Applying rigidity theory to homothetic convex body packings

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In 2019, Connelly, Gortler and Theran utilised techniques from rigidity theory to prove the following result: given n non-overlapping discs in the plane with randomly chosen radii, there will a.s. be at most $2n-3$ contacts between them. They also conjectured that the opposite is true; given a planar graph G where every subgraph on n' vertices has at most $2n'-3$ edges, G can be realised as the contact graph of a random disc packing with non-zero probability. These ideas can be extended to packings of a much larger class of convex body; namely those that are centrally symmetric (c.s.), strictly convex and smooth. As every c.s. convex body generates a unique norm, we can adapt methods from the study of rigidity theory for non-Euclidean norms (e.g. ℓ_p norms, polyhedral norms, etc.) to aid us. During the presentation we shall discuss the following two results: (i) for any strictly convex and smooth c.s. convex body C, every random homothetic packing of C has at most $2n-2$ contacts; (ii) there exists a comeagre subset of c.s. convex bodies where every $(2,2)$-sparse planar graph (i.e. all subgraphs on n vertices have at most $2n-2$ edges) can be realised as the contact graph of a random homothetic packing with non-zero probability. This talk is based on the paper "Homothetic packings of centrally symmetric convex bodies" (https://arxiv.org/abs/2011.03436).