Optimization Algorithms
(ACM 41030)

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Exercises #5

1. Does the OP
$$\min f(\boldsymbol{x}) = (y+100)^2 + \tfrac{1}{100}x^2$$
subject to $y - \cos x \geq 0$ have a finite or infinite number of local solutions? Use the KKT conditions to justify your answer.

2. Let $\boldsymbol{v} : \mathbb{R}^n \to \mathbb{R}^m$ be a smooth vector function, and consider the unconstrained OP
$$\min_{\boldsymbol{x}\in\mathbb{R}^n} f(\boldsymbol{x}),$$
where
$$f(\boldsymbol{x}) = \max_{i\in\{1,2,\cdots,m\}} v_i(\boldsymbol{x}).$$
Reformulate this (generally non-smooth problem) as a smooth constrained problem.

3. Can you perform a smooth reformulation of the previous question when f is defined by:
$$f(\boldsymbol{x}) = \min_{i\in\{1,2,\cdots,m\}} v_i(\boldsymbol{x}).$$
Why or why not?

4. Consider the OP
$$\min(x+y), \qquad \text{subject to } 2 - x^2 - y^2 = 0.$$
Specify two feasible sequences that approach the **maximizing** point $(1,1)^T$ and show that neither sequence is a decreasing sequence for f.

5. If f is convex and the feasible region Ω is convex, show that local solutions of the OP
$$\boldsymbol{x}_* = \arg\min_{\boldsymbol{x}\in\Omega} f(\boldsymbol{x})$$
are also global solutions.

   Hint: Review Theorem 2.8 in the class notes.