^2 y}{dt^2} = \frac{F_y}{m} \]
Newtonian Mechanics

In cartesian coordinates in two dimensions:

\[
\frac{d^2 x}{dt^2} = \frac{F_x}{m} \quad \quad \frac{d^2 y}{dt^2} = \frac{F_y}{m}
\]

In polar coordinates \((r, \phi)\) additional terms appear:

\[
\frac{d^2 r}{dt^2} - r \left( \frac{d\phi}{dt} \right)^2 = \frac{F_r}{m},
\]
\[
r \frac{d^2 \phi}{dt^2} + 2 \frac{dr}{dt} \frac{d\phi}{dt} = \frac{F_\phi}{m}
\]

The equations are **not covariant**.
Velocity in Rotating Frame

A point $x'$ fixed in a rotating frame

$$v = \Omega \times x'$$
Velocity in Rotating Frame

A point $x'$ fixed in a rotating frame

$$v = \Omega \times x'$$

The vector product is the root of the difficulty in understanding the Coriolis effect.
Velocity in Rotating Frame

A point $x'$ fixed in a rotating frame

$$v = \Omega \times x'$$

The vector product is the root of the difficulty in understanding the Coriolis effect.

For a particle with velocity $v'$ in the rotating frame,

$$v = v' + \Omega \times x'.$$

We just add the two contributions to velocity.
O’Brien’s Equation

- Let $A$ be a vector in an inertial frame
- $A'$ the same vector in a frame with rotation $\Omega$.

The rates of change are related:

$$\frac{dA}{dt} = \frac{dA'}{dt} + \Omega \times A'$$

Matthew O’Brien (1814-1855)
O’Brien’s Equation

- Let $A$ be a vector in an inertial frame
- $A'$ the same vector in a frame with rotation $\Omega$.

The rates of change are related:

$$\frac{dA}{dt} = \frac{dA'}{dt} + \Omega \times A'$$

This expression is fundamental. It was first expressed in vector form by Matthew O’Brien.

I will call it O’Brien’s equation.

Applying O’Brien’s equation to the position vectors, 

\[
\frac{dx}{dt} = \frac{dx'}{dt} + \Omega \times x',
\]

or

\[
v = v' + \Omega \times x'.
\]
Applying O’Brien’s equation to the position vectors,

\[ \frac{dx}{dt} = \frac{dx'}{dt} + \Omega \times x', \]

or

\[ v = v' + \Omega \times x'. \]

Now applying the relationship again

\[ \frac{dv}{dt} = \frac{dv'}{dt} + 2\Omega \times v' + \Omega \times (\Omega \times x') + \dot{\Omega} \times x'. \]

The acceleration has three additional terms:

- The Coriolis acceleration $2\Omega \times v'$
- The centrifugal acceleration $\Omega \times (\Omega \times x')$
- The Euler term $\dot{\Omega} \times x'$. 
Applying O’Brien’s equation to the position vectors,

\[ \frac{d\mathbf{x}}{dt} = \frac{d\mathbf{x}'}{dt} + \Omega \times \mathbf{x}', \]

or

\[ \mathbf{v} = \mathbf{v}' + \Omega \times \mathbf{x}'. \]

Now applying the relationship again

\[ \frac{d\mathbf{v}}{dt} = \frac{d\mathbf{v}'}{dt} + \frac{2\Omega \times \mathbf{v}'}{\text{COR}} + \Omega \times (\Omega \times \mathbf{x}') + \dot{\Omega} \times \mathbf{x}' \]

The acceleration has three additional terms:

- The Coriolis acceleration \(2\Omega \times \mathbf{v}'\)
- The centrifugal acceleration \(\Omega \times (\Omega \times \mathbf{x}')\)
- The Euler term \(\dot{\Omega} \times \mathbf{x}'.\)
Assume $\Omega$ constant ($\dot{\Omega} = 0$) and drop the Euler term.

Newton’s equation may then be written

$$m \frac{dv'}{dt} = F' - 2m \Omega \times v' - m \Omega \times (\Omega \times x')$$

where $F'$ is the physical force in the rotating frame.

The two additional terms now appear as forces.
Covariant form of Newton’s equations

We can express Newton’s equations so that they are covariant under rotations.

We define a new time derivative

\[ \frac{DA}{Dt} \equiv \frac{dA}{dt} + \Omega \times A \]
Covariant form of Newton’s equations

We can express Newton’s equations so that they are covariant under rotations.

We define a new time derivative

\[ \frac{DA}{Dt} \equiv \frac{dA}{dt} + \Omega \times A \]

We then write the equation of motion as

\[ p = m \frac{Dx}{Dt} \quad \frac{Dp}{Dt} = F. \]

These equations keep the same mathematical form under all rotational transformations.
Lagrange’s Equations

We define the Lagrangian:

\[ L = \begin{bmatrix} \text{Kinetic Energy} \\ \text{Energy} \end{bmatrix} - \begin{bmatrix} \text{Potential Energy} \end{bmatrix} \]
Lagrange’s Equations

We define the Lagrangian:

\[ L = \begin{bmatrix} \text{Kinetic Energy} \\ \text{Potential Energy} \end{bmatrix} \]

Then Lagrange’s equation of motion are

\[ \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_\rho} = \frac{\partial L}{\partial q_\rho} \]

These equations are in covariant form: They are valid in all frames of reference.
The Principle of Relativity

The equations expressing the laws of physics have the same form in all admissible frames of reference.

Can we use the Principle of Relativity to obtain the Coriolis terms?

Yes!

Warning: Not quite as easy as A, B, C!
The Principle of Relativity

The equations expressing the laws of physics have the same form in all admissible frames of reference.

The Coriolis effect arises through rotation of the reference frame.

Can we use the *Principle of Relativity* to obtain the Coriolis terms?
The Principle of Relativity

The equations expressing the laws of physics have the same form in all admissible frames of reference.

The Coriolis effect arises through rotation of the reference frame.

Can we use the *Principle of Relativity* to obtain the Coriolis terms?

Yes!

**Warning:** Not quite as easy as A, B, C!
Tensorial Formulation of Equations

Three very recent papers in the *Quarterly Journal of the Royal Meteorological Society*:


Fundamental equations in tensor form:

$$T_{\mu\nu} = -\rho h^{\mu\nu} \Phi_{,\nu}$$

where the mass-momentum-stress tensor is

$$T^{\mu\nu} = \rho u^\mu u^\nu + h^{\mu\nu} p + \sigma^{\mu\nu}$$
The General Geodesic Equation

In an inertial frame with cartesian coordinates,

\[ ds^2 = dx^2 + dy^2 = g_{\mu\nu}dx^\mu dx^\nu \]

The line element is invariant.
The General Geodesic Equation

In an inertial frame with cartesian coordinates,

$$ds^2 = dx^2 + dy^2 = g_{\mu\nu}dx^\mu dx^\nu$$

The line element is invariant.

The rotating coordinates $\(X, Y)\$ are

$$X = \cos \Omega t x + \sin \Omega t y$$
$$Y = -\sin \Omega t x + \cos \Omega t y$$

In the rotating frame

$$ds^2 = dX^2 + dY^2 - 2\Omega dx dT + 2\Omega dX dT + \Omega^2 (X^2 + Y^2) dT^2$$
We write this as

\[ ds^2 = g'_{\mu\nu} dX^\mu dX^\nu \]

where the metric tensor is

\[
\begin{pmatrix}
1 & 0 & -\Omega Y \\
0 & 1 & \Omega X \\
-\Omega Y & \Omega X & \Omega^2 (X^2 + Y^2)
\end{pmatrix}
\]

Note that \( g'_{\mu\nu} \) is singular: inverse \( g'^{\mu\nu} \) does not exist.
The geodesic equation is:

\[
\frac{d}{dt} \left( g'_{\sigma \nu} \frac{dX^\nu}{dt} \right) - \frac{1}{2} \frac{\partial g'_{\mu \nu}}{\partial X^\sigma} \frac{dX^\mu}{dt} \frac{dX^\nu}{dt} = 0
\]
The geodesic equation is:

$$\frac{d}{dt} \left( g'_{\sigma\nu} \frac{dX^\nu}{dt} \right) - \frac{1}{2} \frac{\partial g'_{\mu\nu}}{\partial X^\sigma} \frac{dX^\mu}{dt} \frac{dX^\nu}{dt} = 0$$

Writing this explicitly, we get

$$\ddot{X} - 2\Omega \dot{Y} - \Omega^2 X = 0$$
$$\ddot{Y} + 2\Omega \dot{X} - \Omega^2 Y = 0$$

These are the equations derived already by more conventional means.
Why use the Tensor Formulation?

- Tensor equations are **covariant**: they preserve their form in all coordinate systems;
- **Transformations** are handled systematically;
- **Approximations** are derived rigourously;
- **Conservation** properties are preserved.
An alternative equation for the geodesics is
\[
\frac{d^2 X^\rho}{ds^2} + \Gamma^{\rho}_{\mu\nu} \frac{dX^\mu}{ds} \frac{dX^\nu}{ds} = 0
\]

The Christoffel symbols of the first kind are
\[
[\sigma|\mu\nu] = \Gamma_{\sigma|\mu\nu} = \frac{1}{2} \left[ \frac{\partial g'_{\sigma\nu}}{\partial X^\mu} + \frac{\partial g'_{\mu\sigma}}{\partial X^\nu} - \frac{\partial g'_{\mu\nu}}{\partial X^\sigma} \right]
\]

There are ten non-vanishing symbols:

\[
[1, 33] = -\Omega^2 X \quad [2, 33] = -\Omega^2 Y \\
[1, 23] = [1, 32] = -\Omega \quad [2, 13] = [2, 31] = +\Omega \\
[3, 13] = [3, 31] = \Omega^2 X \quad [3, 23] = [3, 32] = \Omega^2 Y
\]

where the variables are \((X^1, X^2, X^3) = (X, Y, T)\).
The Christoffel symbols of the second kind are

\[ \Gamma^\rho_{\mu\nu} = g^{\rho\sigma} \Gamma^\sigma_{\sigma\mu\nu} \]

To regularise \( g_{\mu\nu} \), we write the metric as

\[ ds^2 = dx^2 + dy^2 + \epsilon dt^2 \]

and consider the limiting case \( \epsilon \to 0 \).

The \( \Gamma^\rho_{\mu\nu} \) are independent of \( \epsilon \). The non-zero ones are

\[ \Gamma^1_{23} = \Gamma^1_{32} = -\Omega \quad \Gamma^2_{13} = \Gamma^2_{31} = +\Omega \]
\[ \Gamma^1_{33} = -\Omega^2 X \quad \Gamma^2_{33} = -\Omega^2 Y \]

These yield the same equations as obtained above.

The curvature tensor vanishes: \( R^\rho_{\sigma\mu\nu} \equiv 0 \).
Galileo on Mathematics

[The universe] ... is written in the language of mathematics ... without which it is ... impossible to understand a single word of it.

Without this understanding, one is wandering around in a dark labyrinth.
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Richardson’s Forecast
Lewis Fry Richardson, 1881–1953.

During WWI, Richardson computed the pressure change at a single point. It took him two years!

His ‘forecast’ was a catastrophic failure:

\[ \Delta p = 145 \text{ hPa in 6 hrs} \]

But Richardson’s method was scientifically sound.
Tendency of a Noisy Signal
Initialization of Richardson’s Forecast

Richardson’s Forecast has been repeated.

The atmospheric observations for 20 May, 1910 were recovered from original sources.
Initialization of Richardson’s Forecast

Richardson’s Forecast has been repeated.

The atmospheric observations for 20 May, 1910 were recovered from original sources.

- **ORIGINAL:** \( \frac{dp_s}{dt} = +145 \text{ hPa/6 h} \)
- **INITIALIZED:** \( \frac{dp_s}{dt} = -0.9 \text{ hPa/6 h} \)

Observations: The barometer was steady!
Initialization of Richardson’s Forecast

Richardson’s Forecast has been repeated.

The atmospheric observations for 20 May, 1910 were recovered from original sources.

- **ORIGINAL:** \( \frac{dp_s}{dt} = +145 \text{ hPa/6 h} \)

- **INITIALIZED:** \( \frac{dp_s}{dt} = -0.9 \text{ hPa/6 h} \)

Observations: The barometer was steady!

**BALANCED INITIAL DATA IS ESSENTIAL!**
Thank you