# 7

# Richardson's forecast

*Starting from the table of the initially observed state of the atmosphere . . . the rates of change of the pressures, temperatures, winds, etc. are obtained. Unfortunately this "forecast" is spoilt by errors in the initial data for winds. (WPNP, p. 2)*

We now come to the 'forecast' made by Richardson. We examine what he actually predicted and consider the reasons why his results were so unrealistic. Richardson was acutely aware of the shortcomings of his results. In his Summary (WPNP, Ch. 1) he wrote that errors in the wind data spoilt the forecast. On page 183, he described the computed surface pressure tendency as 'absurd' and, four pages later, called it a 'glaring error'. Yet he believed that the cause of the outrageous results was not the forecasting method itself but the initial data and that, with appropriate smoothing of the data, it would be reasonable to expect the forecast to agree with the actual smoothed weather (WPNP, p. 217).

In §7.1 we review the nature and extent of Richardson's prediction. Considerable insight into the causes of the forecast failure comes from a scale analysis of the equations, in which the relative sizes of the various terms are examined and compared. This is undertaken in §7.2. The initial tendencies that comprise the forecast are then analysed in §7.3. In §7.4, we begin to examine Richardson's discussion of the causes of his forecast failure. Finally, we review the work of Margules, published almost twenty years before WPNP, which pointed to serious problems with Richardson's methodology.

## 7.1 What Richardson actually predicted: twenty numbers

Richardson's forecast amounted to the calculation of *twenty numbers*, the changes of the components of horizontal momentum at an M-point and of pressure, humidity and stratospheric temperature at a P-point. The essentials of the computational

Table 7.1. *Initial values for the two prediction cells. P-point: Pressure $p$ (hPa) at the base of each layer, temperature $T$ (K) for the stratosphere and water content $W$ (kg m$^{-2}$) for the lower layers. M-points: Eastward and northward components of momentum $U$ and $V$ (kg m$^{-1}$ s$^{-1}$) for each layer.*

| P-POINT | | M-POINT | |
|---|---|---|---|
| $p_1 = 205.0$ | $T_1 = 212$ | $U_1 = -5,600$ | $V_1 = -1,800$ |
| $p_2 = 409.0$ | $W_2 = 0.0$ | $U_2 = -14,600$ | $V_2 = -6,200$ |
| $p_3 = 607.9$ | $W_3 = 1.0$ | $U_3 = -9,500$ | $V_3 = +2,900$ |
| $p_4 = 796.0$ | $W_4 = 4.0$ | $U_4 = -5,200$ | $V_4 = +5,800$ |
| $p_S = 962.6$ | $W_5 = 9.0$ | $U_5 = -11,000$ | $V_5 = +5,500$ |

algorithm were given in §5.6 above. A complete appreciation of Richardson's actual computations would require a detailed analysis of his 23 computing forms. The crucial role of these forms was emphasized by Richardson: 'The computing forms . . . may be regarded as embodying the process and therefore summarizing the whole book' (WPNP, p. 181). We leave such a detailed analysis to future investigators and confine our attention here to the primary results obtained by Richardson.

The initial data for the forecast were presented in Table 6.4 on page 115 above. The prognostic variables were the two components of momentum, $U$ and $V$, and the pressure $p$, specified on alternate chequers of a chess-board grid, with five values of each variable in each vertical column. The temperature of the uppermost stratum was also given at P-points, as was the total mass of water in each of the lower four strata. There were thus ten prognostic variables at P-points and ten at M-points, whose initial values are given in Table 7.1 (all values are in SI units). To compute the rates of change of the twenty quantities, their values in a small neighbourhood surrounding the central points were required. It is these values that are presented in Richardson's 'Table of Initial Distribution', on page 185 of WPNP, and reproduced (except for the moisture variable) in Table 6.4 above. Richardson applied his numerical process to these initial data, using the computing forms to organize the calculations. All calculations were performed twice and the results compared to avoid or at least reduce the likelihood of errors. Multiplications were done using a 25 cm slide rule. This gave an accuracy to three significant digits. For more sensitive terms, such as the pressure gradients (see Eq. (5.21) on page 93), five-figure logarithms were found to be essential.

The set of 23 computing forms was in two groups, with nineteen for P-points and four for M-points (see Table 1.3 on p. 24). The P-point selected for the forecast was at 48.6°N, 11°E. This point is in Bavaria, about 65 km north-west of Munich.

Table 7.2. *Richardson's 'forecast': Predicted six-hour changes in pressure $\Delta p$, stratospheric temperature $\Delta T$ and water content $\Delta W$ at the P-point and eastward and northward components of momentum $\Delta U$ and $\Delta V$ at the M-point. The variables and units are the same as in Table 7.1 above.*

| P-POINT | | M-POINT | |
|---|---|---|---|
| $\Delta p_1 = 48.3$ | $\Delta T_1 = 19.6$ | $\Delta U_1 = -73,000$ | $\Delta V_1 = -33,700$ |
| $\Delta p_2 = 77.0$ | $\Delta W_2 = 0.07$ | $\Delta U_2 = -19,600$ | $\Delta V_2 = +23,800$ |
| $\Delta p_3 = 103.2$ | $\Delta W_3 = 0.24$ | $\Delta U_3 = -8,900$ | $\Delta V_3 = +13,800$ |
| $\Delta p_4 = 126.5$ | $\Delta W_4 = 1.49$ | $\Delta U_4 = -15,300$ | $\Delta V_4 = -4,300$ |
| $\Delta p_S = 145.1$ | $\Delta W_5 = 4.02$ | $\Delta U_5 = -17,900$ | $\Delta V_5 = +6,300$ |

The M-point was 200 km to the north (at 50.4°N, 11°E). The vertical velocity, which is computed at P-points, is required also at M-points. Ideally, it would have been computed there by interpolation. However, in view of the limited nature of the forecast, the values of $w$ at the P-point were also used by Richardson in the momentum equations at the M-point.

The calculated tendencies are tabulated on page 211 of WPNP, expressed as changes over a six-hour period centered on the initial time. For example, the computed value of surface pressure tendency $\partial p_S/\partial t$ was multiplied by 21,600 seconds to estimate the change in surface pressure between 0400 UTC and 1000 UTC. The changes are given in Table 7.2. An example of Richardson's wry sense of humour is given by the running header on the page where the forecast values are tabulated; it reads: THE RESULTING "PREDICTION". It is important to stress that Richardson's forecast was confined to the twenty numbers in this table.

The result most frequently quoted from WPNP is the predicted change in surface pressure of 145 hPa in six hours. However, the other forecast changes are also problematical. Richardson wrote below his table of results: 'It is claimed that the above form a fairly correct deduction from a somewhat unnatural initial distribution.' This remarkable claim will be considered in depth below.

A computer program has been written to repeat and extend Richardson's forecast, using the same initial values. We will describe this in more detail in Ch. 9 below. For now we just note that the computer model produces results consistent with those obtained manually by Richardson. In particular, the 'glaring error' in the surface pressure tendency is reproduced almost exactly by the model. These results confirm the essential correctness of Richardson's calculations. Such manual calculations, done with slide-rule and logarithm tables, are notoriously error-prone. Although a number of minor slips were detected in Richardson's calculations, they did not have any significant effect on his results. We may conclude that his unrealistic results were not due to arithmetical blunders, but had a deeper cause.

Richardson's calculations involved an enormous amount of numerical computation. Despite the limited scope of his forecast, it cost him some two years of arduous calculation (see Appendix 4). Moreover, the work was carried out in the Champagne district of France where Richardson served as an ambulance driver during World War I:

> ...the detailed example of Ch. IX was worked out in France in the intervals of transporting wounded in 1916–1918. During the Battle of Champagne in April, 1917 the working copy was sent to the rear, where it became lost, to be re-discovered some months later under a heap of coal (WPNP, Preface).

Richardson described his working conditions with characteristic stoicism: 'My office was heap of hay in a cold rest billet' (WPNP, p. 219). His dedication and tenacity in the dreadful circumstances of trench warfare should serve as an inspiration to those of us who work in more congenial conditions.

## 7.2 Scaling the equations of motion

### *7.2.1 Richardson's scaling*

Although Richardson was reluctant to make any unnecessary approximations in the equations, he was forced by circumstances to replace the equation for vertical acceleration by the hydrostatic balance relation. He could have introduced a number of other simplifications without serious error, but argued that, since the numerical process could handle terms with ease irrespective of their size, they might as well be retained. He did investigate the relative sizes of the terms in the continuity and momentum equations, and set down estimates of the extreme values ordinarily attained by the various terms:

> These figures have been obtained by a casual inspection of observational data and they may be uncertain except as to the power of ten. They are expressed in C.G.S. units. They relate only to the large-scale phenomena that can be represented by the chosen coordinate differences of 200 kilometres horizontally, one fifth of the pressure vertically and by the time-step of six hours (WPNP, p. 22).

The sizes assigned by Richardson to the terms of the continuity equation are given here (they are in SI units, and we omit a small term, as discussed in Ch. 2):

$$\underbrace{\frac{\partial\rho}{\partial t}}_{10^{-6}} + \underbrace{\frac{\partial\rho u}{\partial x}}_{10^{-4}} + \underbrace{\frac{\partial\rho v}{\partial y}}_{10^{-4}} - \underbrace{\frac{\rho v\tan\phi}{a}}_{10^{-5}\tan\phi} + \underbrace{\frac{\partial\rho w}{\partial z}}_{<10^{-4}} = 0 \tag{7.1}$$

Clearly, he was unsure of the appropriate magnitude for vertical momentum. We note a crucial implication of the scaling: the tendency term in (7.1) is two orders of magnitude smaller than the largest terms in the equation, so that the density at a fixed point is close to constant. As Bjerknes and others had found earlier, this is a better approximation than assuming the atmosphere behaves like a liquid; that is,

$$\nabla\cdot\rho\mathbf{v} = 0 \qquad \text{is better than} \qquad \nabla\cdot\mathbf{v} = 0\,.$$

The quasi-nondivergence of specific momentum was the primary reason it was chosen by Richardson as a dependent variable, in preference to velocity (WPNP, p. 24).

The terms of the horizontal equations of motion in flux form were assigned magnitudes as indicated here:

$$\underbrace{\frac{\partial\rho u}{\partial t}}_{10^{-3}} + \underbrace{\frac{\partial\rho u^2}{\partial x}}_{10^{-3}} + \underbrace{\frac{\partial\rho uv}{\partial y}}_{10^{-2}} + \underbrace{\frac{\partial\rho uw}{\partial z}}_{10^{-2}?} - \underbrace{f\rho v}_{10^{-2}} + \underbrace{\frac{2\rho uv\tan\phi}{a}}_{10^{-3}\tan\phi} + \underbrace{\frac{\partial p}{\partial x}}_{10^{-2}} = 0 \quad (7.2)$$

$$\underbrace{\frac{\partial\rho v}{\partial t}}_{10^{-3}} + \underbrace{\frac{\partial\rho vu}{\partial x}}_{10^{-3}} + \underbrace{\frac{\partial\rho v^2}{\partial y}}_{10^{-2}} + \underbrace{\frac{\partial\rho vw}{\partial z}}_{10^{-2}?} + \underbrace{f\rho u}_{10^{-2}} + \underbrace{\frac{(\rho u^2 - \rho v^2)\tan\phi}{a}}_{10^{-3}\tan\phi} + \underbrace{\frac{\partial p}{\partial y}}_{10^{-2}} = 0 \quad (7.3)$$

(an error in Richardson's scaling of the geometric term in (7.3) has been amended). Again, we note the uncertainty regarding terms with vertical momentum. The nonlinear terms involving zonal gradients were assigned smaller scales than those involving meridional gradients. This assumption was presumably inspired by Richardson's 'casual inspection of observational data'. However, despite the predominantly zonal character of the mid-latitude flow, the eastward flux is generally comparable in magnitude to the northward flux.

The sizes of the terms assigned by Richardson are not typical, but represent the extreme values ordinarily attained by the terms. They are such that the nonlinear terms are comparable to the Coriolis and pressure gradient terms, so that geostrophic balance does not obtain. We may set the horizontal and vertical scales $\mathcal{L}$ and $\mathcal{H}$ and the time scale $\mathcal{T}$ to represent synoptic variations:

$$\mathcal{L} = 10^6\,\mathrm{m}\,, \qquad \mathcal{H} = 10^4\,\mathrm{m}\,, \qquad \mathcal{T} = 10^5\,\mathrm{s}\,.$$

Then Richardson's values are consistent with the following scales for the dependent variables:

$$\mathcal{V} = 10^2\,\mathrm{m\,s^{-1}}\,, \quad \mathcal{W} \le 1\,\mathrm{m\,s^{-1}}\,, \quad \mathcal{P}' = 10^4\,\mathrm{Pa}\,, \quad \mathcal{R}' = 10^{-1}\,\mathrm{kg\,m^{-3}}\,.$$

Here, $\mathcal{V}$ and $\mathcal{W}$ are the scales of the horizontal and vertical motion and $\mathcal{P}'$ and $\mathcal{R}'$ are the scales of $p'$ and $\rho'$, the deviations from the mean values $\bar{p}(z)$ and $\bar{\rho}(z)$. Indeed, the chosen scales represent very active atmospheric conditions, more typical of extremes than of average synoptic patterns. The pressure and density variations amount to 10% of their mean values. Moreover, the scale of vertical velocity is unrealistically large (Richardson's question-mark confirms his doubt). Assuming

$w \ll 1\,\mathrm{m\,s^{-1}}$, the dominant terms of the momentum equations yield the well-known diagnostic relationship for the *gradient wind*:

$$\mathbf{v}.\nabla\mathbf{v} + f\mathbf{k}\times\mathbf{v} + (1/\rho)\nabla p = 0\,.$$

This is an extension of geostrophic balance in which the curvature of the flow is taken into account through the advection term.

It is clear from the foregoing that Richardson had an excellent understanding of the concept of scale analysis and a refined capacity to employ it to effect. It is unfortunate that he did not pursue this avenue, as it might have led him to methods of greater utility than the sledge-hammer approach of keeping all terms. Platzman (1967, p. 530) expressed a similar view when he wrote that Richardson's disregard of perturbation theory as a means of clarifying the problems of dynamic meteorology was a major defect in his approach to weather prediction. Scale analysis was the basis of the development of the quasi-geostrophic system some decades later (Charney, 1948).

### 7.2.2 Re-scaling using typical synoptic values

To facilitate further analysis, it is convenient to re-scale the equations using typical synoptic values rather than Richardson's extreme values. We assume characteristic sizes for the independent variables as before, and set the following scales for the dependent variables:

$$\mathcal{V} = 10\,\mathrm{m\,s^{-1}}\,, \quad \mathcal{W} = 10^{-1}\,\mathrm{m\,s^{-1}}\,, \quad \mathcal{P}' = 10^{3}\,\mathrm{Pa}\,, \quad \mathcal{R}' = 10^{-2}\,\mathrm{kg\,m^{-3}}\,.$$

Now the continuity equation scales as follows:

$$\underbrace{\frac{\partial\rho}{\partial t}}_{10^{-7}} + \underbrace{\frac{\partial\rho u}{\partial x}}_{10^{-5}} + \underbrace{\frac{\partial\rho v}{\partial y}}_{10^{-5}} - \underbrace{\frac{\rho v\tan\phi}{a}}_{10^{-6}\tan\phi} + \underbrace{\frac{\partial\rho w}{\partial z}}_{10^{-5}} = 0\,. \tag{7.4}$$

Considering the gas law, the variations in the mass variables are related by

$$p' = \Re(\bar{T}\rho' + \bar{\rho}T')\,. \tag{7.5}$$

Then, noting that $\Re \approx 300\,\mathrm{m^2\,s^{-2}\,K^{-1}}$, and $\bar{T} \approx 300\,\mathrm{K}$, the pressure tendency may be estimated:

$$\frac{\partial p}{\partial t} \approx \Re\bar{T}\frac{\partial\rho}{\partial t} \approx 10^{-2}\mathrm{Pa\,s^{-1}}\,. \tag{7.6}$$

A practical unit for pressure tendency, which is more 'ergonomic' than $\mathrm{Pa\,s^{-1}}$, has been introduced by Sanders and Gyakum (1980) in a study of rapidly intensifying oceanic cyclones (which they called 'bombs'). A deepening rate of 24 hPa in 24 hours (at 60° N) was defined to be one Bergeron (1 Ber) [named for the renowned

Swedish meteorologist Tor Bergeron]. Here we define 1 Ber to be a tendency of *one hectopascal per hour*. Thus, using (7.6), the typical synoptic scale of the pressure tendency is about $10^{-4}$ hPa s$^{-1}$, or 0.36 Ber. Richardson's prediction of 145hPa in six hours is 24 Ber, about two orders of magnitude larger than typical synoptic values.

The horizontal equations of motion, using typical synoptic scales, are as follows:

$$\underbrace{\frac{\partial \rho u}{\partial t}}_{10^{-4}} + \underbrace{\frac{\partial \rho u^2}{\partial x}}_{10^{-4}} + \underbrace{\frac{\partial \rho uv}{\partial y}}_{10^{-4}} + \underbrace{\frac{\partial \rho uw}{\partial z}}_{10^{-4}} - \underbrace{f\rho v}_{10^{-3}} + \underbrace{\frac{2\rho uv \tan\phi}{a}}_{10^{-5}\tan\phi} + \underbrace{\frac{\partial p}{\partial x}}_{10^{-3}} = 0 \quad (7.7)$$

$$\underbrace{\frac{\partial \rho v}{\partial t}}_{10^{-4}} + \underbrace{\frac{\partial \rho vu}{\partial x}}_{10^{-4}} + \underbrace{\frac{\partial \rho v^2}{\partial y}}_{10^{-4}} + \underbrace{\frac{\partial \rho vw}{\partial z}}_{10^{-4}} + \underbrace{f\rho u}_{10^{-3}} + \underbrace{\frac{(\rho u^2 - \rho v^2)\tan\phi}{a}}_{10^{-5}\tan\phi} + \underbrace{\frac{\partial p}{\partial y}}_{10^{-3}} = 0 \quad (7.8)$$

(we do not distinguish between the scales of zonal and meridional fluxes). The Coriolis and pressure gradient terms now dominate everything else, and this fundamental balance corresponds to *quasi-geostrophic flow*:

$$f\mathbf{k} \times \mathbf{v} + \frac{1}{\rho}\nabla p = 0 \qquad \text{or} \qquad \mathbf{v} = \frac{1}{f\rho}\mathbf{k} \times \nabla p\,.$$

It must not be inferred from the re-scaling that Richardson's choice of scales was incorrect or inappropriate. The important point is that he chose values characteristic of atmospheric extremes rather than of typical synoptic conditions. This choice may have made it more difficult for him to develop aproximations applicable to more moderate conditions. We saw in Chapter 6 that the atmospheric conditions prevailing on the occasion for which he made his forecast were quite temperate, with little change over the observation period.

### *7.2.3 The triple compensation effect*

How can the tendencies be small when they are determined by quantities that are orders of magnitude larger? The answer is that, for normal atmospheric conditions, there are several compensating effects that result in strong cancellation between terms. The most prominent is the balance between the Coriolis and pressure gradient terms, corresponding to quasi-geostrophic flow. Geostrophic flow is also quasi-nondivergent, its divergence depending only on the gradient of planetary vorticity, the $\beta$-effect. In general the atmospheric divergence is small relative to its components. The pressure tendency equation follows from integration through the atmosphere of the continuity equation (7.4):

$$\frac{\partial p_S}{\partial t} = -\int_0^{p_S} \frac{1}{\rho}\nabla \cdot \rho\mathbf{v}\, dp\,. \qquad (7.9)$$

With naïve synoptic scaling, the individual components of the integrand are $O(10^{-5})$, so the right-hand side appears to be $O(1)$, or about 36 Ber, comparable to Richardson's computed tendency. But this is not the case, because of the triple compensation effect:

(i) *Horizontal Compensation*: Horizontal influx and out-flow have a strong tendency to off-set each other so that the horizontal divergence is smaller than its component terms.

(ii) *Vertical Compensation*: Horizontal influx at a point tends to correspond to vertical out-flow, and vice-versa. Thus, the three-dimensional divergence is characteristically smaller than the two-dimensional divergence.

(iii) *Dines Compensation*: Inflow at low levels and out-flow at higher levels of a column, or vice-versa, tend to occur simultaneously, resulting in cancellation when the vertical integral is taken.

The first two effects moderate the change in pressure difference across a layer. The first and third act to reduce the change in pressure through a vertical column, that is, the surface pressure tendency. The net effect is that typical values of pressure tendency are about one hundreth of the size that a naïve scaling argument might suggest.

### 7.2.4 The effect of data errors

The quasi-geostrophic and quasi-nondivergent nature of the atmospheric flow are delicate, depending for their persistence on effects that closely cancel each other. If the analysed state were error-free, it would reflect this balance. However, this ideal is never attained, and observational errors can have drastic consequences. Suppose there is a 10% error in the $v$-component of the wind observation at a specific point, $\Delta v \approx 1\,\mathrm{m\,s^{-1}}$. The scales of the terms in the $u$-equation are as before:

$$\underbrace{\frac{\partial \bar{\rho} u}{\partial t}}_{10^{-4}} \quad - \quad \underbrace{f\bar{\rho}(v+\Delta v)}_{10^{-3}} \quad + \quad \underbrace{\frac{\partial p}{\partial x}}_{10^{-3}} \quad = \quad 0 \tag{7.10}$$

(for simplicity, we omit non-linear terms). However, the error in the tendency is $\Delta(\partial\bar{\rho}u/\partial t) \sim f\bar{\rho}\Delta v \sim 10^{-4}\,\mathrm{kg\,m^{-2}\,s^{-2}}$, comparable in size to the tendency itself: the signal-to-noise ratio is 1 (the symbol '$\sim$' represents equality of order of magnitude). The forecast may be qualitatively reasonable, but it will be quantitatively invalid.

A graver conclusion is reached for a 10% error $\Delta p \sim 1\,\mathrm{hPa}$ in the pressure variation: the numerical gradient is not computed using the horizontal scale length

$\mathcal{L}$, but the grid scale $\Delta x$ which is much smaller (say, $\Delta x = \mathcal{L}/10$). Consequently, the error in its gradient is correspondingly large:

$$\frac{\partial p}{\partial x} \sim \frac{\mathcal{P}'}{\mathcal{L}}, \qquad \text{but} \qquad \Delta \frac{\partial p}{\partial x} \sim \frac{\Delta p}{\Delta x} \sim \frac{\mathcal{P}'}{\mathcal{L}} \sim \frac{\partial p}{\partial x},$$

so the error is comparable to the term itself and the error in the wind tendency is now

$$\Delta \frac{\partial \bar{\rho} u}{\partial t} \sim \frac{\partial p}{\partial x} \sim 10^{-3} \gg \frac{\partial \bar{\rho} u}{\partial t}.$$

That is, the error in momentum tendency is larger than the tendency itself; the forecast will be qualitatively incorrect, indeed unreasonable.

Now consider the continuity equation. As we have seen, the pressure tendency has characteristic scale $10^{-2}\,\mathrm{Pa\,s^{-1}}$ or 0.36 Ber, corresponding to a scale for the tendency of density of

$$\frac{\partial \rho}{\partial t} \sim 10^{-7}\,\mathrm{kg\,m^{-3}\,s^{-1}}.$$

The mass divergence $D = \nabla \cdot \rho \mathbf{v}$ is $O(10^{-7})$ but its individual components are $O(10^{-5})$. If there is a 10% error in the northward wind component $v$, naïve scaling would suggest that the resulting error in divergence is $\Delta D \sim \Delta v/\mathcal{L} \sim 10^{-6}$. This error is larger than the divergence itself! As a result, the tendency of density is unrealistic. But matters are worse still: since the numerical divergence is computed using the grid scale $\Delta x$, the error is correspondingly greater:

$$\Delta D \sim \Delta \frac{\partial v}{\partial x} \sim \frac{\Delta v}{\Delta x} \sim \frac{\mathcal{V}}{\mathcal{L}} \sim 10^{-5} \sim 10^{2} \times D.$$

This yields a tendency *two orders of magnitude too large*. By (7.6), this also implies a pressure tendency two orders of magnitude larger than the correct value. Instead of the value $\partial p/\partial t \sim 0.36$ Ber, we get a change of order 36 Ber. This is strikingly reminiscent of Richardson's result.

### 7.3 Analysis of the initial tendencies

Richardson ascribed the unrealistic value of pressure tendency to errors in the observed winds which resulted in spuriously large values of calculated divergence. This is true as far as it goes. However, the problem is deeper: even if the winds were modified to remove divergence completely at the initial time, large tendencies would soon be observed. The close balance between the pressure and wind fields, and the compensations that result in quasi-nondivergence, ensure that the high frequency gravity waves have much smaller amplitude than the rotational part of the flow, and that the tendencies are small. As we have seen from scale analysis, minor

Table 7.3. *Richardson's Computing Form* $\mathrm{P}_{\mathrm{XIII}}$, *giving the horizontal divergence of momentum (in column 5) and the pressure changes (last column). The figure in the bottom right corner is the much-discussed forecast change in surface pressure.*

COMPUTING FORM P XIII. Divergence of horizontal momentum-per-area. Increase of pressure

The equation is typified by: $-\frac{\partial R_{86}}{\partial t} = \frac{\partial M_{E86}}{\partial e} + \frac{\partial M_{N86}}{\partial n} - M_{N86}\frac{\tan\phi}{a} + m_{H6} - m_{H8}{}^{*} + \frac{2}{a}M_{H86}$. (See Ch. 4/2#5.)

* In the equation for the lowest stratum the corresponding term $-m_{G8}$ does not appear

Longitude *11° East*, $\delta e = 441 \times 10^5$ — Latitude *5400 km North*, $\delta n = 400 \times 10^5$ — Instant *1910 May 20ᵈ 7ʰ G.M.T.*, $a^{-1}.\tan\phi = 1{\cdot}78 \times 10^{-9}$ — Interval, $\delta t$ *6 hours*, $a = 6{\cdot}36 \times 10^8$

| REF.:— | | | | previous 3 columns | previous column | | Form P XVI | Form P XVI | equation above | previous column | previous column | previous column |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $h$ | $\frac{\delta M_E}{\delta e}$ | $\frac{\delta M_N}{\delta n}$ | $-\frac{M_N \tan\phi}{a}$ | $\mathrm{div}'_{EN}M$ | $-g\delta t\,\mathrm{div}'_{EN}M$ | Leave the subsequent columns to be filled up after the vertical velocity has been computed on Form P XVI | $m_H$ | $\frac{2M_H}{a}$ | $-\frac{\partial R}{\partial t}$ | $+\frac{\partial R}{\partial t}\delta t$ | $g\frac{\partial R}{\partial t}\delta t$ | $\frac{\partial p}{\partial t}\delta t$ |
| | $10^{-5}\times$ | $10^{-5}\times$ | $10^{-5}\times$ | $10^{-5}\times$ | $100\times$ | | $10^{-5}\times$ | $10^{-5}\times$ | $10^{-5}\times$ | | $100\times$ | $100\times$ |
| $h_0$ | | | | | | | 0 | | | | | 0 |
| | −61 | −245 | −6 | −312 | 656 | | | | −229 | 49·5 | 483 | |
| $h_2$ | | | | | | | −83 | | | | | 483 |
| | 367 | −257 | 2 | 112 | −236 | | | 0·06 | −136 | 29·4 | 287 | |
| $h_4$ | | | | | | | 165 | | | | | 770 |
| | 93 | −303 | −16 | −226 | 478 | | | 0·11 | −124 | 26·8 | 262 | |
| $h_6$ | | | | | | | 63 | | | | | 1032 |
| | 32 | − 55 | −12 | − 35 | 74 | | | 0·07 | −110 | 23·8 | 233 | |
| $h_8$ | | | | | | | 138 | | | | | 1265 |
| | −256 | 38 | − 8 | −226 | 479 | | | 0·03 | − 88 | 19·0 | 186 | |
| $h_G$ | | | | | | | | | | | | 1451 |
| | NOTE: $\mathrm{div}'_{EN}M$ is a contraction for $\frac{\delta M_E}{\delta e} + \frac{\delta M_N}{\delta n} - M_N\frac{\tan\phi}{a}$ | | | | SUM = 1451 $= \frac{\partial p_G}{\partial t}\delta t$ | | | | | | | check by $\Sigma - g\delta t\,\mathrm{div}'_{EN}M$ |

errors in observational data can result in a disruption of the balance, and cause spuriously large tendencies. The imbalance engenders large-amplitude gravity wave oscillations. They can be avoided by modifying the data to restore harmony between the fields. In Ch. 8 we will describe several methods of achieving balance and in Ch. 9 we will apply a simple initialization method to Richardson's data and show that it yields realistic results. Here we examine the initial data used for the forecast and show that they are far from balance.

### *7.3.1 The divergence and the pressure tendency*

Platzman (1967) examined Richardson's results and discussed two problems contributing to the large pressure tendency: the horizontal divergence values are too large, due to lack of cancellation between the terms, and there is a lack of compensation between convergence and divergence in the vertical. We follow the same approach, developing and expanding Platzman's analysis. The Computing Form $\mathrm{P}_{\mathrm{XIII}}$, reproduced in Table. 7.3, contains the results of Richardson's calculations for pressure changes. Table 7.4, constructed using values from Form $\mathrm{P}_{\mathrm{XIII}}$, shows the total divergence and its components. The eastward and northward components

Table 7.4. *Analysis of the components of the mass divergence in each layer and the mean absolute values for the column. $[\rho w]$ denotes a vertical difference across the layer. All values are in SI units (kg m$^{-2}$ s$^{-1}$).*

| | Eastward Component | Northward Component | Horizontal Divergence | Vertical Divergence | Total Divergence |
|---|---|---|---|---|---|
| Layer | $\frac{\partial U}{\partial x}$ | $\frac{\partial(V\cos\phi)}{\cos\phi\,\partial y}$ | $\nabla\cdot\mathbf{U}$ | $[\rho w]$ | $\nabla\cdot\mathbf{U}+[\rho w]$ |
| I | $-0.0061$ | $-0.0251$ | $-0.0312$ | $+0.0083$ | $-0.0229$ |
| II | $+0.0367$ | $-0.0255$ | $+0.0112$ | $-0.0248$ | $-0.0136$ |
| III | $+0.0093$ | $-0.0319$ | $-0.0226$ | $+0.0102$ | $-0.0124$ |
| IV | $+0.0032$ | $-0.0067$ | $-0.0035$ | $-0.0075$ | $-0.0110$ |
| V | $-0.0256$ | $+0.0030$ | $-0.0226$ | $+0.0138$ | $-0.0088$ |
| Mean Abs. Value | 0.0162 | 0.0184 | 0.0182 | 0.0129 | 0.0137 |

Table 7.5. *Analysis of the pressure changes (hPa) across each layer, and the pressure change at the base of each layer ($[p]$ denotes the vertical pressure difference across a layer). Values in column 4 are the products of those in column 3 by $-g\Delta t$ (the time interval is $\Delta t = 21600$ s and for simplicity we assume constant acceleration of gravity, $g = 9.778$).*

| | | Total Divergence | Change in Pressure Thickness | Change in Base Pressure |
|---|---|---|---|---|
| Layer | Level | $\nabla\cdot\mathbf{U}+[\rho w]$ | $\frac{\partial[p]}{\partial t}\Delta t$ | $\frac{\partial p}{\partial t}\Delta t$ |
| I | | $-0.0229$ | $+48.3$ | |
| | 1 | | | $+48.3$ |
| II | | $-0.0136$ | $+28.7$ | |
| | 2 | | | $+77.1$ |
| III | | $-0.0124$ | $+26.2$ | |
| | 3 | | | $+103.2$ |
| IV | | $-0.0110$ | $+23.3$ | |
| | 4 | | | $+126.5$ |
| V | | $-0.0088$ | $+18.6$ | |
| | S | | | $+145.1$ |

are given in Cols. 2 and 3 and their sum, the horizontal divergence in Col. 4. The numbers in the bottom row of the table are the averages of the layer absolute values, and indicate the characteristic magnitudes of the terms. We see that the horizontal divergence is comparable in size to its components: there is no horizontal compensation. The vertical component of divergence (Col. 5) is comparable in size to the horizontal, but there is only a slight tendency for these terms to cancel: the vertical compensation is weak. Finally, Col. 6 of the Table shows that the divergence is negative in all layers: the Dines compensation mechanism is completely absent from the data.

Table 7.5 shows the change in pressure thickness for each layer (Col, 4) and the change in pressure at the base of each layer (Col. 5). As a result of convergence in all layers, the change in the pressure difference across all layers is positive, and the cumulative effect is a huge pressure rise at the surface.

### 7.3.2 The momentum tendencies

For balanced atmospheric flow, the tendencies arise as small residual differences between large quantities. But this is not the case for Richardson's data. The calculated changes in momentum appear on the bottom rows of Computing Forms $\mathrm{M}_{\mathrm{III}}$ and $\mathrm{M}_{\mathrm{IV}}$, reproduced in Table. 7.6. We tabulate the principal terms of the eastward momentum equation in Table 7.7. The pressure gradient and Coriolis terms dominate, but there is little tendency for them to cancel; indeed, for the top two layers they are of the same sign and act to reinforce each other. The numbers in the bottom row of the table are the absolute averages of the layer values. They show that the pressure gradient term is substantially larger than the Coriolis term, and the tendency of $U$ is comparable in size to the pressure term. We conclude that the data are far from geostrophic balance and, as a result, the tendency of momentum is uncharacteristically large. The imbalance is particularly pronounced in the stratosphere, and the momentum change is correspondingly large there. An analysis of the northward momentum equation is shown in Table 7.8. It indicates a similar departure from geostrophic balance and large values for the tendencies.

### 7.3.3 The moisture and stratospheric temperature tendencies

The prediction of changes in the humidity variable are also questionable. For each of the four lower strata, the total mass of water substance per unit area is given in Table 7.1 (p. 117). The saturation values $W_{\mathrm{sat}}$ are given on Richardson's Form $\mathrm{P}_{\mathrm{I}}$. For the lowest stratum, $W_5 = 9\,\mathrm{kg\,m^{-2}}$ and $W_{\mathrm{sat}} = 20\,\mathrm{kg\,m^{-2}}$. The six-hour change for this stratum, given in Table 7.2, is $\Delta W_5 = 4.02$. At this rate, the lowest stratum would reach saturation within 18 hours. The tendencies in the other layers

Table 7.6. *Richardson's Computing Forms* $\mathrm{M_{III}}$, *for the eastward component of momentum (top), and* $\mathrm{M_{IV}}$, *for the northward component (bottom). The values in these tables are in* C.G.S. *units.*

COMPUTING FORM M III. For the Dynamical Equation for the Eastward Component

$$-\frac{\partial M_{EG8}}{\partial t} = \frac{\partial P_{G8}}{\partial e} + \left[p\frac{\partial h}{\partial e}\right]_G^* + \frac{\partial}{\partial e}\left(\frac{M_E^2}{R}\right)_{G8} + \frac{\partial}{\partial n}\left(\frac{M_E M_N}{R}\right)_{G8} + \left[m_E v_H\right]_8 \quad \dagger \quad - 2\omega \sin\phi M_{NG8} + 2\omega\cos\phi M_{HG8} + \left(\frac{3M_E M_H - 2M_E M_N \tan\phi}{aR}\right)_{G8}.$$

* No corresponding term in upper layers. † A term $[m_E v_H]_G$ absent because ground impervious to wind.

Longitude *11° E* Latitude *5600 km N* $2\omega\sin\phi = 1{\cdot}124 \times 10^{-4}$, $2\omega\cos\phi = 0{\cdot}930 \times 10^{-4}$ Instant *1910 May 20ᵈ 7ʰ G.M.T.* $\delta t = 6$ *hours*

$\frac{\tan\phi}{a} = 1{\cdot}897 \times 10^{-9}$; $\delta e = 425 \times 10^5$ for $\delta\lambda = 6°$

| REF. | Term | $h_0$–$h_2$ + | − | $h_2$–$h_4$ + | − | $h_4$–$h_6$ + | − | $h_6$–$h_8$ + | − | $h_8$–$h_G$ + | − |
|---|---|---|---|---|---|---|---|---|---|---|---|
| CH. 4/4 # 14 | $\delta P/\delta e$ | *31 3* | | *4·9* | | *9·4* | | *12·9* | | | *215·5* |
| CH. 4/4 # 14 | $[p\,.\,\delta h/\delta e]_G$ | ——— | | ——— | | ——— | | ——— | | *230·3* | |
| | $\frac{\delta}{\delta e}(M_E^2/R)$ | *0·5* | | | *3·4* | | *2·7* | | *0·5* | *2·6* | |
| | $\frac{\delta}{\delta n}(M_E M_N/R)$ | *0·2* | | | *0·1* | *1·5* | | *0·6* | | | *0·7* |
| Form P XVI | $m_E v_H$ at upper limit | ——— | | | *(0·0)* | | *(0·9)* | | *(0·2)* | | *(0·7)* |
| Form P XVI | $-m_E v_H$ at lower limit | *(0·0)* | | *(0·9)* | | *(0·2)* | | *(0·7)* | | * no term here | |
| | $-2\omega\sin\phi\,.\,M_N$ | *2·0* | | *7·0* | | | *3·3* | | *6·5* | | *6·2* |
| Form P XVI | $+2\omega\cos\phi\,.\,M_H$ | *(0·01)* | | *(0·02)* | | *(0·03)* | | *(0·02)* | | *(0·01)* | |
| Form P XVI | $+3M_E M_H/(aR)$ | | *(0·0)* | | *(0·0)* | | *(0·0008)* | | *(0·0)* | | *(0·0)* |
| | $-2M_E M_N\tan\phi/(aR)$ | | *0·0* | | *0 2* | | *0·1* | | *0·1* | | *0·1* |
| Form M I | viscosity terms | | *0·0* | | *0·0* | | *0·0* | *0·2* | | | *1·4* |
| Form M II | stratosphere, special | | *0·2* | ——— | | ——— | | ——— | | ——— | |
| | sums + and − | *34·0* | *0·2* | *12·8* | *3·7* | *11·1* | *7·0* | *14·4* | *7·3* | *232·9* | *224·6* |
| | $-\partial M_E/\partial t$ | *+33·8* | | *+9·1* | | *+4·1* | | *+7·1* | | *+8·3* | |
| | $+\delta t\,.\,\partial M_E/\partial t$ | $-730 \times 10^3$ | | $-196 \times 10^3$ | | $-89 \times 10^3$ | | $-153 \times 10^3$ | | $-179 \times 10^3$ | |

COMPUTING FORM M IV. For the Dynamical Equation for the Northward Component

$$-\frac{\partial M_N}{\partial t} = -g_N R + \frac{\partial P}{\partial n} + \left[p\frac{\partial h}{\partial n}\right]_G + \frac{\partial}{\partial e}\left(\frac{M_N M_E}{R}\right) + \frac{\partial}{\partial n}\left(\frac{M_N^2}{R}\right) + \left[m_N v_H\right]_8 + 2\omega\sin\phi\,.\,M_E + \frac{3M_N M_H}{aR} + \frac{\tan\phi\{M_E^2 - M_N^2\}}{aR}.$$

The above equation relates to the stratum $h_G$ to $h_8$. For upper strata, omit the term $p\frac{\partial h}{\partial n}$, and subtract a term $m_N v_H$ at the lower boundary.

Longitude *11° E* Latitude *5600 km N* $2\omega\sin\phi = 1{\cdot}124 \times 10^{-4}$, $2\omega\cos\phi = 0{\cdot}930 \times 10^{-4}$ Instant *1910 May 20ᵈ 7ʰ G.M.T.* $\delta t = 6$ *hours*

$\frac{\tan\phi}{a} = 1{\cdot}897 \times 10^{-9}$; $\delta e = 425 \times 10^5$ for $\delta\lambda = 6°$

| REF. | Term | $h_0$–$h_2$ + | − | $h_2$–$h_4$ + | − | $h_4$–$h_6$ + | − | $h_6$–$h_8$ + | − | $h_8$–$h_G$ + | − |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Table in Ch. 4/4 | $-g_N R$ | *2·7* | | *1·5* | | *0·9* | | *0·4* | | *0·1* | |
| Ch. 4/4 # 14 | $\delta P/\delta n$ | *24·1* | | | *1·7* | *2·3* | | *7·7* | | *498·3* | |
| Ch. 4/4 # 14 | $[p\,.\,\delta h/\delta n]_G$ | ——— | | ——— | | ——— | | ——— | | | *487·8* |
| | $\frac{\delta}{\delta e}(M_N M_E/R)$ | | *0·0* | *4·1* | | *2·5* | | *0·4* | | | *2·1* |
| | $\frac{\delta}{\delta n}(M_N^2/R)$ | | *0·4* | *0·5* | | | *1·3* | | *0·4* | *0·3* | |
| Form P XVI | $m_N v_H$ at upper limit | ——— | | *(0·7)* | | | *(0·1)* | *(0·1)* | | *(0·4)* | |
| Form P XVI | $-m_N v_H$ at lower limit | | *(0·7)* | *(0·1)* | | | *(0·1)* | | *(0·4)* | no term here | |
| | $+2\omega\sin\phi\,.\,M_E$ | | *6·3* | | *16·4* | | *10·7* | | *5·8* | | *12·4* |
| Form P XVI | $3M_N M_H/(aR)$ | | *(0·0)* | | *(0·0)* | *(0·0002)* | | *(0·0)* | | *(0·0)* | |
| | $\tan\phi\{M_E^2 - M_N^2\}/(aR)$ | *0·0* | | *0·2* | | *0·1* | | | *0·0* | *0·1* | |
| Form M I | viscosity terms | | *0·0* | | *0·0* | | *0·0* | | *0·0* | *0·2* | |
| Form M II | stratosphere, special | | *3·8* | ——— | | ——— | | ——— | | ——— | |
| | sums + and − | *26·8* | *11·2* | *7·1* | *18·1* | *5·8* | *12·2* | *8·6* | *6·6* | *499·4* | *502·3* |
| | $-\partial M_N/\partial t$ | *+15·6* | | *−11·0* | | *−6·4* | | *+2·0* | | *−2·9* | |
| | $+\delta t\,.\,\partial M_N/\partial t$ | $-337 \times 10^3$ | | $+238 \times 10^3$ | | $+138 \times 10^3$ | | $-43 \times 10^3$ | | $+63 \times 10^3$ | |

Table 7.7. *Terms in eastward momentum equation and tendency of eastward momentum (kg m$^{-1}$ s$^{-2}$).*

| | Pressure Gradient Term | Coriolis Term | Remaining Terms | Tendency of Momentum | Ageostrophic Wind |
|---|---|---|---|---|---|
| Layer | $-\frac{\partial P}{\partial x}$ | $+fV$ | — | $\frac{\partial U}{\partial t}$ | $f(V - V_{\text{geo}})$ |
| I | $-3.13$ | $-0.20$ | $-0.05$ | $-3.38$ | $-3.33$ |
| II | $-0.49$ | $-0.70$ | $+0.28$ | $-0.91$ | $-1.19$ |
| III | $-0.94$ | $+0.33$ | $+0.20$ | $-0.41$ | $-0.61$ |
| IV | $-1.29$ | $+0.65$ | $-0.07$ | $-0.71$ | $-0.64$ |
| V | $-1.48$ | $+0.62$ | $+0.03$ | $-0.83$ | $-0.86$ |
| Mean Abs. Value | 1.47 | 0.50 | 0.13 | 1.25 | 1.33 |

Table 7.8. *Terms in northward momentum equation and tendency of northward momentum (kg m$^{-1}$ s$^{-2}$).*

| | Pressure Gradient Term | Coriolis Term | Remaining Terms | Tendency of Momentum | Ageostrophic Wind |
|---|---|---|---|---|---|
| Layer | $-\frac{\partial P}{\partial y}$ | $-fU$ | — | $\frac{\partial V}{\partial t}$ | $f(U - U_{\text{geo}})$ |
| I | $-2.41$ | $+0.63$ | $+0.22$ | $-1.56$ | $+1.78$ |
| II | $+0.17$ | $+1.64$ | $-0.71$ | $+1.10$ | $-1.81$ |
| III | $-0.23$ | $+1.07$ | $-0.20$ | $+0.64$ | $-0.84$ |
| IV | $-0.77$ | $+0.58$ | $-0.01$ | $-0.20$ | $+0.19$ |
| V | $-1.05$ | $+1.24$ | $+0.10$ | $+0.29$ | $-0.19$ |
| Mean Abs. Value | 0.93 | 1.03 | 0.25 | 0.76 | 0.96 |

are also large and positive and would result in saturation of the entire troposphere within a few days, a highly improbable eventuality.

Finally, the stratospheric temperature prediction is clearly unreasonable. There are spuriously large vertical velocities associated with the strong divergence field.

These give rise, through equation (5.20), to a large change in stratospheric temperature. Richardson's value, given in his Form $\mathrm{P}_{\mathrm{XIV}}$, is $9.2 \times 10^{-4}\,\mathrm{K\,s^{-1}}$, or $19.6\,\mathrm{K}$ in six hours. At this rate, the stratosphere would warm by 80 degrees in a day, an outlandish prediction.

## 7.4 The causes of the forecast failure

Richardson blamed the observed winds for the failure of his forecast. In his Summary (Ch. 1 of WPNP) he wrote of how the forecast was spoilt by errors in the wind data. He continued: 'These errors appear to arise mainly from the irregular distribution of pilot balloon stations, and from their too small number' (WPNP, p. 2). Again, on page 187, he says 'This glaring error is ... traced to errors in the representation of the initial winds'. In his discussion of convergence (WPNP, p. 212) he wrote: 'The striking errors in the "forecast" ... may be traced back to the large apparent convergence of wind.' He then asked whether the spurious convergence arose from the errors of balloon observations, or from the grid resolution being too large, or from the interpolation process from the observation points to the computation grid.

To investigate the calculation of divergence from observations, avoiding errors resulting from interpolation to a regular grid, Richardson computed the divergence directly from observations at three stations, Hamburg, Strasbourg and Vienna. These stations are approximately at the vertices of an equilateral triangle of side 700 km. He obtained a value of $3.05 \times 10^{-2}\,\mathrm{kg\,m^{-2}\,s^{-1}}$ for the total convergence in a column. This corresponds to a rise in pressure of about 60 hPa in six hours, or 10 Ber. He gave the following explanation: 'stations as far apart as 700 kilometres did not give an adequate representation of the wind in the lower layers.' This is a rather surprising conclusion. Elementary scale analysis shows that, for a given error in the observed wind, the error in the computed divergence actually increases as the spatial distance is reduced. Since the wind errors do not depend on the distance between stations, the error in computed divergence would actually be worse for stations more closely located. Richardson did not explicitly consider the compensation between influx and outflow that results in maintaining the divergence at a small value.

Shaw was fully aware of the importance of balance in the atmosphere and had a good understanding of it. In a preface to a Geophysical Memoir (Fairgrieve, 1913) on the closeness of observed winds to geostrophic balance, he wrote:

> ...the balance of [pressure] gradient and wind velocity is the state to which the atmosphere always tends, except in so far as it may be disturbed by the operation of new forces. If we suppose the balance once established, any disturbance by convection or otherwise must act in the free air by infinitesimal stages, and during every stage the tendency to restore the balance is continuously operative.

> Hence the transition from one set of conditions to another must be conducted by infinitesimal stages during which the disturbance of balance is infinitesimal.

This amounts to a clear, qualitative description of the process of geostrophic adjustment which maintains the atmosphere in a state close to balance. Richardson joined the Met Office in 1913, the year this Memoir appeared and he must undoubtedly have studied it. He certainly was aware of the prevalance of quasi-geostrophic balance, but he also understood that the geostrophic wind was not useful for computing the divergence: '... for purposes not connected with finding the divergence of wind, the geostrophic hypothesis appears to serve as well in the stratosphere as elsewhere.' (WPNP, p. 146)

For his barotropic forecast (WPNP, Ch. 2), Richardson chose geostrophic initial winds. However, the implication of Eqn. 4.8 (p. 67) that the pressure disturbances move westward led him to infer that the geostrophic wind is inadequate for the calculation of pressure changes. Purely geostrophic flow implies convergence for poleward flow and divergence for equatorward flow. On the basis of these results, he concluded that geostrophic winds would not serve adequately as initial conditions.

The initial winds tabulated and used by Richardson are wildly out of balance. There is huge disagreement between the pressure gradient and Coriolis terms in Tables 7.7 and 7.8. They should have values approximately equal in magnitude and opposite in sign. In the final columns of the tables, the components of the ageostrophic winds at each level are given (actually $f(U - U_{\mathrm{geo}})$ and $f(V - V_{\mathrm{geo}})$ are tabulated). The figures in the bottom rows show that the ageostrophic wind is comparable in magnitude to the total wind: the initial state is not remotely close to geostrophic balance.

Richardson placed great emphasis on the unrealistic divergence and little on the ageostrophic initial winds. He saw the solution as smoothing of the initial data to produce a realistic divergence field. However, it is insufficient to reduce the divergence to a small level: we saw in §4.6 that setting the divergence to zero at the initial time did not ensure that it remains small. Moreover, Richardson's analysis misses the point that what was needed was a *mutual adjustment* which would ensure harmony between the mass and wind fields.

If the mass and wind are not in balance, large tendencies are inevitable. As a simple example, we consider the barotropic forecast again. If the initial wind field is shifted by 180°, the initial divergence is reversed in sign, so the the initial pressure tendency is unchanged in magnitude but of the opposite sign. However, the fields are now drastically out of balance: instead of cancelling, the pressure gradient and Coriolis terms reinforce each other. The initial momentum tendencies are now

$$\frac{\partial U}{\partial t} = f(V - V_{\text{geo}}) = 2fV \qquad \frac{\partial V}{\partial t} = -f(U - U_{\text{geo}}) = -2fU\,, \tag{7.11}$$

suggesting a time-scale $\mathcal{T} = f^{-1} \approx 10^4\,\text{s} \approx 3\,$hours, a small fraction of the synoptic time-scale. Extrapolating these tendencies over an extended time interval results in a nonsensical forecast. In reality, such extreme initial conditions would engender massive gravity waves with wildly oscillating tendencies.

In summary, the spuriously large magnitude of the divergence of Richardson's initial winds, due to the absence of cancellation of terms arising from the compensation effects described in §7.2.3, made it inevitable that the initial pressure change would be catastrophic or, more correctly, 'anastrophic'. The large departure of his data from geostrophy meant that large initial momentum tendencies were unavoidable.

## 7.5 Max Margules and the 'impossibility' of forecasting

In a short paper published in the *Festschrift* to mark the sixtieth birthday of the renowned physicist Ludwig Boltzmann, Max Margules examined the relationship between the continuity equation and changes in surface pressure (Margules, 1904). A translation of this paper, together with a short introduction, has been published as a Historical Note by Met Éireann, the Irish Meteorological Service. Margules considered the possibility of predicting pressure changes by direct use of the mass conservation principle. He showed that, due to strong cancellation between terms, the calculation is very error-prone, and may give ridiculous results. Therefore, it is not possible, using the continuity equation alone, to derive a reliable estimate of synoptic-scale changes in pressure. Margules concluded that any attempt to forecast the weather was *immoral and damaging to the character of a meteorologist* (Fortak, 2001).

To make his forecast of the change in pressure, Richardson used the continuity equation, employing precisely the method that Margules had shown more than ten years earlier to be seriously problematical. As we have seen, the resulting prediction of pressure change was completely unrealistic. The question of what influence, if any, Margules' results had on Richardson's approach to forecasting was considered by Lynch (2003c). A copy of Margules' article was received and catalogued by the Met Office Library in March, 1905. Thus, it was available for consultation by scientists such as Shaw and Dines. If Shaw, who was fully aware of Richardson's weather prediction project and indeed supported it strongly, knew of Margules' work, he would surely have alerted Richardson to its existence. There was ample opportunity for this between 1913, when Richardson was appointed

Fig. 7.1. Max Margules (1856-1920). Photograph from the archives of Zentralanstalt für Meteorologie und Geodynamik, Wien.

Superintendent of Eskdalemuir Observatory, and 1916, when he resigned in order to work with the Friends Ambulance Unit in France.

There is no reason to believe that Richardson was aware of Margules' paper; certainly, he makes no reference to it in his book. If he had been aware of Margules' results, he might well have decided not to proceed with his trial forecast, or sought a radically different approach (Platzman, 1967). Margules' results are summarized in Exner's *Dynamische Meteorologie*, published in 1917. It is possible that Richardson realized the significance of Margules' results when he read Exner's book. But, since he made no reference to the relevant section of Exner, it seems more likely that he simply overlooked it. At a later stage, Richardson did come to a realization that his original method was unfeasible. In a note contained in the Revision File, inserted in the manuscript version of his book, he wrote 'Perhaps the most important change to be made in the second edition is that the equation of continuity of mass must be eliminated' (Richardson's underlining). Unfortunately, this note, which is reproduced in Platzman, 1967, is undated so we cannot say when Richardson reached this conclusion. He went on to speculate that the vertical component of vorticity might be a suitable prognostic variable.

This was indeed a visionary anticipation of the use of the vorticity equation for the first successful numerical integration by Charney *et al.* in 1950.

Of course, we now know that Margules was unduly pessimistic. The continuity equation is an essential component of primitive equation models which are used in the majority of current computer weather prediction systems. These models support gravity wave solutions and, when changes in pressure are computed using the continuity equation, large tendencies can arise if the atmospheric conditions are far from balance. However, spuriously large tendencies are avoided in practice by an adjustment of the initial data to reduce gravity wave components to realistic amplitudes. This is the process of initialization, which we discuss in detail in the next chapter.