Flow Stability in Point Heated Droplets

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THERMASMART Thermal Management of Microprocessors

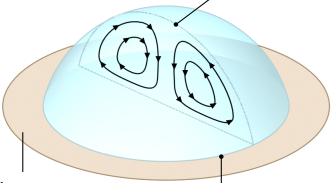
MARIE CURIE ACTIONS

SFI Centre for Research Training in Foundations of Data Science

Marangoni Convection

- Marangoni flow is the flow induced by surface-tension gradients.
- Gradient can be caused by a temperature difference.
- Ehrhard and Davis[a] studied droplets with homogeneous heating.

[a] P. Ehrhard and S. H. Davis, Journal of Fluid Mechanics 229, 365–388 (1991).



Axisymmetric Marangoni current – **homogeneously heated** substrate

Applications?

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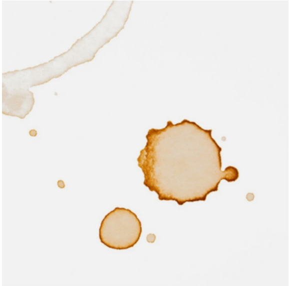
Hu and Larson. "Marangoni Effect Reverses Coffee-Ring Depositions" (2006).



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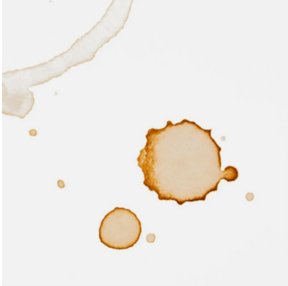
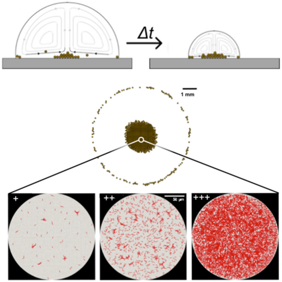
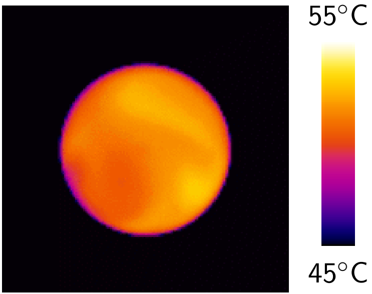


Pearlman et. al., "Controlling Droplet Marangoni Flows to Improve Microscopy-Based TB Diagnosis" (2021).



Motivation

- Localised heating via a laser at the centre of the droplet.
- Twin vortical flow parallel to the substrate was observed.
- Contact angle = 104°, radius = 1.4mm.

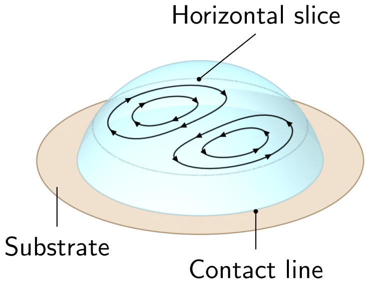


Askounis et al. (2017)[a]

[a] A. Askounis, Y. Kita, M. Kohno, Y. Takata, V. Koutsos, and K. Sefiane, Langmuir 33, 5666 (2017), pMID: 28510453

Problem Statement

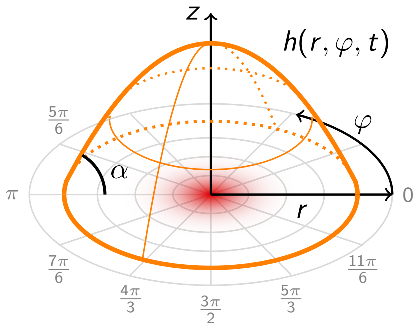
- Can localised heating at the centre induce twin vortical flow parallel to the substrate?
- Deformation of the droplet interface in the case of inhomogeneous heated substrate.



Schematic description of the twin vortical flow

Lubrication Theory

Dramatic assumption – for theoretical understanding of problem.



Cylindrical polar coordinates (r, φ, z)

- When α is small, the Navier–Stokes equations can be simplified.
- The thin-film equation describes the evolution of the interface height h.
- The internal flow (velocity field) can be recovered from h.

Temperature Model

Diffusion in the vertical direction

$$\partial_{zz} T(r, \varphi, z, t) = 0,$$

with boundary conditions

$$\begin{aligned} T &= T_s(r, \varphi), && \text{at } z = 0, \\ -\partial_z T &= \mathrm{Bi}(T - T_{ambient}), && \text{at } z = h. \end{aligned}$$

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The surface tension is a linear function of the (interface) temperature

$$\gamma(\vartheta) = \gamma_0 - \gamma_1 \vartheta(r, \varphi), \qquad \vartheta = T|_{z=h}.$$

Thin-Film Equation

At lowest order in the expansion, we get

$$\partial_t h + \nabla \cdot \left\{ -\tfrac{1}{2} h^2 \underbrace{\mathrm{Ma} \nabla \vartheta}_{\text{Marangoni Stress}} - \tfrac{1}{3} h^3 \nabla \left(\underbrace{-\nabla^2 h}_{\text{Pressure}} + \underbrace{\phi}_{\text{Ext. Potential}} \right) \right\} = 0,$$

where the temperature at the interface is given by

$$\vartheta(r, \varphi) = \frac{T_s(r, \varphi) + \Theta \mathrm{Bi} h}{1 - \mathrm{Bi} h}.$$

We model the substrate temperature with a Gaussian profile.

$$T_s(r) = \mathrm{e}^{-r^2/s^2}.$$

Equilibrium Solution

When $\partial_t h = 0$, the TFE becomes

$$h''' = \tfrac{3}{2}\mathrm{Ma}\frac{\vartheta'}{h} - \frac{h''}{r} + \frac{h'}{r^2}, \qquad r \in [0,1].$$

Solved using the shooting method with boundary conditions

$$h'(0) = 0, \qquad h(1) = 0, \qquad h'(1) = -\alpha.$$

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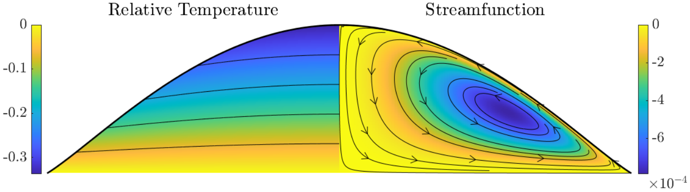
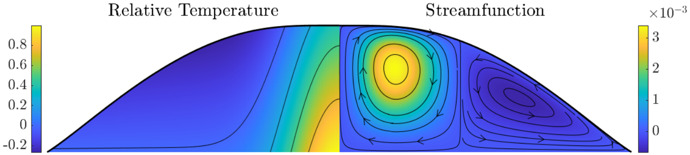
The Stokes stream function is given by

$$\psi(r,z;h) = \tfrac{1}{2}\mathrm{Ma}z^2 r\psi' - \left(\tfrac{1}{2}hz^2 - \tfrac{1}{6}z^3\right) r\frac{\partial}{\partial r}\left(h'' + \frac{h'}{r}\right).$$

Equilibrium Solution



(a) Homogeneous heating $T_s(r) = 0$.



(b) Localized heating $T_s(r) = \mathrm{e}^{-r^2/0.2^2}$.

Linear Stability Analysis

Introduce small perturbation to the equilibrium solution h_0,

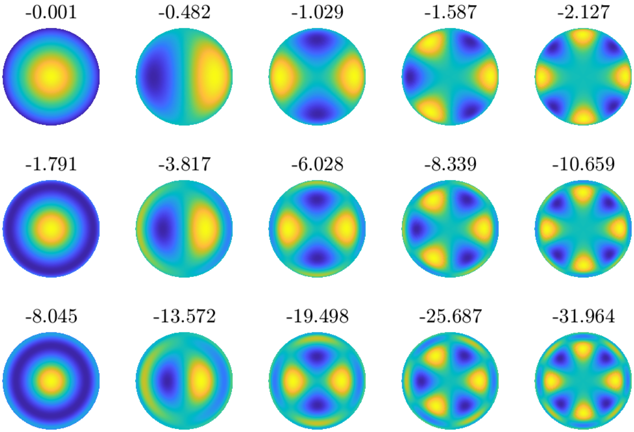
$$h(r,\varphi,t) = h_0(r) + h_1(r)\mathrm{e}^{\sigma t + \mathrm{i}k\varphi}, \qquad k = 0, 1, 2, \ldots.$$

The linearized TFE becomes

$$\mathcal{L}h_1 = A_4 h_1'''' + A_3 h_1''' + A_2 h_1'' + A_1 h_1' + A_0 h_1 = \sigma h_1.$$

This eigenvalue problem can be solved using a Chebyshev method with appropriate boundary conditions.

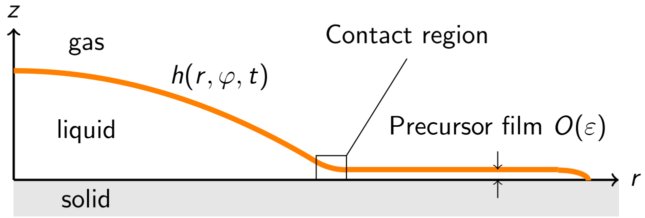
Eigenmodes and Eigenvalues



The equilibrium solutions are stable!

Precursor Film

To understand the onset of the twin vortices, we perform transient simulations. But need to avoid the contact-line singularity. Hence, we introduce a **precursor film**.



The potential has the form:

$$\phi(h) = \mathcal{A}(\varepsilon^2 h^{-2} - \varepsilon^3 h^{-3}).$$

With this, we can simulate **off-centered heating**.

Off-centered Heating

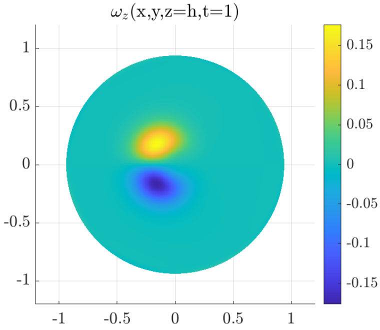
The velocity field can be computed with

$$(u, v) = -\mathrm{Ma} z \nabla \vartheta + \left(\tfrac{1}{2} z^2 - hz\right) \nabla(-\nabla^2 h + \phi),$$

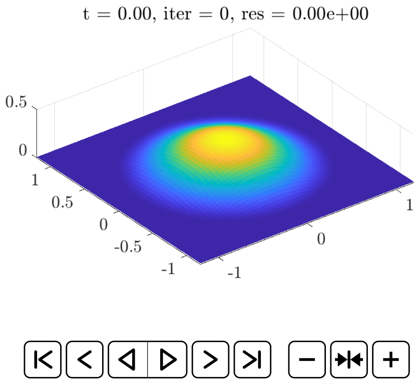
and the z-component of the vorticity is given by

$$\omega_z(x, y, z, t) = \partial_x v - \partial_y u.$$

Simulation of off-centered heating revealed twin vortical flow on the droplet interface.



Rupturing

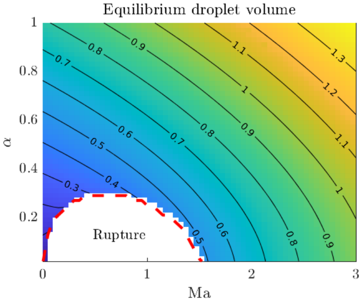


Rupturing occurs for certain parameter values. Physically:

- Low CA α
- High heating power

We provide a precise criterion for rupture in terms of α and Ma next.

Rupture – Parameter Space



Parameter space with regions for droplet solutions and ring-shaped solutions ($r_* = 1$)

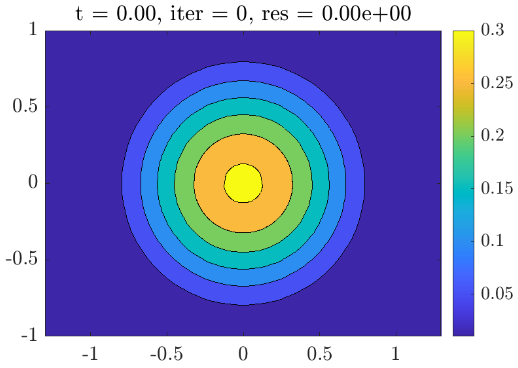
Analysis of the base-state ODE reveals a necessary condition for no rupture in the case $\mathrm{Bi} = 0$:

$$\alpha^2 \geq 3\mathrm{Ma}\left[T_s(0) - T_s(r_*)\right].$$

- $\alpha \sim \mathrm{Ma}^{1/2}$ for small Ma;
- Transition curve 'curves back down towards zero' for finite Bi.

Plateau–Rayleigh Instability

- Slightly off-centred heating and parameter values corresponding to ring rupture.
- Leads to droplet breakup into smaller and larger regions.



Conclusion

- Axisymmetric heating is linearly stable to small perturbation (in the lubrication theory).
- Twin vortical flow can be induced by slight off-centred heating.
- For certain parameter combinations, a ring rupture occurs.

Acknowledgments

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