Numerical Weather Prediction
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Text for the Course

The lectures will be based closely on the text

Atmospheric Modeling, Data Assimilation and Predictability
by Eugenia Kalnay
• NWP is an initial/boundary value problem
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• Given
  – an estimate of the present state of the atmosphere (initial conditions)
  – appropriate surface and lateral boundary conditions

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The first objective analysis systems were developed (independently) in Sweden and in USA in the 1950s.
ECMWF Data Coverage - SYNOP/SHIP
28/FEB/1999; 00 UTC
Total number of obs = 12688
ECMWF Data Coverage - BUOY
28/FEB/1999; 00 UTC
Total number of obs = 1568
ECMWF Data Coverage - AIRCRAFT
28/FEB/1999; 00 UTC
Total number of obs = 18964
ECMWF Data Coverage - TOVS (120km)
28/FEB/1999; 00 UTC
Total number of obs = 11005
ECMWF Data Coverage - SATOB
28/FEB/1999; 00 UTC
Total number of obs = 91405
ECMWF Data Coverage - ERS-2
28/FEB/1999; 00 UTC
Total number of obs = 107939
Insufficiency of Data Coverage

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Present-day operational systems typically use a 6-h cycle performed four times a day.
Typical 6-hour analysis cycle.
Suppose the **background field** is a model 6-h forecast:

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The difference between the observations and the background,

\[ y_o - H(x_b) \]

is called the **observational increment or innovation**.
The analysis $x_a$ is obtained by adding the innovations to the background field with weights $W$ that are determined based on the estimated statistical error covariances of the forecast and the observations:

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Earlier methods such as the SCM used weights which were determined empirically.

The weights were a function of the distance between the observation and the grid point, and the analysis was iterated several times.
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Lorenc (1986) showed that OI and the 3D-Var approach are equivalent if the cost function is defined as:

$$ J = \frac{1}{2} \left\{ (y_o - H(x))^T R^{-1} (y_o - H(x)) + (x - x_b)^T B^{-1} (x - x_b) \right\} $$
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The cost function \( J \) measures:

- The distance of a field \( x \) to the observations (first term)
- The distance to the background \( x_b \) (second term).
The distances are scaled by the observation error covariance $R$ and by the background error covariance $B$ respectively. The minimum of the cost function is obtained for $x = x_a$, which is defined as the \textit{analysis}. 
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- In OI, the weights $W$ are obtained for each grid point or grid volume, using suitable simplifications.
- In 3D-Var, the minimization of $J$ is performed directly, allowing for additional flexibility and a simultaneous global use of the data.
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However, why should an observation in New Zealand be used to determine the pressure pattern in Ireland?
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A quadratic in $x$ and $y$ was defined at each grid point:

$$z(x, y) = a_{00} + a_{10}x + a_{01}y + a_{20}x^2 + a_{11}xy + a_{02}y^2$$
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The coefficients were determined by minimizing the mean square difference

$$\min_{a_{ij}} E = \min_{a_{ij}} \left[ \sum_{k=1}^{K_z} p_k (z_k^o - z(x_k, y_k))^2 \right.$$ 
$$\left. + \sum_{k=1}^{K_v} q_k \left\{ [u_k^o - u_g(x_k, y_k)]^2 + [v_k^o - v_g(x_k, y_k)]^2 \right\} \right]$$

Here $p_k, q_k$ are empirical weighting coefficients and $K_z$ is the total number of observations within the radius of influence.
Figure 5.1.1: Schematic of grid points (circles), irregularly distributed observations (squares), and a radius of influence around a grid point $i$ marked with a black circle. In 4DDA, the grid-point analysis is a combination of the forecast at the grid point (first guess) and the observational increments (observation minus first guess) computed at the observational points $k$. In certain analysis schemes, like SCM, only observations within the radius of influence, indicated by a circle, affect the analysis at the black grid point.
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* * * * *

**Exercise:** Consider the Gilchrist and Cressman scheme. What does the analysis look like if there is (i) a single pressure observation; (ii) two pressure observations close together; (iii) two pressure obs. far apart?
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There are many **new types of data**, such as satellite and radar observations, but:

- they **don’t measure the variables** used in the models
- their distribution in space and time is **very nonuniform**.
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If a forecast is unavailable (e.g., if the cycle is broken), we may have to use climatological fields . . .

. . . but they are normally a poor estimate of the initial state.
Global 6-h analysis cycle (00, 06, 12, and 18 UTC).
Regional analysis cycle, performed (perhaps) every hour.
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The model is able to transport information from data-rich to data-poor areas.
Exercise:  Simple chart analysis.