# Potential Flows in a T-Junction

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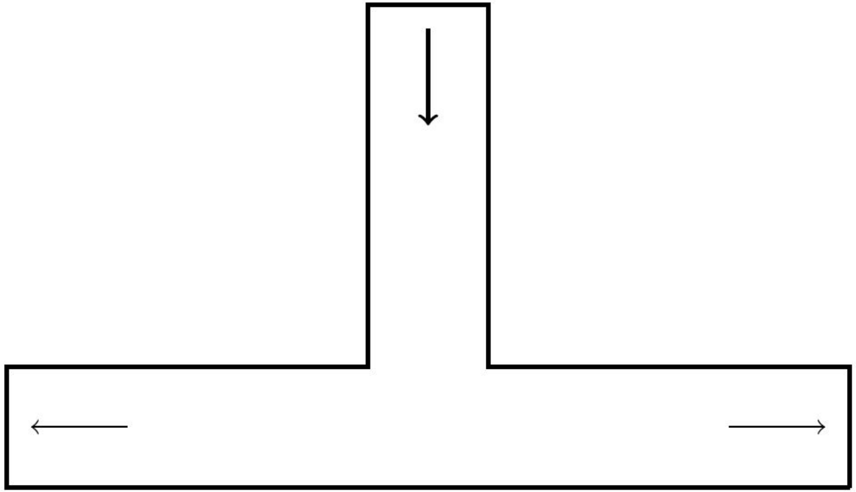
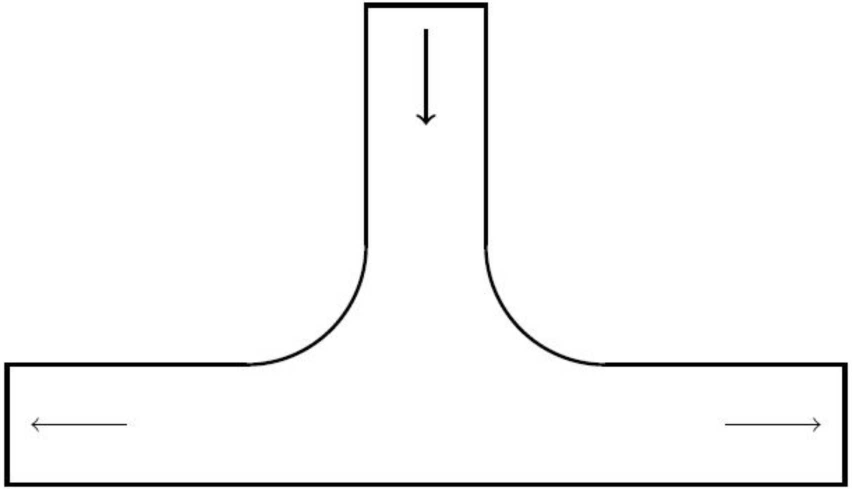
Summer Undergraduate Research Project

# The T-Junction

Sharp corners



Rounded Corners



# Potential Flows

- Irrotational and incompressible fluid flows which satisfy $\nabla \times \vec{v} = 0$ and $\nabla \cdot \vec{v} = 0$ where $\vec{v}$ is the flow velocity.

- This allows for a potential function $\varphi$ which satisfies $\vec{v} = \nabla\varphi$ and $\nabla^2\varphi = 0$

- We can use a solver for Laplace's equation to obtain the potential function for this kind of flow for some given boundary condition.
- We obtain the fluid velocity $\vec{v} = \nabla\varphi$ from this by taking derivatives of $\varphi$

$$\vec{v} = (u, v) = (\frac{\partial\varphi}{\partial x}, \frac{\partial\varphi}{\partial y})$$

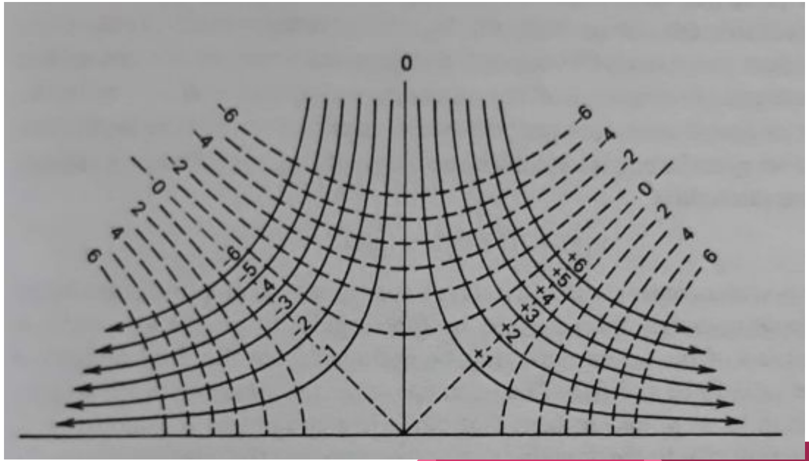
- We look at 2D flows with complex potential functions $w = \varphi + i\psi$

# Stagnation Point Flow

- T-junction flows are often compared to stagnation point flows.
- This is the flow a fluid colliding with a wall will take.

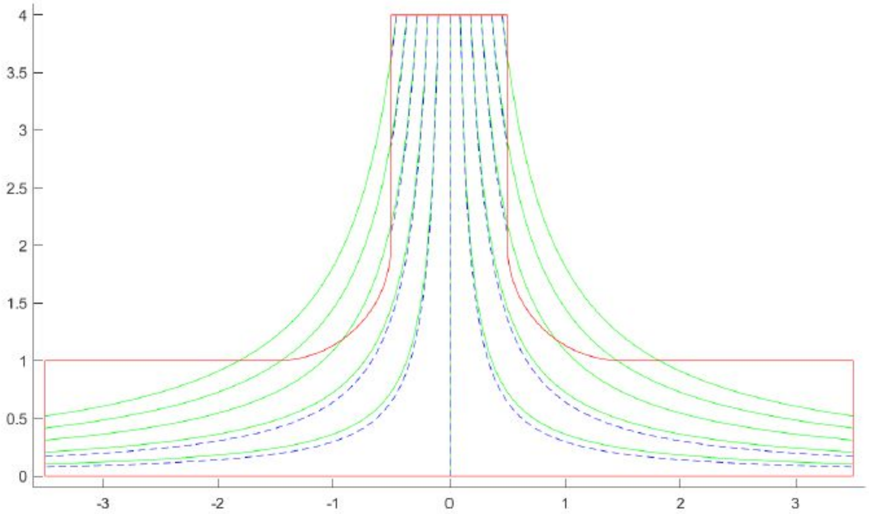
$$w = z^2 = x^2 - y^2 + 2ixy$$
$$z = x + iy$$

$$\varphi = x^2 - y^2,\ u = 2x,\ v = -2y$$



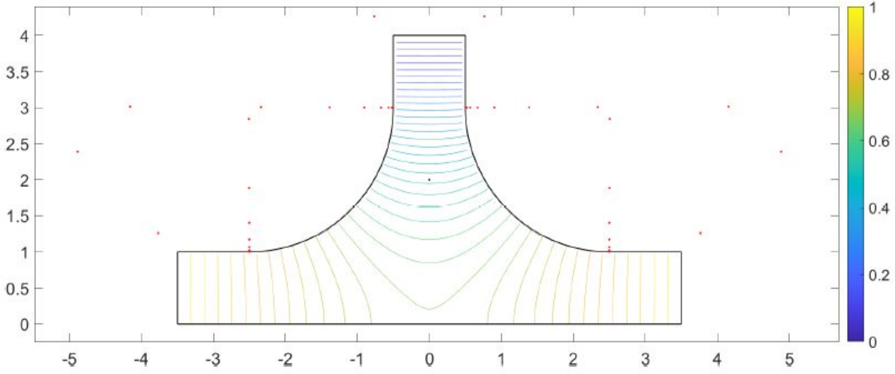
# Stagnation Point Flows

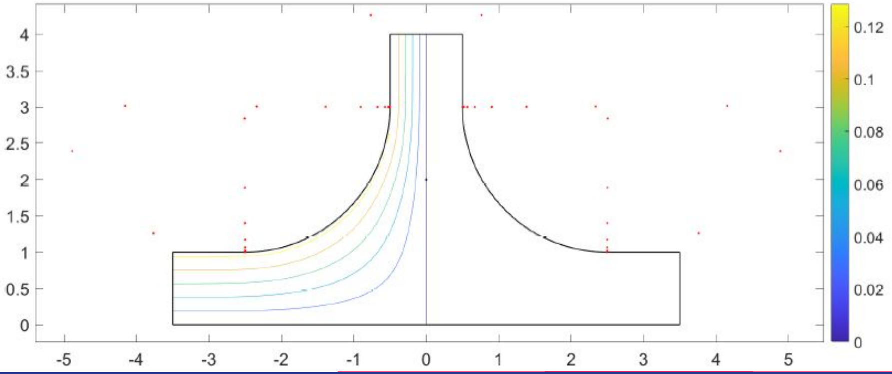
- The streamlines of the stagnation point flow pass through the walls of the pipe.
- Most particles in this flow hit the side of the side of the pipe before reaching the junction.
- Any particles that make it through end up coming out near the bottom of the outlet.



# The Lightning Laplace Solver

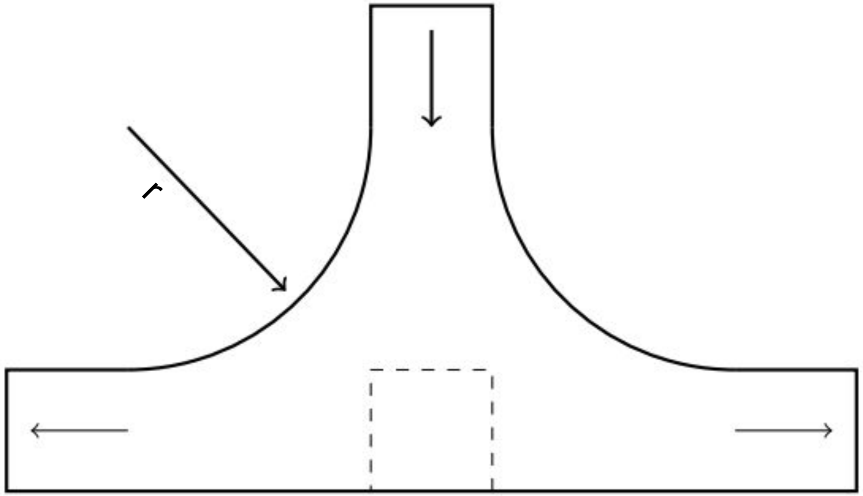
- Finds the complex potential function for any flow with polygonal or circular boundary conditions.
- Uses this to get potential lines and streamlines.
- Allows us to find the flow velocity for any point in the T-junction.
- Singularities in the flow will arise near any sharp corners.



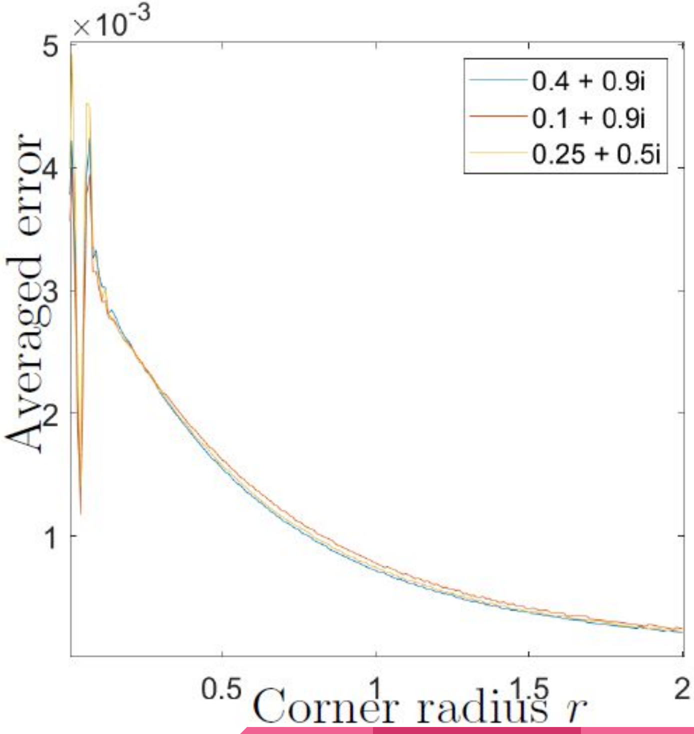
# Comparison with Stagnation Point Flow

- We find the potential function, f, for various r values
- We adjust this function so f(0,0) = 0 so it matches up with the stagnation point flow.
- We compare this function to the stagnation point flow in the dotted region.



# Comparison with Stagnation Point Flow

- We compare f with $w = kz^2$
- k is scaled so it matches f at some point $z = x + iy$
- The error is found by taking $|f(x,y) - w(x,y)|$ at 500 points in the dotted region and getting their average.
- We plot the error vs r and repeat for different z-values.

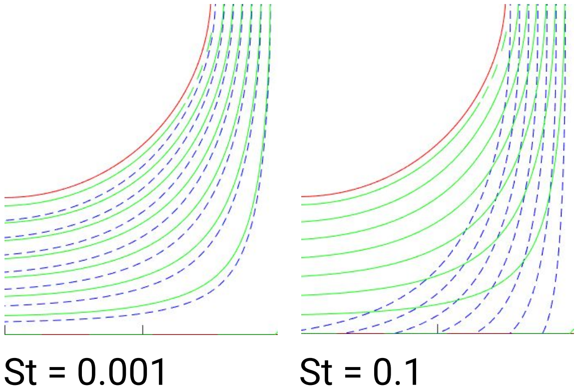


# Particle Paths

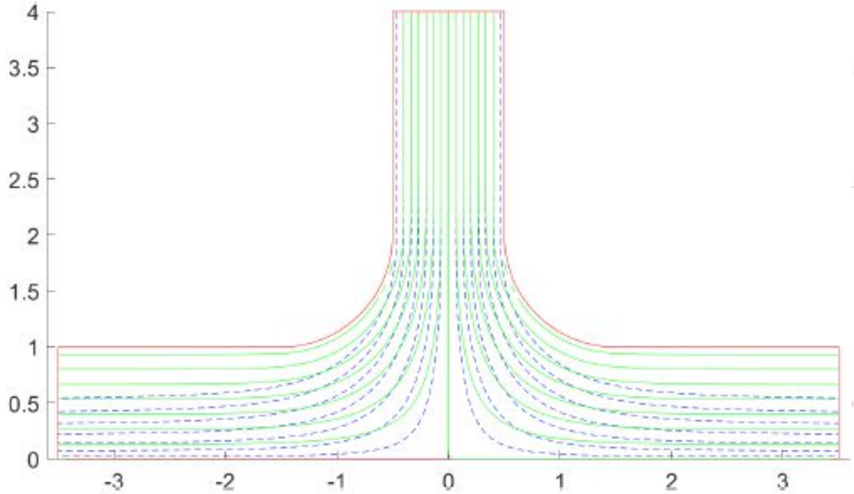
- The main force on particles in the flow is drag force by the fluid viscosity

$$St\,\ddot{\mathbf{x}}_p = \frac{C_D}{2}(\mathbf{u} - \dot{\mathbf{x}}_p)|\mathbf{u} - \dot{\mathbf{x}}_p|$$

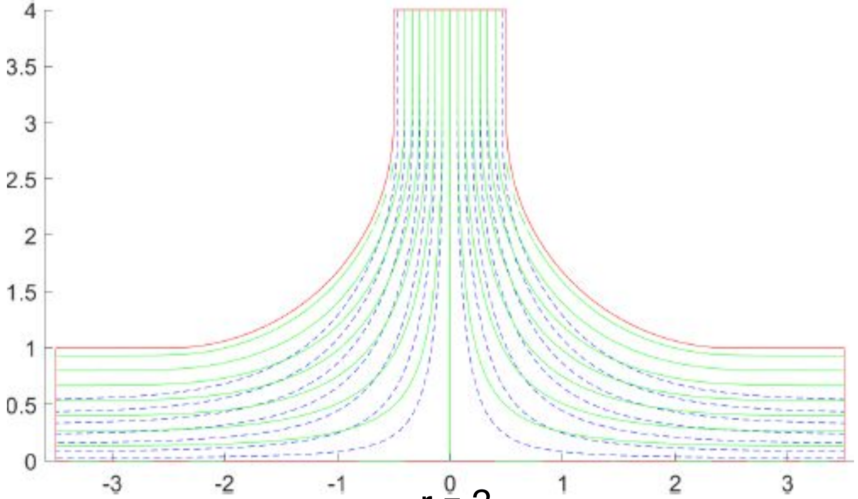
- This is equivalent to Newton's 2nd Law, F = ma.
- St is the particle's Stokes number in the fluid, and is describes the inertia of the particle which it can use to deviate from the streamlines in a flow that is changing direction.



# Particle Paths

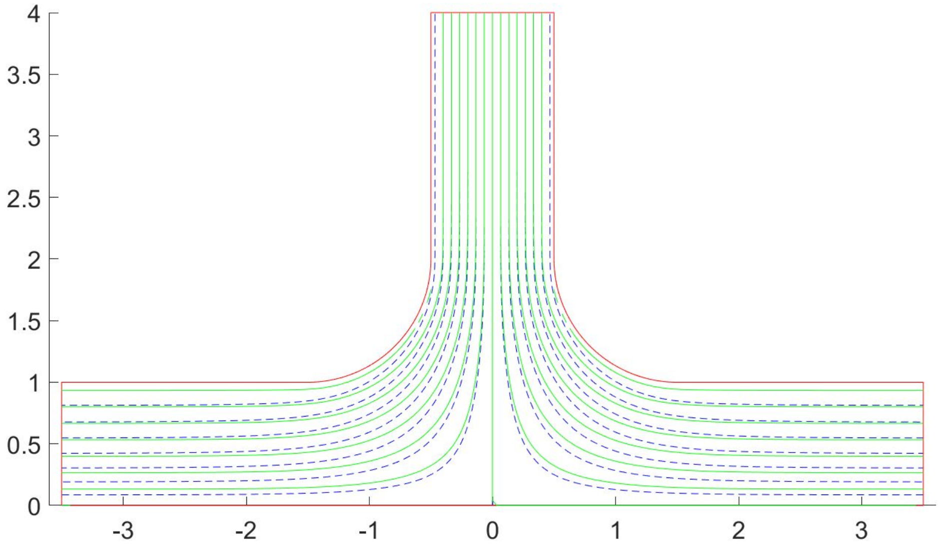



r = 1

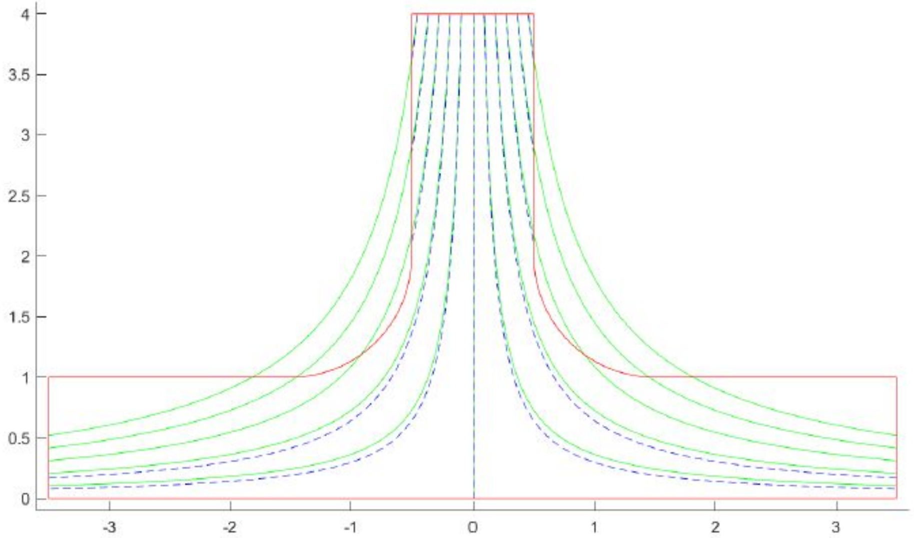



r = 2

# Comparing with Stagnation Point Flow



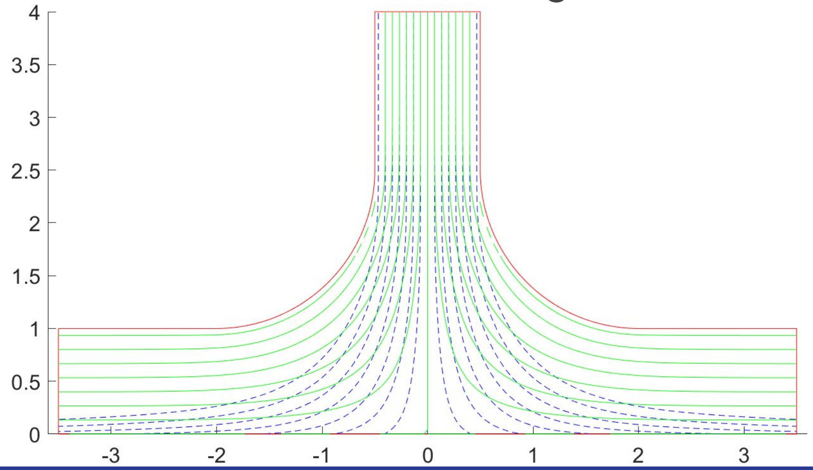

Laplace Solver Solution

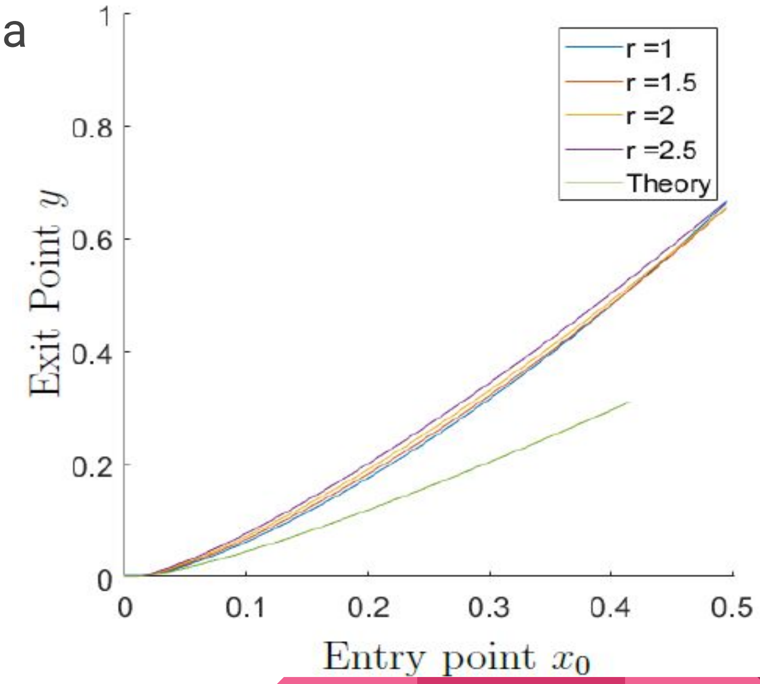



Stagnation Point Flow

# Particle Paths

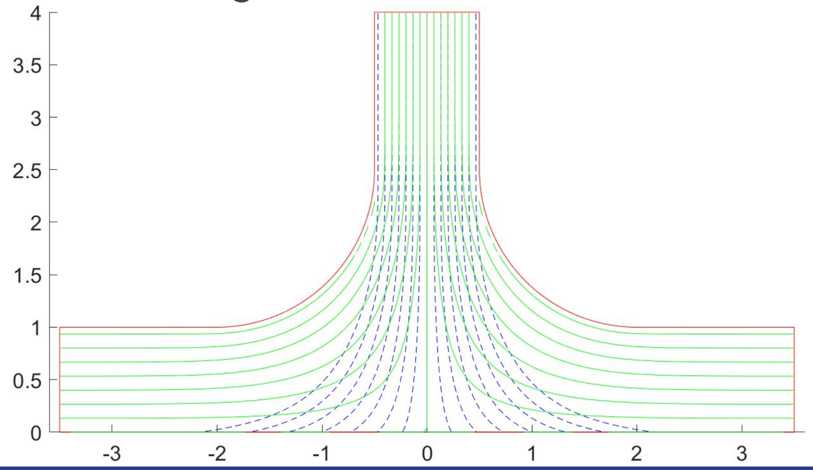
- Particles starting closer to the edge exit at a higher point.
- Points near the center collide with the bottom wall before exiting.

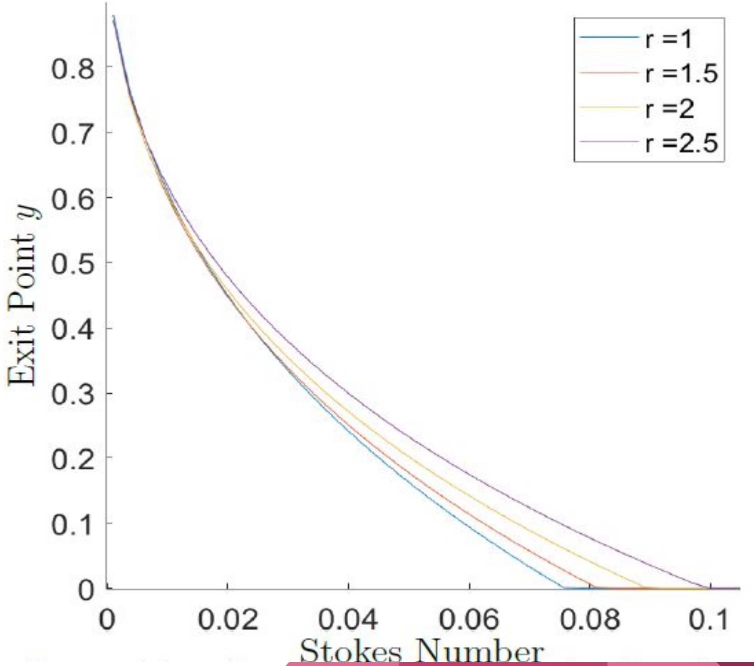


# Stoke Numbers

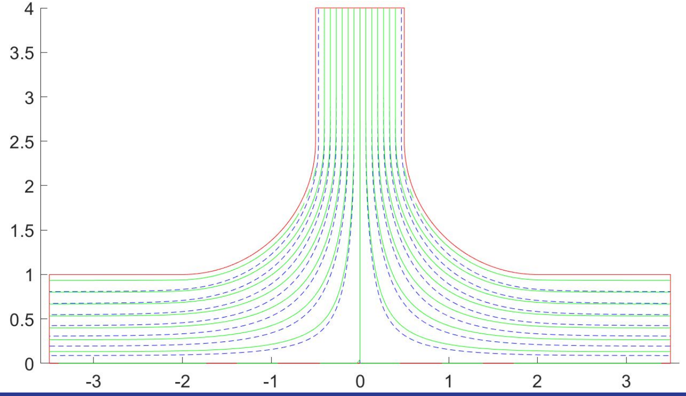
- Lower Stokes numbers let particles follow the fluid flow easier.
- All particles will hit the bottom wall first for high Stokes numbers.

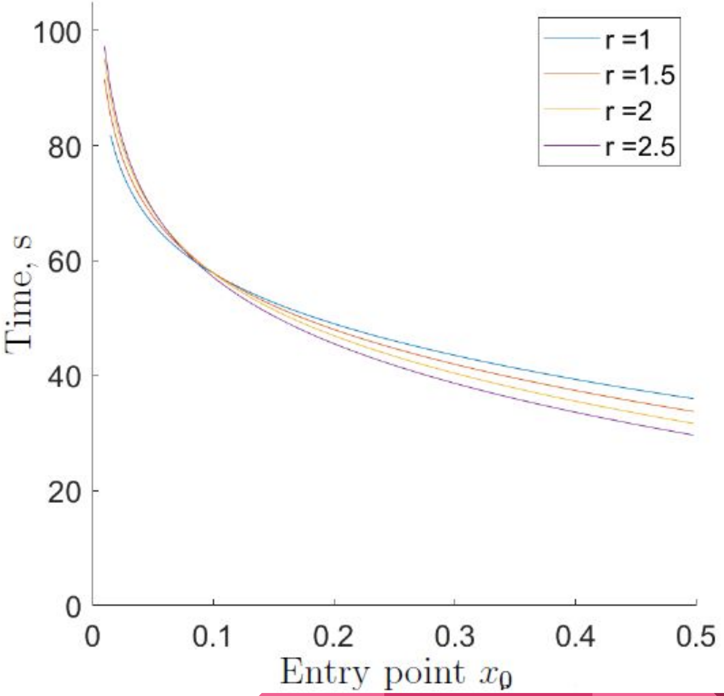


# Particle Hang Times

- Particles closer to the side will exit sooner.
- Particles near the center of the pipe take longer as they are closer to the stagnation point.



# Conclusion

- Stagnation point flows are a less accurate model for sharp corner flows.
- Particle paths are mainly affected by their Stokes number, and more weakly by the corner radius of the junction.
- More accurate models for flows in T-junction geometries yield better physics simulations for particle paths.