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Book Reviews

Edited by Robert E. O'Malley, Jr.

Featured Review: Geometric Mechanics.

Geometric Mechanics, Part I: Dynamics and Symmetry. *By Darryl D. Holm.* Imperial College Press, London, 2008. $68.00. xx+354 pp., hardcover. ISBN 978-1-84816-195-5.

Geometric Mechanics, Part II: Rotating, Translating and Rolling. *By Darryl D. Holm.* Imperial College Press, London, 2008. $68.00. xvi+294 pp., hardcover. ISBN 978-1-84816-155-9.

Lagrange was reputedly proud to say that his book *Méchanique Analytique* did not have a single diagram or illustration. His approach to the fundamental principles of mechanics was severely analytical. In stark contrast, Hamilton's formulation was immersed in a geometric framework. Modern workers recognize the power and beauty of the geometric approach, which gives us great insight into the behavior of mechanical systems and provides a unifying paradigm that guides us in tackling new and recondite problems.

Professor Holm's two-volume work introduces geometric mechanics through the consideration of a range of explicit examples. Part 1 is subtitled *Dynamics and Symmetry*, and Part 2 *Rotating, Translating and Rolling*. The book is suitable for graduate students and (talented) advanced undergraduates.

The underlying ideas of geometric mechanics first arose in the study of optics by, among others, Fermat and Huygens. There is a duality between tangents to least-time paths (Fermat) and normals to wave fronts (Huygens) in classical optics. This corresponds to the duality between velocities and momenta in geometric mechanics. Fermat's principle leads naturally to Hamilton's principle, phase space, symplectic transformations, and momentum maps. Fermat's principle provides a guiding example of the principles of geometric mechanics that is beautifully developed in this book.

Hamilton's unified treatment of optics and mechanics was inspirational in the development of quantum mechanics and continues to provide a gateway to modern physics. The geometric approach was further greatly advanced by Poincaré. In the present work, the dynamics of coupled resonant oscillators are treated as flows along curves on manifolds, in the manner of Poincaré. Poincaré's geometric approach is applied through Lie symmetry. By Noether's theorem, each Lie symmetry is associated with a conservation law. The text provides an introduction to the basic tools of Lie derivatives and exterior calculus, both of which are needed to take advantage of the geometric framework.

The book treats several key examples in classical mechanics to provide a concrete application of the mathematical methods that are introduced. These examples include Fermat's principle of ray optics, bifurcations in resonant oscillators, polarized wave pulses in optical fibers, the elastic pendulum, and other mechanical systems. In each case, the geometric analysis reduces to divergence-free flow on $\mathbf{R}^3$.

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As an example of the unifying nature of the geometric approach, Chapters 5 and 6 treat two problems of very different physical nature, the spherical swinging spring and the Maxwell–Bloch equations for self-induced transparency in laser-matter interaction. In both cases, the averaged Lagrangian method results in a Lie symmetry that reduces the dimensionality of the dynamics, making the problems tractable. Bifurcations are topological transitions in the intersections of level surfaces of orbit manifolds of the Hamiltonian or momentum map. Holm shows that the geometric approach reveals how bifurcations arise under changes of parameter values in, for example, the system Hamiltonian. The method allows a visual classification of bifurcations. An application of the method to optical traveling wave pulses clarifies this.

A formidable arsenal of mathematical techniques is built up in the text. We have differential forms using wedge products on local bases, push-forward and pull-back operators on differential forms under smooth invertible maps, contraction of a vector field to a differential form, Lie derivatives, Nambu brackets, the Hodge star operator, and tangent and cotangent bundles.

A very thorough analysis of the elastic pendulum or swinging spring is presented. This provides a lucid application of the theoretical machinery developed earlier. The dynamical equations reduce, under Lagrangian averaging, to the three-wave equations. These equations are equivalent to the Maxwell–Schrödinger equations for interaction between radiation and a two-level medium in a microwave cavity. The three-wave system describes a wealth of physical phenomena and so is another valuable unifying paradigm.

In the second volume, *Rotating, Translating and Rolling*, more difficult non-holonomic problems are addressed, such as Chaplygin's top and Euler's disk. The Euler–Poincaré equations are the underlying context for all these applications. These equations arise from Lie symmetry reduction in the Lagrangian context. The corresponding system on the Hamiltonian side is the Lie–Poisson equation. Much of Part 2 is devoted to elucidating this relationship.

Students who work carefully through the material in these volumes will amass a formidable armory of mathematical techniques and will be well equipped to attack new and challenging problems in mechanics. But this mastery will not come easily. Readers require a solid background in linear algebra, calculus, and generalized classical mechanics. The pace of development is intense: new ideas, definitions, and notation are introduced in rapid-fire mode. Students will have to apply themselves with diligence and energy to keep up. But they will be rewarded by the achievement of mastering a body of theory and techniques that is elegant and elegantly presented in these books.

The appendices contain a collection of valuable example problems that are suitable for both homework and enhanced coursework. There are also numerous exercises scattered throughout the text to allow readers to evaluate their progress.

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Algebraic Models in Geometry. *By Y. Félix, J. Oprea, and D. Tanré.* Oxford University Press, Oxford, 2008. $55.00. xxii+460 pp., softcover. ISBN 978-0199-2065-20.

Postnikov resolutions, or Postnikov towers as they are often known, show how to construct the homotopy type of a topological space one homotopy group at a time. The