Operator Algebras and Geometric Rigidity; Places, People, Influences

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IWOTA, Lancaster, August 2021, Special Session:
Emerging Topics: From Operator Algebras to Geometric Rigidity

Imperial College, London, 1970-73

The "old Huxley building" (now part of the V&A)

G.E.H. Reuter (Analysis, Functional Analysis),
Walter Hayman and Jim Clunie (Complex Analysis),
Yael Dowker (Measure Theory).

The mathematics library!

Functional Analysis tracts ... *"Riesz and Nagy"*, *"Dunford and Schwarz"*,
Ringrose's purple paperback: *"Nonselfadjoint Operators"*.

Edinburgh, 1973-76

Minto House, and Frank Bonsall

Edinburgh, 1973-76

Frank Bonsall, Allan Sinclair, Sandy Davie, Alastair Gillespie ...

visitors, some via NBFAS: George Elliot, Bill Arveson, Peter Rosenthal ...

Influential books: "Complete normed Algebras" (FFB),
"Invariant subspaces" (Radjavi and Rosenthal), and especially
"Banach Algebra Techniques in Operator Theory" (Ron Douglas).

University Library, and R.S.E. library in town.

Hot topics of the 70s: H^∞, Douglas algebras, Toeplitz Operators,
Toeplitz C*-algebras, compact perturbations, "BDF", K-theory, ...

Dalhousie University, Nova Scotia. “Dal”, 1976-78

Peter Filmore, Heydar Radjavi,
postdocs: John Phillips, Iain Raeburn, John Borwein, ...

Key conferences !

“COSy”, Guelph, Ontario: Man-Duen Choi CP maps !
Ed Effros, Eric Nordgren, Peter Rosenthal, ... , Ken Davidson

Function Theory on the Unit Circle, Blacksburg, Virgina, June 1978
masterclass lecture series from Donald Sarason !
100 participants .., Allen Shields ...

1978-79 California Institute of Technology

visit to Berkeley (Sarason, Arveson, Helson, ...)

Lancaster, and the 1980s

1980: Dublin conference Trevor West, Gerard Murphy, Donal O'Donovan, John Erdos, ...

extended visits to North America:

Michigan Sheldon Axler, Joel Shapiro, Allen Shields, Tim Feeman,..

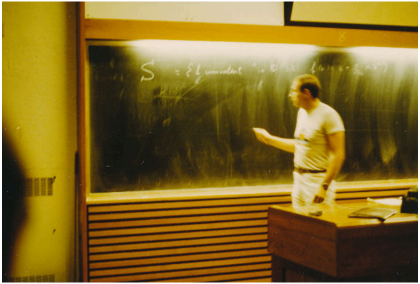
Houston, Texas Vern Paulsen, ...
Tuscaloosa, Alabama Alan Hopenwasser, Cecelia Laurie, Bob Moore
Waterloo/Kitchener Ken Davidson, et al

Above: Tim Feeman, ?, David Pitts, and Vern Paulsen: SEAM 1885

Lancaster 1984 : “Operators and Function Theory”

Gohberg, Sarason, Conway, Young and Douglas. And Nikolai Nikolskii.

Lancaster 1984 : “Operators and Function Theory”



Left: Ken Davidson, Paul Halmos, Alan Hopenwasser.
Right: Vladimir Peller presents de Branges proof of the Bieberbach conjecture.

Ambleside 1997: Limit Algebras

Ambleside 2004

Greek and Irish connections: Operator Algebras

Aristides Katavolos.

And Martin Mathieu, Donal O'Donovan, and postgrads Derek Kitson et al.

Geometric Rigidity 2000-...

John Owen: Imperial College '73-76.
CAD software developer: D-cubed Cambridge, Siemens.

GR is multifaceted, with many application areas!

commutative algebra and differential geometry methods

matroid methods, combinatorics, inductive constructions

low rank matrix completion

CAD, distance geometry

Chemistry, molecule conformation

GR with respect to non-Euclidean norms

zero modes for (infinite) crystals,.... Fourier analysis techniques

rigidity operator theory, $R(G, p)$

Algebraically soluble CAD equations ?

CAD issue: Edge-lengths are "inputs" for a 2D drawing, and a precise realisation or output is to be computed (very rapidly).

THM (Owen-P, 2007) Let G be a finite planar Laman graph together with generic edge-length inputs. If G is 3-connected then the realisation is not solvable by radicals.

Open Problem. Can one replace "planar graph" by "graph" ?

Lancaster 2010

Lancaster Workshop 2010

Left: Bob Connelly, Franz Wegner, Shin-Ichi Tanigawa.
Right: Bernd Schulze, Elissa Ross, Tony Nixon, BillJackson.

The rigidity of a crystal framework $\mathcal{C}$

THM. (Kastis-P, 2021.) The following are equivalent:

i) $\mathcal{C}$ is first-order rigid (ie nullspace of rigidity operator is trivial).

ii) The transfer function $\Psi_{\mathcal{C}}(z_1, \ldots, z_d)$ has maximal rank for each $(z_1, \ldots, z_d)$ in $(\mathbb{C} \backslash \{0\})^d$, and there are no periodic flexes in the flexible lattice sense.

Hard proof: Commutative algebra, discrete group spectral synthesis, ...

Need to understand shift-invariant subspaces of

$$C(\mathbb{Z}^d) \otimes \mathbb{C}^r$$

ie. vector-valued multisequences (bi-infinite) $(\mathbb{Z}^d)^{\mathbb{C}^r}$

Case $r = 1$: Marcel Lefranc, 1958.
General r: Kastis-P, 2019.

Time off from Maths

Lefteris Kastis (and his brother-in-law)

Zero modes for quasicrystals

Background: Crystal framework $\mathcal{C} \rightarrow$ Zero mode (RUM) spectrum:

Badri-Kitson-P: It is a subset of $[0, 2\pi)^d$ playing the role of a "Bohr spectrum" for the space of almost periodic flexes of $\mathcal{C}$.

Problem: Define such spectra for quasicrystals. (eg Penrose)

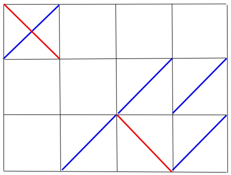
P-2021: *Parallelogram tilings and flexible quasicrystals.* (arXiv)

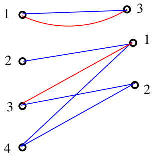
A method: Characterise bracing patterns which make a Penrose rhomb framework rigid.

Bracing non-Euclidean grids rigidly.

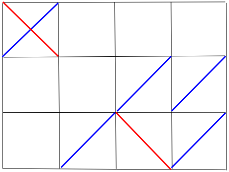
1977: Bolker-Crapo Theorem: braces graph (simple) needs to be spanning and connected.

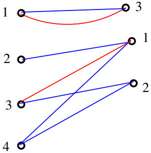
THM. (P-2020) For a strictly convex smooth norm:
the necessary and sufficient conditions are
(a) A doubly braced square is first-order rigid and
(b) The braces graph is spanning with an "independent cycle" in each path-connected component.





Thank you for listening





(Math. Proc. Royal Irish Acad., 2020.)