Determining the Time and Date from Shadows in a Photograph

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Abstract. A photograph of Kingstown (Dun Laoghaire) taken from the East Pier includes some children. Their shadows in the morning Sun enable us to determine the time of day when the photograph was taken and also permit us to estimate the season. The time of day (local solar time) determined by the azimuthal angle of the shadows can be estimated with confidence. The season is determined by the azimuth and elevation together, and is subject to greater uncertainty. Moreover, since any combination of these angles can occur twice each year, we must use other evidence to decide which of two dates is more likely.

The conclusion of the analysis is that the photograph was taken at approximately 08:20 Local Solar Time. The most probable times of year are mid-April or late August, approximately nine weeks before and nine weeks after the Summer solstice. The ample foliage on the trees visible in the photograph might be used as evidence in support of an August date.

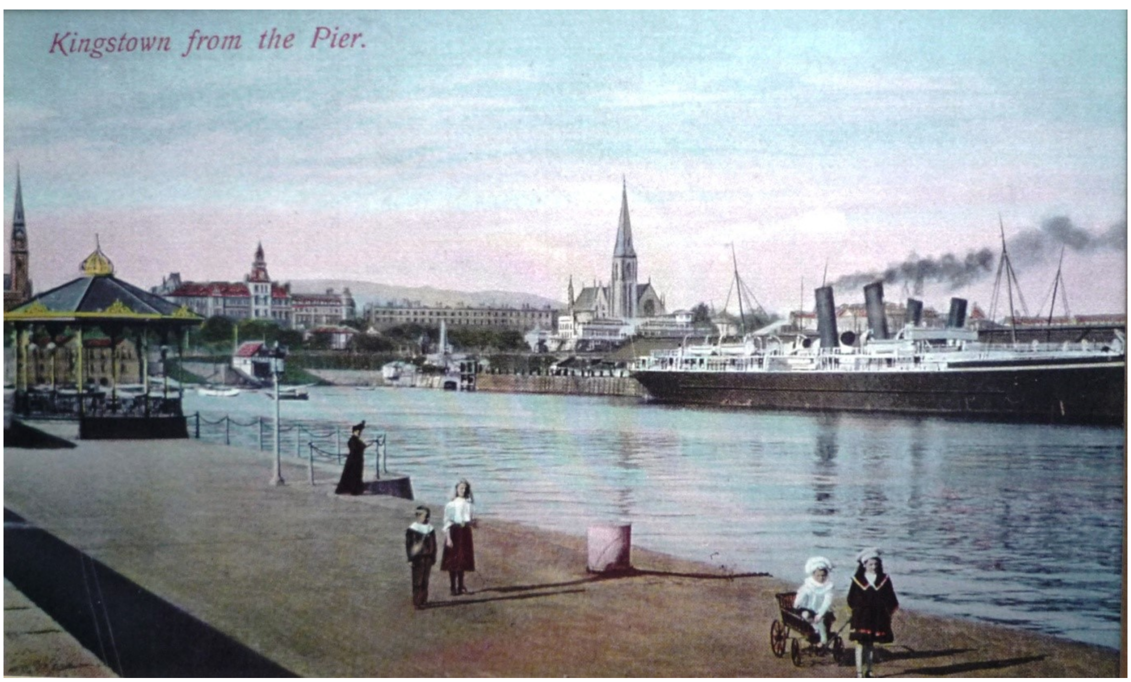

Figure 1: Postcard photograph of Kingstown from the East Pier, taken in about 1905.

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The Post Card

The photograph of Kingstown taken in about 1905 (Fig. 1) includes a wealth of interesting details. In the background are some prominent buildings: from left to right are the spire of the Mariners' Church, which now houses the National Maritime Museum of Ireland, the tower of the Royal Marine Hotel, the spire of Saint Michael's Church and — obscure behind the funnels of the ships — the tower of the Town Hall. Other details include the bandstand on the East Pier, the old RNLI Lifeboat House (now replaced by a newer building) and the George IV Obelisk, in front of which we see the entrance to a tunnel under Carlisle Pier.

Two ships are docked alongside Carlisle Pier. They are two of four sister ships of the City of Dublin Steam Packet Company, named after the four provinces of Ireland. Unfortunately, the name of the closest ship is near the stem, outside the range of the picture, so it cannot be identified with certainty. The ship appears to be building up steam ready for departure. The scheduled sailing time was 9:00 AM and, since the weather was fair and severe penalties were imposed for late departures, a prompt departure would be expected.

In the foreground, there are five figures on the pier. A lady stands near a lamp-post (this is now gone). A boy and girl stand in the centre of the image, close to a mooring bollard (still *in situ*) from which a chain hangs over the edge of the pier. There are two more children closer to the camera. All the children cast clear shadows in the morning Sun, and the lengths and angles of these are the key to deducing the time and date of the photo. The children appear to be standing rather than walking; presumably this was deliberate, allowing the photographer to use a long exposure time; the photo is clearly choreographed.

Location of the Camera

The position from which the photograph was taken can be determined accurately from vertical alignments. From the segment of the upper pier deck in the lower left corner of the picture, it is clear that the photographer was standing on the upper deck. There are several vertical lines that can be used to identify the camera location. We focus on two clear alignments:

- The vertical line through the spire of Saint Michael's Church passes through the lower right point of the mooring bollard on the lower deck of the pier.
- The spire of the Mariners' Church aligns with the left-most pillar of the bandstand.

Using these alignments, it is quite easy to determine the precise location of the photographer. We conclude from a site inspection that the photo was taken from a point where there is a groove in the upper deck that passes under the northernmost of two benches, at a point about 2 metres in from the edge of the upper deck. A photo taken from this point corresponded closely to the original photograph. It is not obvious from the picture, but this point is well beyond the perpendicular to the end of Carlisle Pier.

Pencil of Lines on the Pier

The following three lines are parallel:

1. The edge of the upper deck, visible in the lower left corner of the image.
2. The shadow of the upper deck, visible on the lower deck.
3. The outer edge of the lower deck.

Due to perspective, all these lines meet at a vanishing-point off to the left of the image. All the lines through this vanishing point form a pencil. Thus, other lines parallel to the edge of the pier, and having the same direction as the pier itself, can be determined. The direction of the pier was established using an OSI map: the compass bearing is 30 degrees East of True North.

The boy in the centre of the photograph — let us call him Reggie — was chosen for closer examination, as his shadow was defined most clearly in the image. A line parallel to the pier was drawn through the feet of this child. This line, of known direction, was used as a reference from which the angle of the boy's shadow was measured.

Measurement of the Angles

Fig. 2 is a schematic diagram showing the principal lines and angles. Point **O** is at Reggie's feet, his head is at **V** his shadow is **OS**. A Sun beam passes along the line **VS**. The center-line of the pier is $\mathbf{C'C}$. The angle from shadow to centre-line (in the horizontal plane) is **a** and the angle (in the vertical plane) between the Sun beam and shadow is **e**.

Angles measured on the photo must be adjusted for the camera viewing angle. Accurate representation of an angle in the horizontal plane would require the camera to be vertically above it. In fact, the camera is at a low viewing angle. The upper deck is about 2 metres above the lower one. If the camera was mounted at eye level on a tripod, its height above the lower deck would have been about 3.5 metres. The distance from camera to Reggie was measured to be 20 metres. This implies a viewing angle of $\kappa = 10°$. We denote the complement of this angle by $\gamma = 90° - \kappa$.

The line $\mathbf{H'H}$ was drawn perpendicular to the vertical **OV**. Lengths parallel to $\mathbf{H'H}$ are represented accurately in the image, whereas distances perpendicular to it are foreshortened by a factor $\cos\gamma = \sin\kappa$.

Angles can be measured using a protractor, but it was found that better results were obtained by measuring lengths and computing tangents. Assuming $\tan\psi$ is the tangent of an angle having one arm parallel to $\mathbf{H'H}$, the adjustment for foreshortening is

$$\tan\Psi = \frac{\tan\psi}{\cos\gamma}\,. \qquad (1)$$

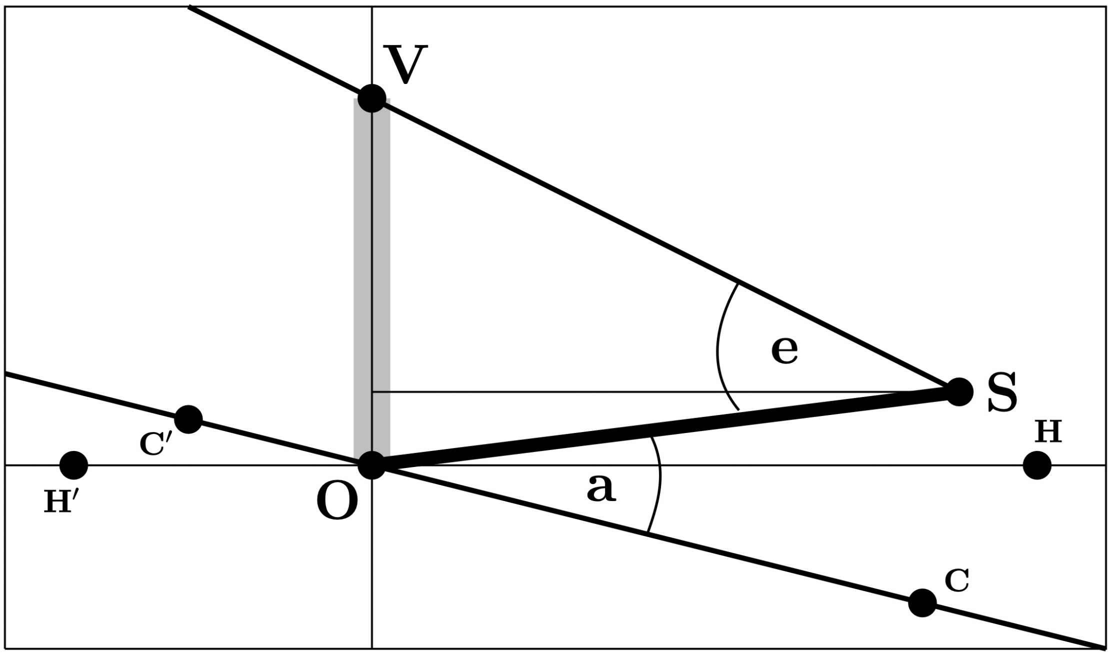

Figure 2: Point **O** is at Reggie's feet, his head is at **V** his shadow is **OS**. A Sun beam passes along **VS**. The center-line of the pier is **C′C**. The line **H′H** is perpendicular to the vertical **OV**. The angle from shadow to centre-line (in the horizontal plane) is **a** and the angle of the Sun beam (in the vertical plane) is **e**.

Then Ψ, which is greater in magnitude than ψ, is the true angle. The angle **a** was expressed as the sum of two angles relative to the horizontal line **H′H**. These components were adjusted using (1) to give the true angle A. The numerical value obtained as $A = 98.0°$, so the bearing of the shadow was $30° - 98° = -68°$, or $282°$. The Sun was directly opposite to this, so its azimuth was $a = 112°$.

A similar adjustment was made for the angle **e** but, since it is in the vertical plane, the correction factor $\sin\gamma = \cos\kappa$ was used in place of $\cos\gamma$, to give an angle E. The numerical value obtained for E was $30.9°$. Thus, the elevation of the Sun was approximately $e = 31°$.

There are many sources of error, difficult to quantify. Assuming that the uncertainty is of order of magnitude $10°$, we estimate from analysis of the photograph that the position of the Sun, given by the azimuth and elevation, is $(a, e) = (112 \pm 10°, 31 \pm 10°)$.

Determination of the Time of the Photo

The local solar time when the photo was taken can be determined once the azimuth and elevation of the Sun have been found. We calculate the solar hour angle h from one or other of two formulas, one using the elevation e and one using the azimuth a. The first is

$$\cos h = \frac{\sin e - \sin\varphi \sin\delta}{\cos\varphi \cos\delta}$$

and the second is

$$\sin h = \frac{\cos e \sin a}{\cos\delta}\,. \tag{2}$$

The angle δ is the *declination* (see below). The two formulas were found to give very similar results. With h in degrees, the local solar time is then found:

$$t = 12 - \frac{h}{15}\ \text{hours}$$

(15° corresponds to 1 hour). Using the values $a = 112°$, $e = 31°$, $\varphi = 53.5°$ and $\delta = 13°$, both of these formula give the time as $t = 8.35$ hours or, in more familiar form, 8:21 LST (local solar time). Solar time in Dun Laoghaire/Kingstown is very close to Dunsink Time, or Dublin Mean Time (DMT), the legal time for Ireland during this period. DMT was about 25 minutes behind Greenwich Mean Time, now usually denoted UTC. We conclude that the photograph was taken at approximately 08:45 UTC. It is also possible to make a correction for the variation of solar time from mean time, using the relationship called the equation of time. However, this is never more than about fifteen minutes, so no adjustment was made.

Possible Positions of the Sun

The position of the Sun in the sky depends on where we are and on the time of day. Due to the Earth's rotation, the Sun appears to cross the celestial sphere each day along a path called the *ecliptic*. The observer's position on Earth is given by the geographic latitude and longitude. The path of the Sun depends on the latitude while the time when the Sun crosses the local meridian is determined by the longitude.

The elevation e is the angle of a line to the Sun above the horizontal plane. The azimuth a is (essentially) the compass bearing of the Sun from the point of the observer. Plotting these angles on a polar diagram, we obtain a plot of the Sun as it moves across the sky.

We consider the daily path of the Sun as observed from a point at latitude 53.5° on three dates, the Summer solstice ($\delta = 23.5°$N), the Winter solstice ($\delta = -23.5°$N) and the Equinoxes ($\delta = 0°$N). The declination δ is the latitude at which the Sun appears vertically overhead at local Noon. We plot the zenith angle (the complement of the elevation) and azimuth for the three cases on a polar diagram (Fig. 3, left panel). The course of the Sun across the sky is shown by the curves.

The light area in the figure illustrates the region of the celestial dome where the Sun may be seen from the latitude of Dun Laoghaire/Kingstown. It also gives the maximum elevation and the azimuth at sunrise and sunset, when the elevation is zero. Between the Winter solstice and Summer solstice, the declination of the sun, δ, increases from $-23.5°$ to $+23.5°$ and the solar elevation at noon, $\theta + \delta = 90° - \varphi + \delta$, increases from 13° to 60°. Between

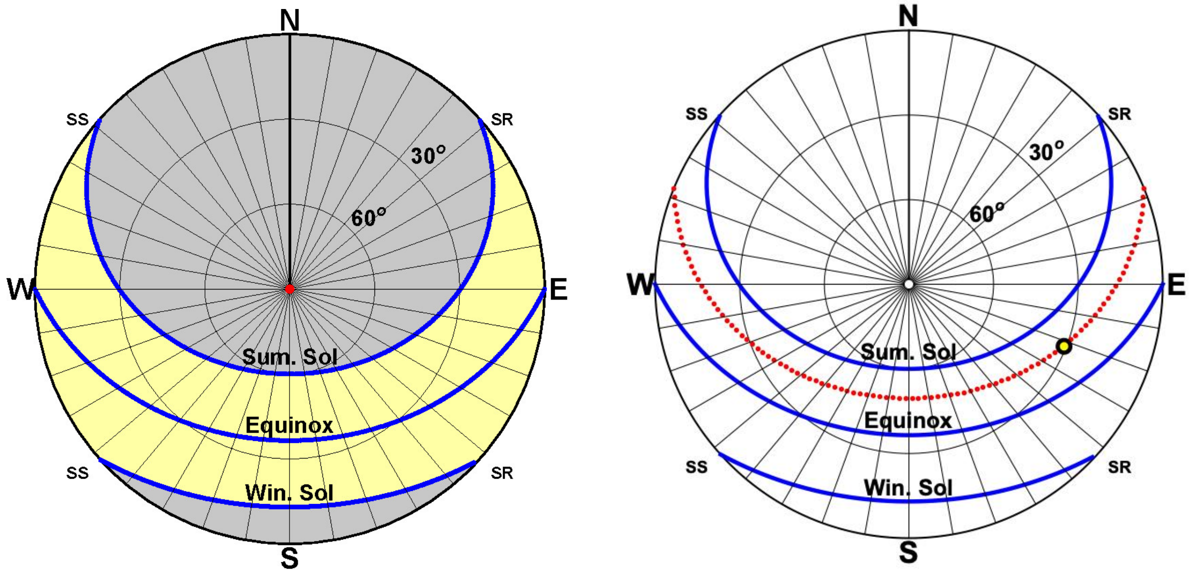

Figure 3: Left: Path of the Sun on the solstices and equinoxes. The light region represents all possible positions of the Sun. Right: Additional (dotted) line is the path for declination $\delta = 13°$.

mid-summer and mid-winter, the midday elevation of the Sun decreases again, from 60° to 13°.

If we know the azimuth and elevation, (a, e), can we deduce the date? This cannot be done unambiguously: for every point within the light region of Fig. 3 (left panel), there are *two* times each year when the azimuth and elevation have values corresponding to that point. In Fig. 4, the annual variation of the declination angle is shown. For a fixed value of δ, as indicated by the horizontal line, there are two days on which the Sun has this declination.

Sun's Path when $\delta = 13°$.

Given the estimate $(a, e) = (112°, 31°)$ we could use trigonometric relationships to deduce the solar declination and, therefrom, the possible dates. However, with so many uncertainties, a simpler "shooting method" was used. We plotted the track for a first-guess value of δ and, based on how close the path passed to (a, e), adjusted the declination repeatedly until the path fitted with the estimated values. This required only one or two iterations. The numerical result was a declination of $\delta = 13°$.

In Fig. 3 (right panel) we add a fourth path to the three already shown in the left panel. The dotted curve is the path for the estimated declination $\delta = 13°$. To give more fine detail, we plot the Sun's course during the morning, as seen from Dun Laoghaire/Kingstown, in Fig. 5. The radial lines are spaced five degrees apart, and we see that the azimuth at sunrise is close to 67°.

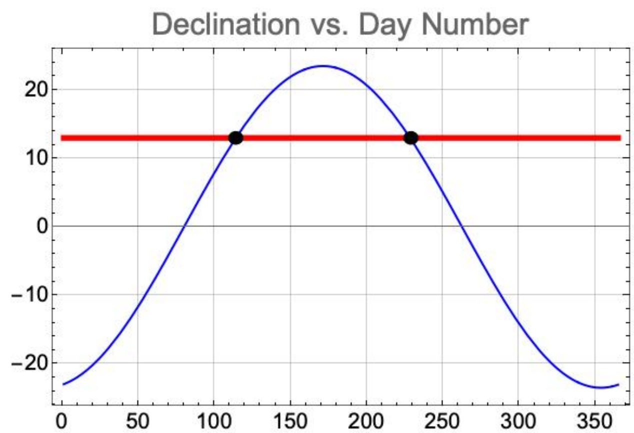

Figure 4: Declination of the Sun as a function of the day number. The red line is for a declination $\delta = 13°$. The days on which this value is found are marked by black spots; they are $d = 114$ and $d = 230$. Midsummer's Day is at $d = 172$.

Deducing the date (or dates)

The next step is to deduce the date from the declination. It is a reasonably good approximation to assume that δ varies sinusoidally over a year, between its minimum of $-23.5°$ at the Winter solstice and its maximum of $23.5°$ at the Summer solstice, passing through $0°$ at the equinoxes. Therefore, we can write

$$\delta = 23.5° \times \sin\left[\left(\frac{d - d_0}{365}\right) 360°\right] \tag{3}$$

where d is the day number in question and $d_0 = 80$ is the day number of the Spring equinox. Thus, given the declination, we can invert this to deduce the day number. We get

$$d = d_0 + \left(\frac{365}{360}\right) \arcsin\left(\frac{\delta}{23.5}\right) .$$

With the exception of the solstices, there are two possible solutions; the 'arcsin' function has two values (see Fig. 4). Using the values $d_0 = 80$ and $\delta = 13°$, we find the value $d = 80 + 34 = 114$ which is April 24. This is 58 days before the Summer solstice. The other solution is 58 days after the solstice, which is August 18. Again, we must acknowledge the uncertainty, estimated to be of the order of 15 days.

Comparing Calculated and Analysed Solar Positions

We have used the angles measured in an image to deduce the position (azimuth and elevation)of the Sun and, from that, the time and date. To verify the results, the process can be reversed: using the time and date, the position of the Sun can be calculated using standard astronomical formulas. This provides a check on the analysis of angles in the postcard image.

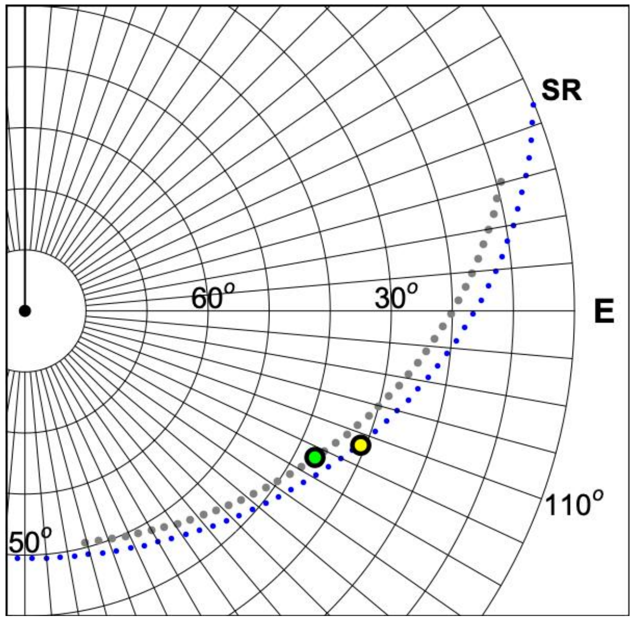

Figure 5: Path of the Sun for a declination $\delta = 13°$. The position estimated from the photograph, $(a, e) = (112°, 31°)$, is shown by a small yellow circle. The green circle gives the position for the reconstructed image.

An online system for solar position, the NOAA Solar Calculator, is available at `https://gml.noaa.gov/grad/solcalc/`. Using this application, and entering the position (53.5°N, 6°W), time (08:45 UTC) and date (21/04/2022), we obtained the following values for declination, azimuth and elevation:

$$\delta = 12.9° \qquad a = 112.3° \qquad e = 31.2°\,,$$

in excellent agreement with the values obtained from analysis of the postcard, $\delta = 13°$, $a = 112°$ and $e = 31°$. However, this is a check only on the solution of the equations (2) and (3) and does not allow for several other sources of error, such as angle adjustments to allow for perspective. We have not undertaken a formal error analysis, but the precision of the estimated time and date are probably of the order of 30 minutes (for the time) and 15 days (for the date).

Replicating the Photograph

To support the above analysis, a photograph was taken from the same location on the East Pier, close to the estimated date and time of day. The image in Fig. 6 was taken at 09:48 IST on 21 April, 2022. Angles were measured as described above and indicated in the schematic diagram Fig. 2. These were adjusted for the camera viewing angle, as already described.

The numerical value obtained for the adjusted angle between shadow and pier centre-line was $A = 93.0°$, so the bearing of the shadow was $30° - 93° = -63°$. The Sun was directly

Figure 6: Photograph of Dun Laoghaire from the East Pier, taken at 09:48 IST, 21 April, 2022.

opposite to this, with azimuth $a = 117°$. This differs by 5° from the analysis of the postcard image, or by about 20 minutes, well within the expected range of error.

The numerical value obtained for the elevation of the Sun was 37.7°. This is 7° greater than the value for the postcard image. The local solar time calculated using a solar declination of 13°, was 8:55 LST (10:20 IST). But since the known time of the photograph was 09:48 IST or 08:23 LST, we can use (2) to calculate the declination. As a check, since the day number d is known (21 April has $d = 111$), the declination can also be calculated from (3). The value obtained was $\delta = 11.95°$, about 1° less than for 24 April.

The higher solar elevation (by 7°) in the photograph taken on a known date (21 April) makes it clear that the postcard image was taken on an earlier date, probably in mid-April, or or in late August.

The path of the Sun (for declination $\delta = 13°$) was shown in Fig. 5. The position estimated from the photograph, $(a, e) = (112°, 31°)$, was shown by a small yellow circle. The green circle gives the position $(a, e) = (117°, 38°)$, estimated using the modern photograph. The solar elevation in this photograph is visibly greater than that in the postcard image, so the date for the postcard must have been farther from the Summer Solstice. Given the magnitude of the discrepancy, we estimate this to be about one week. Therefore, we conclude that the postcard photo was taken in mid-April or late August.

Conclusion

From an examination of the shadows in the postcard in Fig. 1, the position of the Sun was estimated. Further analysis then enabled the determination of the local solar time and the time of year when the photograph was taken. We conclude from this analysis that the local time was about twenty minutes past eight, and the date was in mid-April or late August (about nine weeks before or after the Summer Solstice).

Further evidence is needed to decide which of these dates is more likely. The ample foliage on the trees visible in the photograph might be used as evidence in support of the August date.

References

• Lynch, Peter, 2018: Pedro Nunes and the Retrogression of the Sun. *Bull. Irish Math. Soc.*, **81**, 23–30.

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• Wikipedia articles: (1) Position of the Sun. (2) Solar zenith angle. (3) Solar azimuth angle. (4) Declination.