Integrating Experiments, Simulation, and Theory into Fluids Mechanics Teaching

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Introduction

- Learning Fluid Mechanics in a Maths Department is usually very theoretical. This is not a bad thing - this is the point of a Maths Department.
- Often simulations are used to illustrate the fluid phenomena. As a very important bonus, this teaches students marketable skills in coding, CFD, numerical PDEs, etc.
- But at some point, it is nice to see the equations 'come alive' and manifest themselves through a tangible physical system.
- In a Maths Department, this will be hampered by resource constraints (space, budgets, technical expertise).
- In this talk I will show some case studies where experiments, simulations, and theory can be combined.
- Projects can be offered as final-year projects or potentially combined into a module on 'experimental fluid mechanics'.
- Huge potential here – the smartphone as a 'lab in the pocket'.

Talk Overview

- Two case studies:
  - Tabletop flume
  - Droplet Impact
- Future possibilities.

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  - Droplet Impact
- Future possibilities.

Research Intern — Nicolas Farault (Polytech Lyon)
CFD

Final-Year Project Students — Conor Quigley (UCD), Joseph Anderson (UCD), Patrick Murray (UCD)
High-speed camera work, CFD

Summer Research Student — Nicola Young (UCD – now MSc Applied Mathematics at Imperial)
Theoretical Analysis, Lego

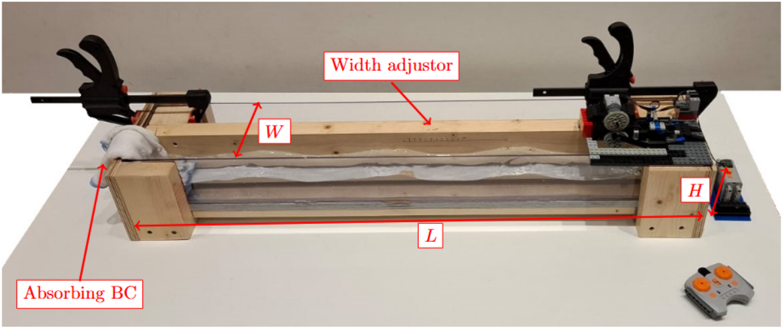
Other students — ACM 40890 Class of 2025 (4th-year BSc / 1st-year PhD UCD)

Case study #1: Tabletop Flume

In this case study... develop a low-cost, integrated framework for studying small-amplitude water waves, combining theory, experiments, and CFD simulations.

- Experiment: A 1-meter tabletop flume with a variable-frequency piston wavemaker made from LEGO.
- Data acquisition: wave motion captured using a smartphone camera.
- Theory: Linear water-wave theory with inlet forcing and appropriate boundary conditions.
- Simulation: OpenFOAM volume-of-fluid
- Goals:
  - Compare experimental observations with linear theory predictions and OpenFOAM CFD simulations.
  - Demonstrate both research and educational value: the setup is accessible for students while producing high-quality, reproducible results.

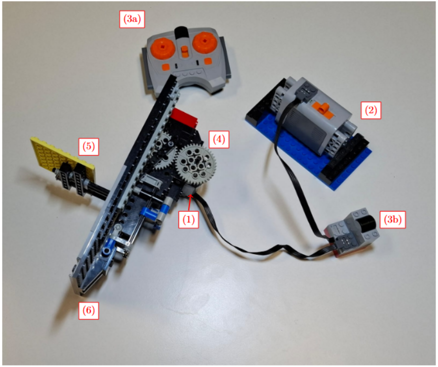
Wave Tank



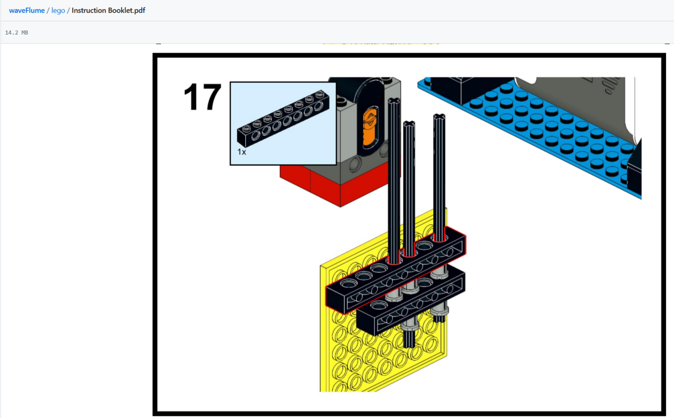
- Simple perspex box, open at top, $L = 1\,\mathrm{m}$, $H = 10\,\mathrm{cm}$, $W = 16\,\mathrm{cm}$.
- Divider to reduce width, for 2D wave train.
- Wooden housing for support, also to mount wavemaker.

Wavemaker

1. Electric motor;
2. Battery pack;
3. Variable RPM controller (parts (3a) and (3b) in figure);
4. Transmission system, with 1:1 gear ratio;
5. Oscillating piston;
6. Housing.



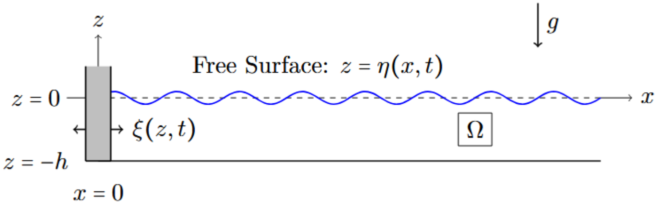
Online presence



- Instructions in Bricklink, downloadable from Github[1] (Credit: Nicola Young).
- Wavemaker in action – YouTube

[1] onaraighl/waveFlume

Theoretical Analysis



- Disturbance at $x = 0$:

$$\xi(z,t) = \Re\left[-\frac{1}{\mathrm{i}\omega} f(z)\mathrm{e}^{-\mathrm{i}\omega t}\right].$$

- Inviscid irrotational flow:

$$\nabla^2\Phi = 0, \qquad x > 0, \qquad -h < z < \eta.$$

- Bottom boundary condition:

$$w = \frac{\partial \Phi}{\partial z} = 0, \qquad z = -h$$

Further Boundary Conditions

- Inlet forcing:

$$u = \frac{\partial \Phi}{\partial x} = \frac{\mathrm{d}\xi}{\mathrm{d}t}, \qquad x = 0.$$

- Boundary condition at $z = \eta$ linearized on to $z = 0$:
  - Kinematic Condition:

$$\frac{\partial \eta}{\partial t} = w \qquad z = 0.$$

  - Dynamic Condition:

$$\rho \frac{\partial \Phi}{\partial t} + \rho g \eta = \gamma \eta_{xx}, \qquad z = 0.$$

Eigenvalue Problem

Standard techniques (harmonic disturbances $\propto \mathrm{e}^{-\mathrm{i}\omega t}$, separation of variables) give an eigenvalue problem for $\Phi(x,z,t) = \mathrm{e}^{-\mathrm{i}\omega t}\phi(x,z) = \mathrm{e}^{-\mathrm{i}\omega t}X(x)Z(z)$:

$$Z'' + k^2 Z = 0, \qquad Z'(-h) = 0, \qquad Z'(0) = \alpha_k Z(0),$$

where

$$\alpha_k = \frac{\omega^2}{g - \frac{\gamma}{\rho}k^2}.$$

Solution:

$$Z = \frac{\cos[k(z+h)]}{\cos kh},$$

with solvability condition $k\tan(kh) = -\alpha_k$, or:

$$k\tan(kh) = -\frac{\omega^2}{g - \frac{\gamma}{\rho}k^2}.$$

Solutions labelled as k_n, where $n \in \{0, 1, 2, \cdots\}$.

Eigenvalue Problem (wrap-up)

- $n = 0$, hence k_0. Purely imaginary, write $k_0 = \pm \mathrm{i}\kappa$, DR reduces to classical form:
$$\kappa \tanh(\kappa h) = \frac{\omega^2}{g + \frac{\gamma}{\rho}\kappa^2},$$
- Otherwise, $\neq 1$. Graphical analysis, infinitely many real positive roots.
- Sommerfeld outgoing boundary condition gives general solution:
$$\phi(x, z) = \sum_{n=1}^{\infty} a_n \chi_n(z) \mathrm{e}^{-k_n x} + a_0 \chi_0(z) \mathrm{e}^{\mathrm{i}\kappa x}.$$

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Coefficients:

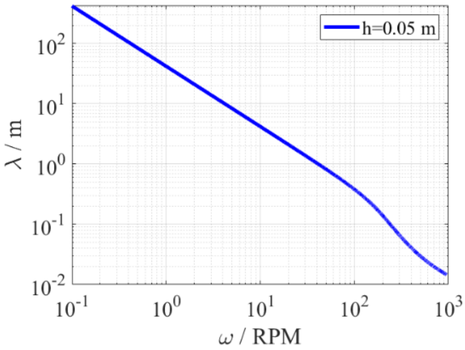
$$a_0 = \frac{1}{(\mathrm{i}\kappa) C_0} \int_{-h}^{0} f(z)\chi_0(z)\mathrm{d}z, \qquad a_n \overset{n\geq 1}{=} \frac{1}{(-k_n) C_n} \int_{-h}^{0} f(z)\chi_n(z)\mathrm{d}z.$$

For piston wavemaker with amplitude f_0:

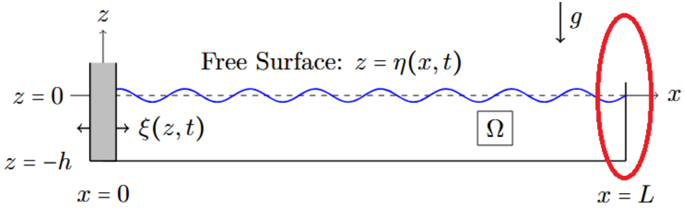
$$a_0 = \frac{f_0}{(\mathrm{i}\kappa) C_0} \frac{1}{\kappa} \frac{\sinh(\kappa h)}{\cosh(\kappa h)}.$$

Dispersion Relation

- **Dispersion relation:** for a given ω, there is a uniquely determined k, hence $\lambda = 2\pi/k$.
- Parameter values:
  - $h = 0.05\,\mathrm{m}$,
  - $\rho = 1000\,\mathrm{kg}\cdot\mathrm{m}^{-3}$,
  - $g = 9.8\,\mathrm{m}\cdot\mathrm{s}^{-2}$,
  - $\gamma = 0.072\,\mathrm{N}\cdot\mathrm{m}^{-1}$.



Reflecting Boundary Condition



New BC:

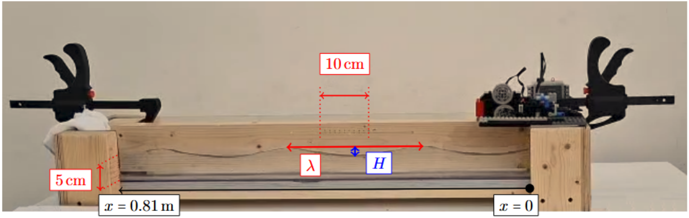
$$\frac{\partial \phi}{\partial x} \propto \frac{\mathrm{d}\xi}{\mathrm{d}t}, \text{ at } x = 0, \qquad \underbrace{\frac{\partial \phi}{\partial x} = 0, \text{ at } x = L}_{\text{Reflecting BC}}$$

Standing-wave solution (off-resonance):

$$\eta(x,t) = \Re\left[\frac{\mathrm{i} f_0}{\omega}\mathrm{e}^{-\mathrm{i}\omega t}\right]$$

$$+\frac{2}{L}\sum_{n=1}^{\infty} \frac{\Re(\mathrm{i} f_0 \omega \mathrm{e}^{-\mathrm{i}\omega t})}{k_n \cosh(k_n h)\left\{\omega^2 - \left[g + (\gamma/\rho)k_n^2\right] k_n \tanh k_n\right\}} \sinh(k_n h) \underbrace{\cos(n\pi x/L)}_{\text{Standing wave}}.$$

Summary Results



From mobile-phone video analysis @$30\,\mathrm{fps}$:

- $\omega = (140 \pm 10)\mathrm{RPM}$
- $\lambda = (0.28 \pm 0.01)\,\mathrm{m}$.

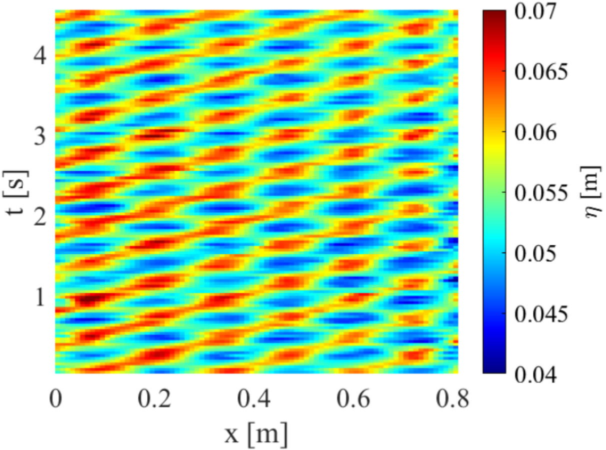
Plug $\omega = (140 \pm 10)\mathrm{RPM}$ into the theoretical dispersion relation:

$\lambda(\omega = 130\,\mathrm{RPM})$	$\lambda(\omega = 140\,\mathrm{RPM})$	$\lambda(\omega = 150\,\mathrm{RPM})$
$0.27\,\mathrm{m}$	$0.25\,\mathrm{m}$	$0.22\,\mathrm{m}$

- $\lambda_{measured}$ falls within the range of possible predicted values.

A more in-depth analysis

Each frame is extracted and a spacetime picture of $\eta(x,t)$ is built up:



Evidence of superposition of standing wave (reflection) and travelling wave (outgoing / absorbed) BC.

Non-Linear Least-Squares

We fit a superposition of standing- and travelling-waves to the data:

$$\eta_{model}(x,t) = h_0 + A_1\cos(\omega t - k_1 x + \varphi_1) \\ + A_2\cos(\omega t + \varphi_2)\cos(k_2 x) + A_3\cos(\omega t + \varphi_3)$$

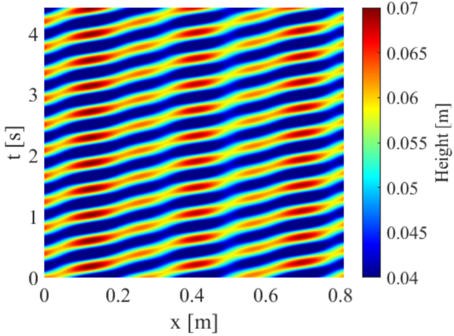
Non-linear least-squares fitting minimizes the cost function:

$$J(h_0, A_1, \omega, \phi_1, \cdots) = \sum_i \sum_j \left[\eta_{model}(x_i, t_j) - \eta(x_i, t_j)\right]^2 .$$

Results

Results include candidate solution and parameter estimates, together with confidence intervals.

- $\omega = 14.7\,\mathrm{rad}\cdot\mathrm{s}^{-1}$
  $(= 140\,\mathrm{RPM})$
  CI: $(14.4, 14.9)\,\mathrm{rad}\cdot\mathrm{s}^{-1}$
- $k_1 = 24.6\,\mathrm{m}^{-1}$
  CI: $(23.4, 26.5)\,\mathrm{m}^{-1}$
  Theoretical value: $24.2\,\mathrm{m}^{-1}$
- Height-over stroke ratio:
  $H/S = 1.07$
  CI: (0.41,1.29).
  Theoretical value: 1.27.



Numerical Simulations

We use the `OpenFOAM` *interFoam* solver (two-phase, incompressible Navier–Stokes).

Governing equations:

$$\nabla \cdot \mathbf{u} = 0, \qquad \rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \nabla\cdot\left[\,\mu(\nabla\mathbf{u} + \nabla\mathbf{u}^T)\,\right] + \rho\mathbf{g}.$$

- **Interface capturing:** Volume-of-Fluid (VOF) method using phase fraction α.
- **Surface tension:** Continuum Surface Force (CSF) model

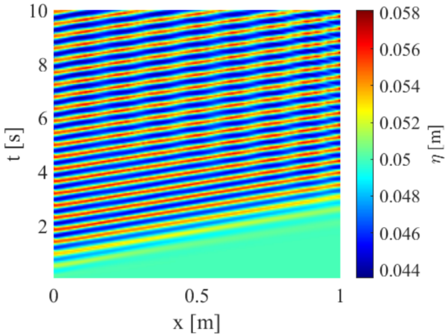
$$\mathbf{f}_\sigma = \sigma\,\kappa\,\nabla\alpha.$$

- **Domain & BCs:** 2D flume matching experiment; moving-wall boundary for piston motion.
  - Moving-wall boundary using OpenFOAM add-on phicau/olaFlow.
- **Outputs:** Free-surface elevation $\eta(x,t)$ can be extracted for comparison with theory & experiment.

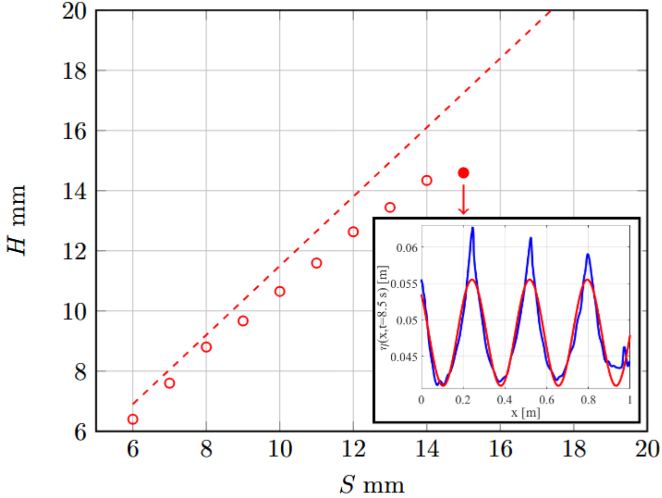
Results

Results include candidate solution and parameter estimates, together with confidence intervals.

- Simulations employ **absorbing BC** at outlet.
- Excellent agreement between linear theory (unbounded domain) and simulations.
- Simulations consistent with travelling wave passing through system (albeit some residual wave reflection remains).



Height-to-Stroke Ratio



Dashed line: linear theory; dots: DNS. Increasing $S = 2f_0$ means waves become non-linear.

Student Experiences – ACM 40890 Class of 2025

- As part of ACM 40890 *Advanced Fluid Mechanics* in UCD in Spring 2025, students participated in a combined theoretical, experimental, and computational project.
- Activities: using the tabletop flume, recording wave motion, making measurements, running corresponding CFD simulations, project writeup.
- Following approval from the UCD Human Research Ethics Committee, students were surveyed to understand their experiences.
- **Survey feedback highlights:**

  > *I learned a lot from how we used the data from the experiments to help us set up the simulations*

- Exposure to OpenFOAM: valuable but challenging, especially for students with limited programming experience.

Student Experiences – Reflections & Insights

- **Positive outcomes:**
  - Increased confidence in connecting mathematical modelling, physics, and numerical simulations.
  - Appreciation of making experimental measurements, quantifying uncertainty, etc.
- **Challenges:**
  - Computational setup sometimes frustrating.
  - Students suggested more guided support for CFD simulations.
- **Overall conclusion:** The integrated approach enhances understanding, engagement, and curiosity in Fluid Mechanics, especially for Maths-focused students.
- **Last word:** I think the approach needs more fine-tuning. Longer *final-year projects* give students the opportunity to grapple with these challenges over a longer time period.

Case Study #2: Droplet Impact

Second case study... droplet impact on to a hard surface, combining theoretical modelling, CFD, and experiments.

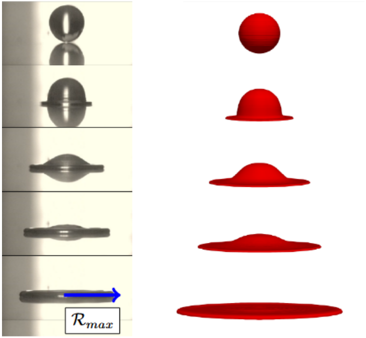
- Experiment: A droplet-impact rig with a high-speed **Chronos 1.4** camera.
- Data acquisition: SD card and mobile-phone photography for calibration (lengths scales)
- Theory: Theoretical and semi-empirical correlations for the **spreading radius**, theoretical bounds on the spreading radius
- Simulation: OpenFOAM volume-of-fluid with **contact-angle model**
- Goals:
  - Cross-validation of CFD simulations and theoretical correlations.
  - Provide interesting final-year projects to test students' skills and introduce new ones.
- **Spoiler:** This project is very well suited for final-year projects (difficulty level, complexity, time required).

Context

- I will look at droplet impact on a smooth surface.
- **Impact, Spread, Retraction**
  In the land of splashes, what the scientist knows as Inertia and Surface Tension are the sculptors in liquids, and fashion from them delicate shapes none the less beautiful because they are too ephemeral for any eye but that of the high-speed camera [Yarin, Annu. Rev. Fluid Mech. (2006)]
- Highlights the importance of **parameters** in such studies; key parameters are the **Weber number**, $\mathrm{We} =$ Inertia/Surface Tension, and the **Reynolds number** – *sculptor has two tools*.

Splash Threshold

- Droplet spreading below **splash threshold** (no splash), $K \lesssim 3,000$, where $K = \mathrm{We}\sqrt{\mathrm{Re}}$
- At low We, droplet spreads out into a pancake structure – rim and lamella.
- Of interest is the **maximum spreading radius** $\mathcal{R}_{max}$ and its dependence on We and Re.



Droplet impact study. Left: high-speed camera. Right: OpenFOAM simulations. Credit: Conor Quigley. Parameters: $\mathrm{Re} = 1700$ and $\mathrm{We} = 20$.

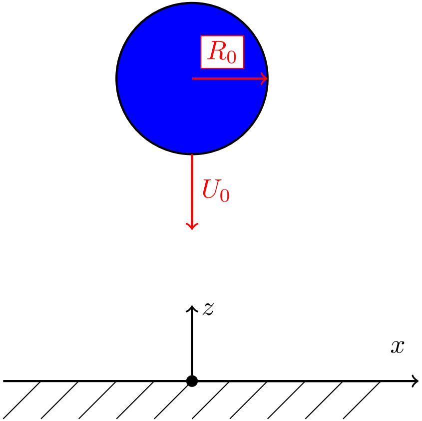
For the avoidance of doubt...

We use the following definitions for the Reynolds and Weber numbers:

$$\mathrm{Re} = \frac{\rho U_0 R_0}{\mu},$$

$$\mathrm{We} = \frac{\rho U_0^2 R_0}{\gamma},$$

where ρ is the fluid density, μ the viscosity, and γ the surface tension.



Droplet-Impact Rig

- Chronos 1.4 Camera mounted on base.
- Max: $1280 \times 1024@1069\,\mathrm{fps}$, Min: $320 \times 96@40,413\mathrm{fps}$.
- YouTube link (only slight digression)
- Perspex or aluminum substrates.
- Light source (ScrewFix) and diffuser (A4 paper)
- Droplet deposited via syringe, held in place by clamp.
- Clamp height can be adjusted through a height h – controls Weber number.
- $U_0 = \sqrt{2gh}$ is a good approximation ($\mathrm{We} \sim h^{1/2}$).

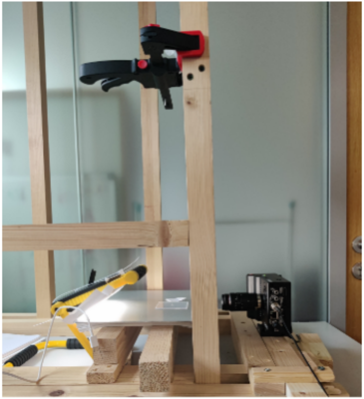


Image credit: Patrick Murray

Calibration

- Droplet base at equilibrium (post impact) can be measured with a ruler, fixing N in the relation $1\,\mathrm{cm} = N(\mathrm{pixels})$. **L**
- Frame rate of high-speed gives fixed **T**.
- Distance travelled between successive frames gives U_0.
- $U_0 = \sqrt{2gh}$ is a good approximation.
- Density of water $\rho = 1000\,\mathrm{kg}\cdot\mathrm{m}^3$. **M**

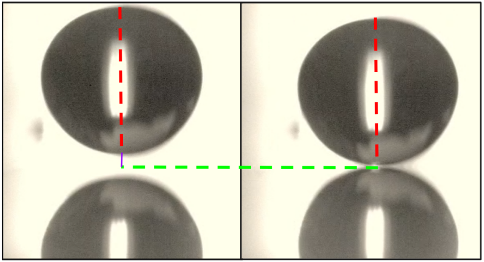
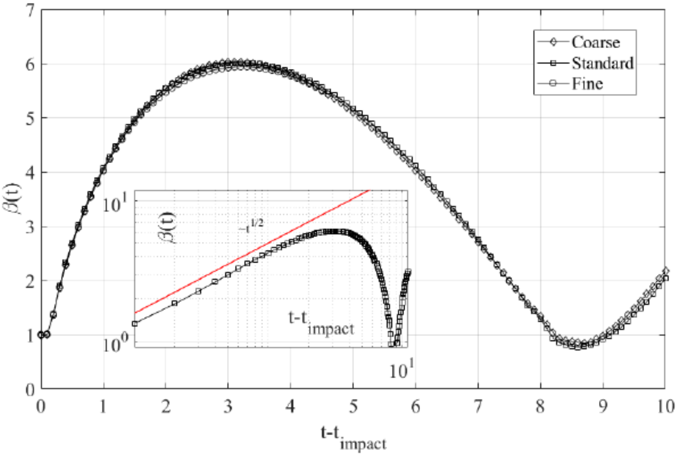


Image credit: Conor Quigley

What we expect

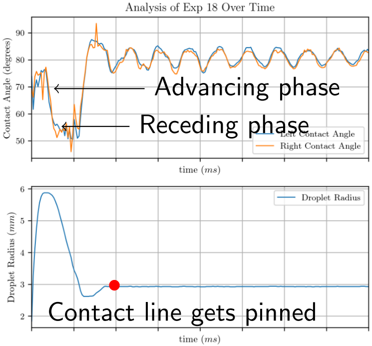
Typical behaviour of the spreading radius $\beta(t) = R(t)/R_0$ in a **simulation**.[2]

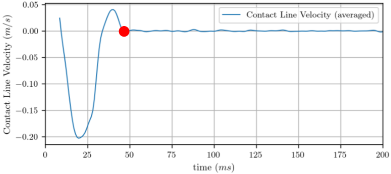


[2] Ó Náraigh, L. and **Mairal, J.**, 2023. Analysis of the spreading radius in droplet impact: The two-dimensional case. Physics of Fluids, 35(10).

What we get

Image-processing (Matlab) on each frame is used to extract the interface profile. Maximum spreading radius tracked over time.





Contact-line velocity

Credit: Patrick Murray

Contact-line model

Fit dynamic contact angle to model:

$$\theta(t) = \begin{cases} \theta_0 + \left(\theta_a - \theta_0\right)\tanh(U_\parallel/U_\theta), & U_\parallel > 0, \\ \theta_0 + \left(\theta_r - \theta_0\right)\tanh(-U_\parallel/U_\theta), & U_\parallel < 0, \end{cases}$$

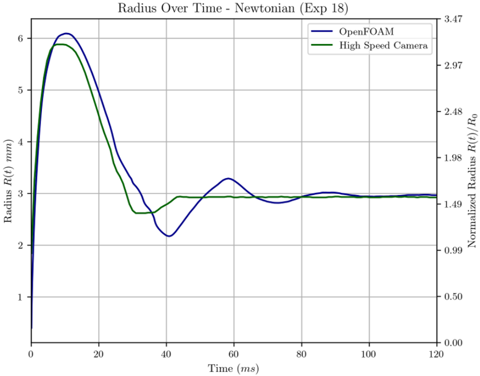
where U_θ is a **fitting parameter**.

We again use OpenFoam for the simulations:

- `interFoam` Volume of Fluid method to capture the interface (marker function α).
- One-fluid Navier–Stokes formulation.
- Surface tension modelled as continuous surface force, $\widehat{n} = \nabla\alpha/|\nabla\alpha|$, $\kappa \propto \nabla \cdot \widehat{n}$.
- Standard boundary conditions... except
- α adjusted at bottom boundary to account for contact angle $\theta(t)$.
- Sample test case in Github repository

Comparison between simulations and experiment

- $\mathcal{R}_{max}/R_0 =$ 2.91 (experiment) 3.01 (simulation)
- Relative error 3.4%.
- Issues with pinned droplet not captured by simulation.



Credit: Patrick Murray

Comparison with theory

- Semi-empirical correlation due to Roisman:[3]

$$\frac{\mathcal{R}_{max}}{R_0} = 1.0\mathrm{Re}^{1/5} - 0.37\,\mathrm{We}^{-1/2}\mathrm{Re}^{2/5}.$$

  - Maximum droplet spreading limited by viscous boundary layer, hence $\mathcal{R}_{max}/R_0 \sim \mathrm{Re}^{1/5}$.

- Bounds due to Bustamante and Ó Náraigh:[4]

$$k_1\mathrm{Re}^{1/5} - k_2(1-\cos\vartheta_a)^{1/2}\mathrm{Re}^{2/5}\mathrm{We}^{-1/2} \le \frac{\mathcal{R}_{max}}{R_0} \le k_1\mathrm{Re}^{1/5}.$$

  - Derived via *a priori* estimates on Rim-Lamella model (Gronwall's inequality).
  - Consistent with Roismann's formula.

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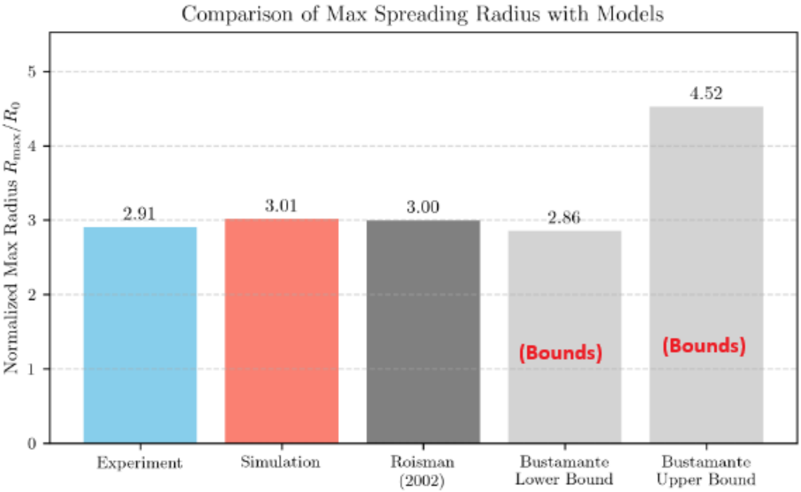
[3] Roisman, I.V., 2009. Inertia dominated drop collisions. II. An analytical solution of the Navier–Stokes equations for a spreading viscous film. Physics of Fluids, 21(5).

[4] Bustamante, M.D. and Ó Náraigh, L., 2025, May. Bounds on the spreading radius in droplet impact: the viscous case. In Proceedings A (Vol. 481, No. 2313, p. 20240791). The Royal Society.

Nice illustration



Credit: Patrick Murray

Conclusions from Case Study #2

- Case Study #2 is ideal for individual final-year projects (Conor Quigley, Joseph Anderson, Patrick Murray).
- Too difficult to include in a 12-week Advanced Fluids module (ACM 40890)
- In both case studies, new skills acquired:
  - OpenFoam simulations (mesh generation, algorithm selection, mesh refinement study...)
  - Data acquisition, image processing
  - Nonlinear least-squares fitting, optimization.
  - Interpreting experimental data in context of recent literature (#2).

Conclusions

- Integrating simulations and experiments into Fluid Mechanics teaching in a Maths Department – well received by students.
  - Creates a 'buzz' about Fluid Mechanics.
  - Steals a march on our peers.
- Some projects can be readily integrated into a 12-week module, e.g. e.g. #1 (wavemaker).
- Other projects are more suited for a final-year project, e.g. #2 (droplet impact).
- For projects in a 12-week module, documentation (including OpenFoam cases) is essential.
- Suggestions? Preferably inexpensive, **amenable to data acquisition by mobile phone**.