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A RAPIDLY TRAVELLING PLANETARY-SCALE INSTABILITY IN THE ATMOSPHERE

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Abstract. The hydrodynamic instability of the zonally averaged circulation of the atmosphere is investigated using a primitive equation model on a global domain. The classical baroclinically unstable modes are examined and a new mode of instability is found. This mode has phase-speed greater than the maximum zonal flow velocity (such a solution is impossible within the framework of quasi-geostrophic theory). It draws energy from the mean flow in the troposphere through baroclinic conversion and it penetrates deeply into the middle atmosphere, its growth there being supported by convergence of vertical wave energy flux. With zonal wavenumber one it has a period of two days and an e-folding time of six days. Such instabilities may play an important role in the dynamics of the middle atmosphere.

Introduction

Until recently almost all the studies of baroclinic instability in the atmosphere have been based on the quasi-geostrophic equations, with β-plane geometry. With this system the phase-speeds of instabilities are restricted by the Miles-Howard theorem to lie within a certain semicircle in the complex c-plane (Miles, 1961; Howard, 1961). Furthermore, the real part of the phase-speed must be less than the maximum eastward mean flow velocity $\bar{u}_{max}$. Thus, rapidly travelling instabilities cannot be described by the quasi-geostrophic system. The present note describes some results of an instability study based on the primitive equations on a sphere. The solutions of this system are no longer subject to the constraints of the Miles-Howard theorem, and indeed an instability is found with phase-speed much greater than $\bar{u}_{max}$. The mode has zonal wavenumber one and planetary meridional scale. It has a period of about two days and an e-folding time of 6 days. A further instability with zonal wavenumber two has also been found.

The Model

The primitive equations are linearized about a zonally averaged flow which is a function of latitude and height. The independent variables are longitude λ, latitude φ, log-pressure $Z=-\ln p$ and time t. The equations are nondimensionalized using length- and time-scales a and $(2\Omega)^{-1}$, where a and Ω are the radius and rotation rate of the earth. If u', v', W' and Φ' represent the perturbation zonal, meridional and vertical velocities and geopotential, the linearized nondimensional primitive equations may be written:

$$\begin{aligned} u'_t+\bar{\omega}u'_\lambda-f_1v'+\bar{u}_ZW'+(1/\sigma)\Phi'_\lambda &= 0 \\ v'_t+\bar{\omega}v'_\lambda+f_2u' \quad + \quad \Phi'_\phi &= 0 \\ \Phi'_{Zt}+\bar{\omega}\Phi'_{Z\lambda}+\bar{\Phi}_{Z\phi}v'+N^2W' &= 0 \\ u'_\lambda/\sigma+(\sigma v')_\phi/\sigma+e^Z(e^{-Z}W')_Z &= 0 \end{aligned} \qquad (1)$$

where $\mu=\sin\phi$, $\sigma=\cos\phi$, $\bar{u}=\sigma\bar{\omega}$ is the zonal mean windspeed, $\bar{\Phi}$ the corresponding mean geopotential, $f_1=\mu(1+2\bar{\omega})-\sigma\bar{\omega}_\phi$, $f_2=\mu(1+2\bar{\omega})$, and N^2 is the Brunt-Väisälä frequency. The perturbation quantities are assumed to have harmonic dependence upon λ and t, $q'(\lambda,\phi,Z,t)=q(\phi,Z)\exp(i(m\lambda-\nu t))$, where the zonal wavenumber m is a positive integer and the frequency ν is real for neutral modes and complex for growing or decaying modes.

We now derive from the primitive equations, without further approximation, a single equation for the perturbation geopotential, Φ':

$$A\Phi'_{\phi\phi}+B\Phi'_{\phi Z}+C\Phi'_{ZZ}+D\Phi'_\phi+E\Phi'_Z+F\Phi' = 0 \qquad (2)$$

where the coefficients are complex functions of φ and Z. Their explicit form is given in Lynch (1982). We solve this equation on a global domain with full spherical geometry. The perturbation geopotential must vanish at the poles and a radiation condition is applied at 100 km. We force a vertical velocity with specified complex frequency at the lower boundary, and solutions of the homogeneous eigensystem are determined by searching in the frequency plane for resonant response to the forcing. (Resonances are found using coarse scans in ν; a triangular steepest descent method (Hollingsworth, 1975) is then used to locate the eigenfrequencies accurately). The grid resolution is $\Delta\phi=5^\circ$ and $\Delta Z=2\frac{1}{2}$km (37 × 41 points), and we use the method of Lindzen and Kuo (1969) to solve the system.

The zonal winds are represented by simple polynomial functions of sinφ and Z. The mean wind and mass fields are in geostrophic and hydrostatic balance. The zonal wind cross-section is shown in figure 1. It is based on the solsticial cross-section of Murgatroyd (1969) and is representative of Northern winter conditions.

Results

Zero Zonal Flow

To test the system a scan is made over real frequencies for an isothermal motionless basic state and a norm of the response is plotted against forcing frequency. Strong peaks occur at points corresponding to the eigenfrequencies of the known analytical solutions, the Hough modes (figure 2). The solutions at these points agree closely with the theoretical structures. Imposition of a non-zero zonal flow modifies these modes to varying degrees, as found in previous studies (Schoeberl and Clark, 1980; Salby, 1981).

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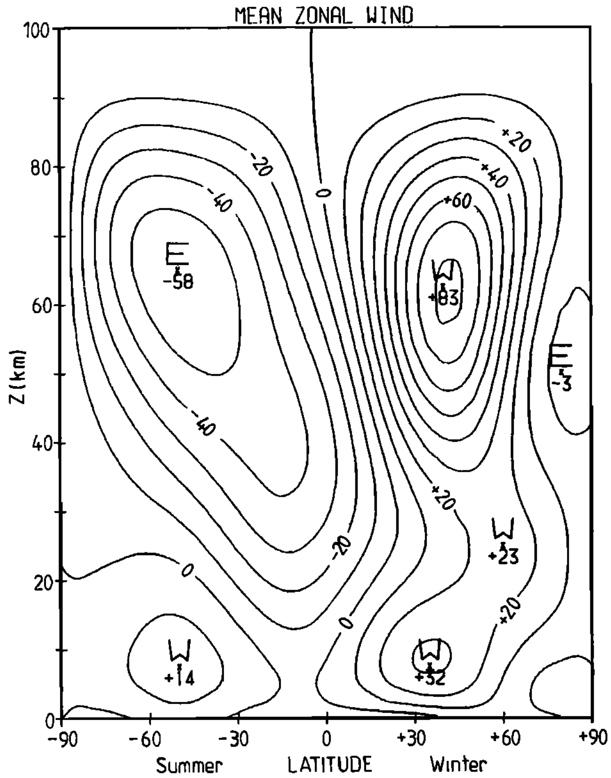

Fig. 1. Zonal average wind distribution $\bar{u}$ (m/s).

With the zonal flow in figure 1 various complex resonances are detected. Some of these are described below. All solutions shown have zonal wavenumber one and no dissipation is included.

Charney Mode

A strong resonant response is obtained for frequency ν=(0.012,0.020), corresponding to a period of 42 days and a growth-rate (e-folding time) of 8 days. The modulus of the geopotential perturbation is shown in figure 3. The structure is typical of solutions first investigated by Charney (1947). It is strongly bound to the surface and energy calculations reveal the source of the instability to be the meridional temperature shear below the winter tropospheric jet. Because of its relatively slow growth compared to the synoptic scale instabilities it is likely to be swamped by them in the troposphere.

Green Mode

Green (1960) discovered instabilities with internal vertical structure and significant amplitude in the stratosphere. A resonance peak was found with the present model for ν=(-0.029, 0.008) (period 17 days and growth rate 19 days). The solution had a maximum above the surface, strong penetration into the stratosphere, planetary meridional scale and an energy cycle typical of baroclinic instabilities. It is like a Green mode but with such a slow growth it is questionable whether it could maintain itself against dissipation. (The solution is shown in Lynch (1982)).

New Mode

In view of the bounds on phase-speed and growth rate of unstable waves implied by quasi-geostrophic theory previous studies have sought solutions within the resulting parameter range. Furthermore, since the main source of energy is in the horizontal temperature shear and since this is very small near the equator most models have been in a hemispheric domain. The present global primitive equation model is not subject to these limitations and is capable of simulating unstable motions which are not represented in more restricted models. A strong resonance peak is found to occur when the frequency ν = (0.244,0.026); this corresponds to a wave with period of 2 days and e-folding time of 6 days. The Doppler shifted phase-speed is $(c_r-\bar{u})$ = $(a\sigma/m)\times(\nu_r-m\bar{\omega})$ and since m=1 and $\bar{\omega}_{max}$ = 0.127 this is everywhere positive. That is, the phase-speed is greater than the maximum zonal windspeed and there is no steering level for the wave. Such a motion cannot be described within the ambit of quasi-geostrophic theory. It represents a wave with rapid eastward progression and substantial growth rate. The structure of the wave is shown in figure 4. The maximum value of the geopotential occurs at the earth's surface, near the equator. There is another surface maximum near 30°N, and there is a further maximum near the equatorial stratopause. Comparing figures 3 and 4, we see that the penetration of the wave into the upper atmosphere is very strong (recall that the values plotted in figures 3 and 4 must be scaled by exp(Z/2) to get the geopotential amplitudes). The horizontal scale decreases away from the surface; this behaviour of unstable ultra-long waves was also noted by Hartmann (1979). The source of energy for the wave is the baroclinic conversion of potential energy in the troposphere; the conversion takes place near the surface, between about 15°N and 30°N latitude where the temperature shear below the tropospheric jet is very strong. The energy released near the surface is carried upward and equatorward. There is strong convergence of wave energy flux in the middle atmosphere.

The meridional structure of the mode in figure 4 is similar (except near the surface) to that of

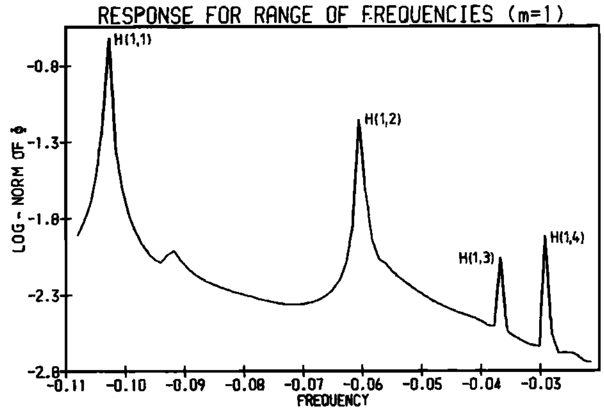

Fig. 2. Log-norm of geopotential response as a function of frequency for a motionless basic state. H(1,1) is the '5-Day Wave'; other peaks correspond to higher rotational Hough modes.

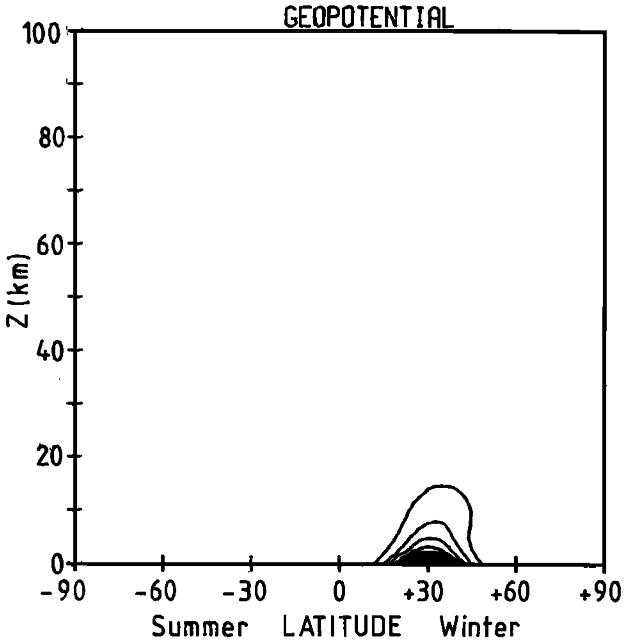

Fig. 3. Amplitude of geopotential, $\hat{\Phi}=\Phi'\exp(-Z/2)$, for a forcing frequency ν = (0.012,0.020).

the Kelvin Mode, the gravest eastward-travelling free mode. A relationship between the two modes was sought. The zonal flow was scaled by an intensity factor which was gradually reduced to zero. The growth rate of the instability decreased to zero as the zonal flow was 'switched off' in this way (in accordance with physical requirements), and its resonance frequency approached a (real) value ν= 0.290 (for $\bar{T}$ = 300K). The Kelvin wave was found to have a frequency ν= 0.418 (for zero zonal wind and $\bar{T}$ = 300K). It remained stable as the zonal intensity factor was increased and its structure and frequency were not strongly modified. Thus, there does not appear to be any relationship between the two modes. The resonance peak at ν = 0.290 for zero zonal wind was very weak. This frequency does not correspond to that of any of the Hough modes, but this is not surprising if we regard the problem of switching on the wind as a singular perturbation problem: for such a system there is no guarantee that solutions of the perturbed problem will correspond to, or continuously approach, solutions of the unperturbed problem as the perturbation parameter is reduced to zero.

Boyd (1982) has found that the Kelvin wave is unstable in the presence of meridional shear (though the growth rate is very small). The present instability was modified, but remained qualitatively unchanged as the zonal wind in figure 1 was gradually replaced by an analytical wind profile without horizontal shear. The growth rate is an order of magnitude greater than that found by Boyd. Therefore, there does not appear to be any relationship between these two forms of instability. Neither does the present instability bear any obvious relationship to those discussed by Boyd and Christidis (1982).

An instability with wavenumber two and frequency ν = (0.512,0.041) was found. It has a period of one day and e-folding time of 4 days, and its structure is similar to that of the wavenumber one instability.

Conclusions

A realistic zonal flow provides a source of energy for the growth of instabilities. Unstable modes of the types first discovered by Charney (1947) and by Green (1960) have been located. These solutions are in general agreement with the findings of previous studies.

A new mode of instability, not discussed by previous investigators, has been found. This solution has a phase-speed greater than the maximum eastward zonal flow velocity; the Doppler shifted phase-speed is everywhere positive. Such a solution is impossible within the framework of quasi-geostrophic theory. The mode has zonal wavenumber one, a period of two days and growth rate of 6 days. Its energy source is in the troposphere but it penetrates deeply into the middle atmosphere, its growth there being supported by vertical wave-energy flux convergence.

Rapidly travelling instabilities of the type discussed here are likely to play an important role in the dynamics of the middle atmosphere. Such motions have not yet been isolated with certainty in observational studies; it is probable that they are very common, since the tropospheric energy sources are a permanent feature of the zonal flow. However, the penetration of such instabilities into the middle atmosphere must be sensitive to the detailed structure of the mean flow and to the effects of radiative damping. The mean flow is subject to variations over a wide range of time-scales, and its intensity at a given moment is likely to be greater than that depicted in figure 1.

The results discussed in this study were confined to a few solutions for wavenumber one. It would be of interest to investigate solutions for

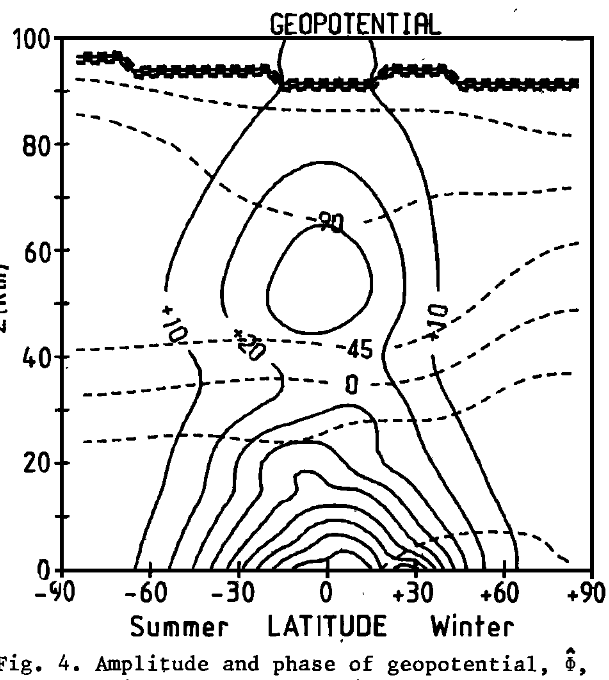

Fig. 4. Amplitude and phase of geopotential, $\hat{\Phi}$, for a forcing frequency ν = (0.244,0.026).

other wavenumbers and for other zonal flows. The effects of dissipation should also be investigated. The energetics of the instabilities must be studied in greater detail. Such studies are necessary for the assessment of the importance of the instabilities in the overall dynamics of the middle atmosphere.

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