Techniques of Initialization

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The story of Lewis Fry Richardson's remarkable numerical forecast, made about seventy years ago, is well known and is recounted in a recent biography by Ashford. Richardson forecast the change in surface pressure and other parameters at a point in central Europe, using the mathematical equations and observations valid at 0700 GMT on 20 May, 1910. He described his methods and results in his book "Weather Prediction by Numerical Process". His results implied a change in surface pressure of 145 millibars in 6 hours. As Sir Napier Shaw remarked, "the wildest guess ... would not have been wider of the mark ..." Yet, Richardson claimed that his forecast was "... a fairly correct deduction from a somewhat unnatural initial distribution"; he ascribed the unrealistic value of pressure tendency to errors in the observed winds, leading to a spuriously large value of the calculated divergence. This large tendency reflects the fact that the atmosphere can support motions with a great range of timescales.

Multiple Timescales

As a simple example of a system with multiple timescales, consider a tide-gauge in open water, measuring the depth as a function of time. The gauge will measure the slow changes between low and high water, which have periods of about twelve hours. It will also register the changes due to sea and swell, which have periods of less than a minute, but whose amplitude may equal or exceed the tidal range. Clearly, the instantaneous readings of the gauge cannot be used for tidal analysis until the 'noise' due to the short-period waves has been removed in some way. Furthermore, the rate-of-change observed at an instant is no guide to the long-term movement of the tide. The problem arises because the water level is governed by processes with widely differing time-scales.

The atmosphere also has motions with different timescales, and the "primitive equations" used for numerical weather prediction have solutions in two different categories. The solutions of meteorological significance are low frequency motions with phase speeds of the order of ten metres per second, and periods of a few days. These 'rotational' motions are generally close to geostrophic balance, and are quasi-nondivergent. There are also very fast gravity inertia wave solutions, with phase speeds of hundreds of metres per second; these high frequency waves do not interact strongly with the slow rotational motions, are generally of small amplitude, and may be regarded as noise.

The Problem of Initialization

A subtle and delicate state of balance exists in the atmosphere between the wind and pressure fields, ensuring that the fast gravity waves have much smaller amplitude than the slow rotational part of the flow. However, when we make a forecast starting from analysed mass and wind fields, large high frequency oscillations occur; these are due to slight imbalances in the initial data resulting from observational and other errors and from imperfections in the forecast model. (It was the presence of unrealistically large gravity wave components in the initial data which led to Richardson's "glaring error" in his forecast of pressure tendency).

The solid curve in figure 1 is a trace of the surface pressure at a particular gridpoint during the first 24 hours of a forecast made with the ECMWF model. It is a vivid illustration of the problems caused by high frequency noise. In particular, the initial pressure tendency calculated by the model is utterly unrealistic, just as Richardson's calculated tendency was.

One of the long-standing problems in numerical weather prediction has been to overcome the problems associated with high frequency motions. This is achieved by the process known as "initialization", the principal aim of which is to define the initial fields in such a way that the gravity inertia waves remain small throughout the forecast. If the fields are not initialized the spurious oscillations which occur in the forecast can lead to various problems. In particular, new observations are checked for accuracy against a short-range forecast. If this forecast is noisy, good observations may be rejected or erroneous ones accepted. Thus, initialization is essential for satisfactory data assimilation. Another problem occurs with precipitation forecasting. A noisy forecast has unrealistically large vertical velocity. This interacts with the humidity field to give hopelessly inaccurate rainfall patterns. To avoid this, we must control the gravity wave oscillations.

The Filtered Equations

The first computer forecast was made in 1950 by Charney, Fjortoft and Von Neumann. In order to avoid Richardson's error, they modified the prediction equations in such a way as to eliminate the high frequency solutions. This process is known as "filtering". The basic filtered system is the set of "quasi-geostrophic" equations. These equations were used in operational forecasting for a number of years. However, they involve approximations which are not always valid, and this can result in poor forecasts at times.

A more accurate filtering of the primitive equations leads to the "balance equations". This system is more complicated than the quasi-geostrophic system, and more accurate, but is still free from high frequency solutions. In 1960 Charney read a paper at the Tokyo WMO/IUGG symposium entitled "Integration of the Primitive and Balance Equations". It would seem that he was unsure at that time which way the ball would bounce.

In the meantime, Hinkelmann of the German Weather Service had successfully integrated the primitive equations, using a very short timestep. He estimated the initial winds using the geostrophic relation. Forecasts made with the primitive equations were soon shown to be clearly superior to those using the quasi-geostrophic system. The pendulum of interest swung towards the primitive equations and filtered systems fell out of favour. I don't think that the full balance equations have ever been used for operational forecasting.

Early Initialization Methods

Hinkelmann had avoided the inaccuracies of filtered systems by returning to the primitive equations – but he had not solved the noise problem. This problem had now to be squarely faced, and methods of specifying appropriate initial data had to be devised. Geostrophic initial winds are inadequate, as was shown already by Richardson in the "introductory example" in Chapter 2 of his book. Charney proposed that a better estimate of the initial wind field could be obtained by using the nonlinear balance equation. This equation – part of the balance system – is a diagnostic relationship between the pressure and wind fields. It implies that the wind is nondivergent. It was later argued by Phillips that a further improvement would result if the divergence of the initial field were set equal to that implied by quasi-geostrophic theory. Each of these steps represented some progress, but the noise problem still remained essentially unsolved.

Another approach, called dynamic initialization, uses the forecast model itself to define the initial fields. The dissipative processes in the model can damp out high frequency noise as the forecast procedes. We integrate the model first forward and then backward in time, keeping the dissipation active all the time. We repeat this forward-backward cycle several times until we finally obtain fields, valid at the initial time, from which the high frequency components have been damped out. The forecast starting from these fields is noise-free. However, the procedure is expensive in computer time, and damps the meteorologically significant motions as well as the gravity waves, so it is no longer popular.

Normal Mode Methods

We consider now a totally different approach. In certain circumstances, the solution of the model equations can be separated, by a process of spectral analysis, into two sets of components or "linear normal modes". The slow rotational components, often called Rossby modes, are the ones we are interested in, whereas the high frequency gravity modes constitute the noise which we wish to banish, or at least control. Let us suppose that the initial fields are separated into slow and fast parts, and that the latter are removed so as to leave only the Rossby waves. It might be hoped that this process of "linear normal mode initialization" would ensure a noise-free forecast. However, the results of the technique are disappointing: the noise is reduced initially, but soon reappears; the forecasting equations are nonlinear, and the slow components interact nonlinearly in such a way as to generate gravity waves. The problem of noise remains.

A satisfactory solution to the problem was finally found, about ten years ago, independently by Machenhauer in Copenhagen and Baer in Michigan. Machenhauer reasoned as follows: the gravity waves are small to begin with, but they grow rapidly; to control this growth, we set their initial rate-of-change to zero, in the hope that they will remain small throughout the forecast. Machenhauer's idea worked like a charm: the forecast, starting from initial fields modified in this way, is very smooth and the spurious gravity wave oscillations are completely removed. The method takes account of the nonlinear nature of the equations, and is referred to as "nonlinear normal mode initialization" (NNMI).

Temperton and Williamson have used the NNMI method to initialize the data for the forecast model of the European Centre for Medium range Weather Forecasts. In figure 1 we see a trace of surface pressure for two forecasts starting from the same initial time. The solid curve is for a forecast from uninitialized data; the high frequency gravity wave noise is all too apparent. The dashed curve shows the surface pressure evolving from initial fields which have been balanced using NNMI. The contrast with the uninitialized forecast is spectacular: the spurious oscillations no longer appear, and the calculated tendencies are realistic. Despite some drawbacks, NNMI is today the most popular method of initialization, and is used in many forecast centres. The method is so successful that we can say that the long-standing problem of how to define initial fields has now been solved.

Some Recent Work

The nonlinear normal mode method is remarkably successful. However, there are circumstances in which it is difficult to apply. This is particularly the case for limited area models (LAMs): for these models the artificial boundaries introduce mathematical complexities such that the normal modes are, in general, unknown. Much recent work has been devoted to the problem of initialization for these models, and constitutes variations on the normal mode theme. Bourke and McGregor, working in Melbourne, have derived balance condidions in an intuitive way: they set to zero the initial tendencies of divergence and departure from linear balance; these conditions differ from the classical balance equation in the important respect that they provide an appropriate divergence field. Their method has recently been rigourously justified and shown to be an implicit form of the normal mode method.

Another approach, proposed by this author, uses the properties of Laplace transforms to isolate the slowly varying part of the solution. The initial fields are transformed, and a modified inverse Laplace transform is then applied, which selectively filters out the high frequency components of the flow. This method is also closely related to the normal mode method, and gives very similar results, but it may be applied without knowledge of the model normal modes.

Finally, I mention the bounded derivative method. This method is based on the fact that, if a solution of the equations is to vary slowly, a number of its time derivatives must remain small. From this observation, we can derive constraints which must be satisfied by the initial data, and applying these constraints we get the balanced initial fields. The bounded derivative method has been applied to the initialization of LAMs, and the results are very similar to those using other methods. The method differs in an important way from NNMI: there is a degree of arbitrariness in the distribution of the changes between the mass and wind fields. This is a strength in that it allows us freedom to choose, but a weakness in that we have no idea of the optimal choice.

The Future

Richardson used the primitive equations for his forecast. Charney et al. changed over to a filtered system, but a return was later made to the primitive equations. Will the pendulum swing over once again? Using the ideas of normal mode initialization, it is possible to derive a set of equations (I will call them the "slow equations"), which forecast the low frequency motions while diagnosing appropriate gravity wave components. These equations are in physical space, obviating the need for costly transformations. The slow equations are similar to the balance system, but not identical to it. They would appear to provide us with a means of selectively forecasting those components of the atmospheric motion which are important, while avoiding the troubles associated with the gravity waves. They would also seem to provide a suitable means for the assimilation of observational data during a forecast, as they can absorb inserted data without suffering high frequency shocks. Only time will tell whether these, or other similar equations, have a role to play in the future of numerical weather prediction.

Recently, forecast models with very fine grids have been designed, which represent local "mesoscale" phenomena. Here some components of the flow, which might be treated as noise on the larger scale, are important for the local dynamics; the separation into slow and fast components is no longer clear. Initialization for these mesoscale models is an unsolved problem, and it seems that a radically new approach is needed.

Richardson's Forecast Factory

Richardson had a dream that numerical weather prediction would someday become a practical proposition, a dream which has now come true. He also "played with a fantasy" about a forecast factory for carrying out this dream. He estimated that 64,000 people would be needed to keep pace with the weather. This enormous number, together with his disappointing results, ensured that the forecast factory was never built.

I will "play with a different fantasy". Let us suppose that Richardson had used initial data which was in balance, or that he had been able to initialize it in some way. He would then have obtained values for the tendencies which were accurate, or at least reasonable (the tendencies depend only on the initial fields, and are quite independent of any stability criteria). I suggest that the response of the meteorological community would have been overwhelmingly favourable, and that the subsequent history of numerical weather prediction would have been dramatically different to what has actually happened. It is amusing to imagine that Richardson's fantasy, as well as his dream, might have been realised, and that a forecast factory (in some form) might have been built.

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References

Ashford, O.M. (1985): Prophet or professor? The life and work of Lewis Fry Richardson. Adam Hilger, Bristol and Boston, 304 pp.

Baer, F. (1977): Adjustment of initial conditions required to supress gravity oscillations in nonlinear flows. Beitr. Phys. Atmos., 50, 350-366.

Bourke, W. and J.L. McGregor (1983): A nonlinear vertical normal mode initialization scheme for a limited area prediction model. Mon. Weather Rev.,111, 2285-2297.

Charney, J.G. (1955): The use of the primitive equations of motion in numerical prediction. Tellus, 7, 22-26.

Charney, J.G., R Fjortoft and J. von Neumann (1950): Numerical integration of the barotropic vorticity equation. Tellus, 2, 237-254.

Hinkelmann, K. (1959): Ein numerisches experiment mit den primitiven gleichungen. The Atmosphere and the Sea in Motion, Rossby Memorial Volume, Rockefeller Institute Press, 486-500.

Lynch, Peter (1985): Initialization using Laplace Transforms. Q. J. Roy. Met. Soc., 111, 243-258.

Machenhauer, B. (1977): On the dynamics of gravity oscillations in a shallow water model, with applications to normal mode initialization. Beitr. Phys. Atmos., 50, 253-271.

Phillips, N.A. (1960): On the problem of initial data for the primitive equations. Tellus, 12, 121-126.

Richardson, L.F. (1922): Weather prediction by numerical process. Cambridge University Press, Cambridge, reprinted Dover, New York, 1965, 236 pp.

Temperton, C. and D. Williamson (1979): Normal mode initialization for a multilevel gridpoint model. Tech. Rep. No. 11, European Centre for Medium range Weather Forecasts, Reading, England.