Financial Modeling

An introduction to financial modelling and financial options

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Outline

Introduction
- Financial Modeling
- Basics of Financial Modeling
- Derivatives

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A Quick Look at Bubbles

The Tulip Crash Netherlands, 1634-1637

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- The true bulb buyers filled up their inventories, so increasing scarcity and demand.
- Soon prices were rising so fast and high that people were trading their land, life savings to get more tulip bulbs.
- The originally overpriced tulips enjoyed a twenty-fold increase in value - in one month.

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- The government attempted to step in and halt the panic and the crash, but that did not work out well.
- Even the people who locked in their profit early suffered under the following depression.

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The simplest concept in finance is the time value of money.

- $1 today is worth more than $1 in a year's time.
- There are several types of interest
  - There is simple and compound interest. Simple interest is when the interest you receive is based only on the amount you initially invest, whereas compound interest is when you also get interest on your interest.
  - Interest typically comes in two forms, discretely compounded and continuously compounded.
  - Invest $1 in a bank at a discrete interest rate of r (assumed to be constant), paid once per year.
  - At the end of one year your bank account will contain $1 \times (1 + r)$.

Interest Rates

- Now suppose you receive m interest payments at a rate of $\frac{r}{m}$ per annum.
- After one year you will have $(1 + \frac{r}{m})^m$.
- Suppose these interest payments come at increasingly frequent intervals, but at an increasingly smaller interest rate (we will take the limit $m \to \infty$). This will give a continuously paid rate of interest.
- The expression above becomes

$$(1 + \frac{r}{m})^m = e^{m \log(1+r/m)} \to e^r$$

- That is how much money you will have in the bank after one year if the interest is continuously compounded.
- And similarly, after a time t you will have an amount e^{rt}.

Interest Rates

- Suppose $M(t)$ in the bank at time t, how much does this increase with time?
- If you check your account at time t and again a short period later, time $t + dt$, the amount will have increased by

  $$M(t+dt) - M(t) = \frac{dM}{dt} \times dt + \ldots \text{(Taylor series expansion)}.$$

  The interest you receive must be proportional to the amount you have, M, the interest rate r and the time-step, dt. Thus,

  $$\frac{dM}{dt} dt = rM(t)dt \Rightarrow \frac{dM}{dt} = rM(t).$$

- If you have $\$M(0)$ initially, then the solution is $M(t) = M(0)e^{rt}$.
- Conversely, if you know you will get \$1 at time T in the future, its value at an earlier time t is simply $e^{-r(T-t)}$.

Financial Instruments

Equities

A basic financial instrument often referred to as *equity, stock or shares*.

- This is the ownership of a small piece of a company.
- The price is determined by the value of the company and by the expectations of the performance of the company.
- These expectations are seen in the bid and ask behaviour in the market.
- The expectations give an uncertainty to the future price development of the stock.
- The exact profit is known only at the date of selling.
- The real value of the stock is sometimes a bit higher, sometimes a bit lower than the expected value.
- The amount in which the stock price development can differ from the expected value is determined by the so-called *volatility*.

Volatility

What does it mean?

- A statistical measure of the tendency of a market or security price to rise or fall sharply within a period of time. Volatility is typically calculated by using variance of the price or return. A highly volatile market means that prices have huge swings in very short periods of time.
- Security: An instrument representing ownership (stocks), a debt agreement (bonds), or the rights to ownership (derivatives).
- Return: The gain or loss of a security in a particular period. The return consists of the income and the capital gains relative on an investment. It is usually quoted as a percentage.

Volatility

Figure: Volatility

Exchanges

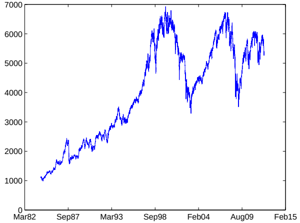
Shares of larger companies are quoted on regulated stock exchanges, so that they can be bought and sold freely.

Figure: London Stock Exchange

Security Prices

- Prices have a large element of randomness. This does not mean that we cannot model stock prices, but it does mean that the modelling must be done in a probabilistic sense.
- A well known and often used model for generating asset prices via a stochastic differential equation is referred to as geometric Brownian motion.

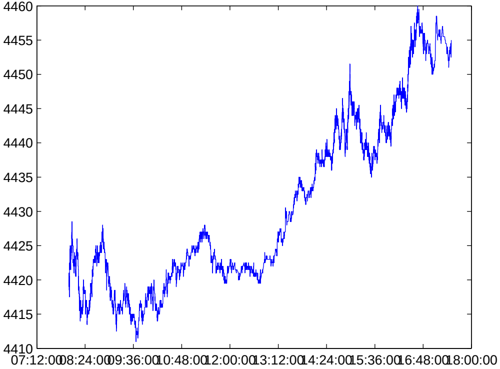
Figure: FTSE 100 Stock Index Over Last 30 Years



FTSE 100

- The graphs below depicts the FTSE over the course of a single trading day (11/05/2004)!

Figure: FTSE 100 Stock Index



Modeling Asset Prices

- When investing, the main concern is that the return on the investment is satisfactory. Suppose we have given asset S_t , then

$$\text{Return} = \frac{\text{Stock tomorrow - Stock today}}{\text{Stock today}} = \frac{S_{t+\delta t} - S_t}{S_t}$$

Lets see this in practise. Below is the returns of the FTSE 100 over last 30 years.

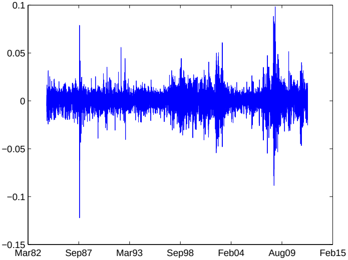


Figure: FTSE 100 Stock Index Returns

Modeling Asset Prices

- From the data in this example we find that the mean is 0.00028543 (0.0285%) and the standard deviation is 0.0121 (1.21%).

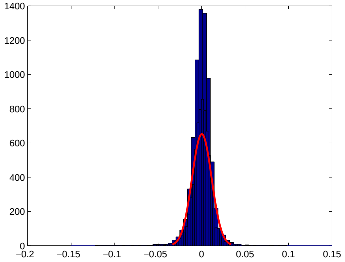


Figure: FTSE 100 Histogram

Randomness of the stock prices

Daily returns for assets look like noise!

- What can be then done?
- We can model the noise!
- Definition: Wiener Process:
  A stochastic process W_t for $t \in [0, \infty)$ is called a Wiener Process (or Brownian motion) if the following conditions are satisfied:
  - It starts at zero: $W_0 = 0$,
  - It has stationary, independent increments,
  - For every $t > 0$, Wt has a normal distribution with mean 0 and variance t,
  - It has a.s. continuous paths with NO JUMPS.

Weiner Processes

- A sample of trajectories from a Weiner process

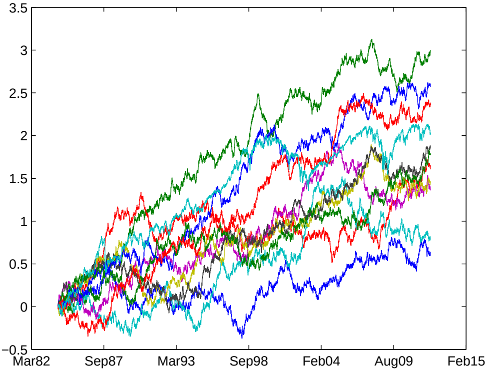


Figure: FTSE 100: Simulations

Weiner Processes

- A sample of trajectories from a Weiner process (real FTSE highlighted)

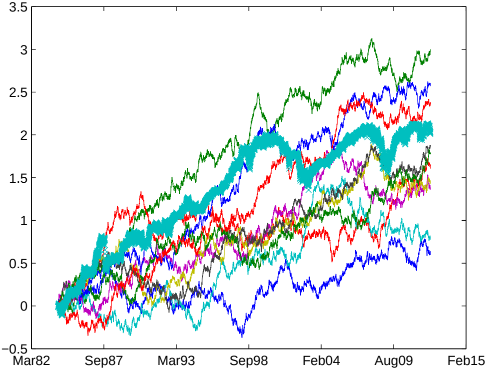


Figure: FTSE 100: Simulations

Stochastic Processes

Suppose we observe the stock price of Company Y at every fixed instance t from some initial time t_0 till today t_n and we denote $T = [t_0, t_n]$.

- We can interpret the observed stock values as a realisation $X_t(\omega)$ of the random variable X_t .
- We need a model which takes into account almost continuous realisations of the stock prices.
- Definition: Stochastic Process
  A stochastic process X_t is a collection of random variables

$$(X_t, t \in T) = (X_t(\omega), t \in T, \omega \in \Omega)$$

Stochastic Processes

We note that a stochastic process X_t is a function of two variables:

- for a fixed time t it's a variable $X_t = X_t(\omega), \omega \in \Omega$
- for a fixed random outcome $\omega \in \Omega$, it's a function of time $X_t = X_t(\omega), t \in T$

Modeling Asset Prices

The most popular Stochastic Process for generating prices is the Geometric Brownian Motion process (GBM):

$$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

which can be translated to:

$$\frac{S_{t+\delta t} - S_t}{S_t} = \mu \delta t + \sigma (W_{t+\delta t} - W_t)$$

where

- $\mu \delta t$ is the deterministic return
- σdW_t is the random change with dW_t a sample from a normal distribution with mean 0 and variance δt.

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Derivatives

- It was only on 26th April 1973 that options were first officially traded on an exchange. It was then that The Chicago Board Options Exchange (CBOE) first created standardised, listed options.
- Initially there were just calls on 16 stocks. Puts weren't introduced until 1977.
- In the US options are traded on CBOE, the American Stock Exchange, the Pacific Stock Exchange and the Philadelphia Stock Exchange.
- Worldwide, there are over 50 exchanges on which options are traded.

Derivatives

Option

Is a contract written by a seller, that gives the right (but not the obligation) to the holder to trade the underlying asset in the future at a previously agreed price.

Option styles:

- European option- an option that may be only exercised on expiration;
- American option - an option that may be exercised on any trading day (also on the expiration);
- Barrier option - option which is exercised, for example, only if security's price reaches some trigger level during the life of the option.

Derivatives

Most popular options are Call and Put options: At a prescribed time in the future, (maturity: T):

- Call Option: The holder of the option may purchase a prescribed asset (shares, stocks : S) for a prescribed amount (strike: K) and the writer of the contract must sell the asset, if the holder decides to buy it.
- Put Option: The holder of the option may sell a prescribed asset (shares, stocks : S) for a prescribed amount (strike: K) and the writer of the contract must buy the asset, if the holder decides to sell it.

Option Payoffs

- The value of European call option at expiry T is given by:

$$C(T, S_T) = \max(S_T - K, 0)$$

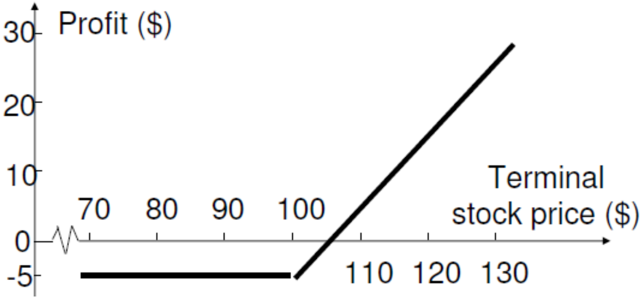
- The value of European put option at expiry T is given by:

$$P(T, S_T) = \max(K - S_T, 0)$$

Option Payoffs

Long Call

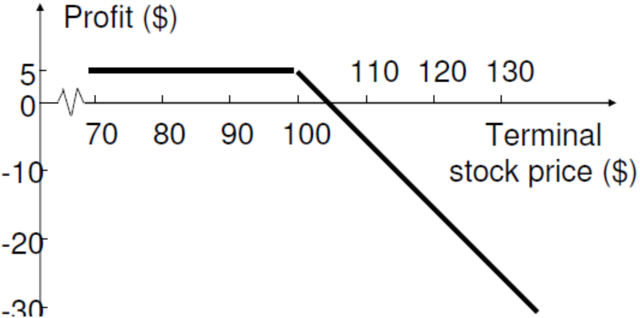
Profit from buying one European call option: option price = $5, strike price = $100, option life = 2 months



Option Payoffs

Short Call

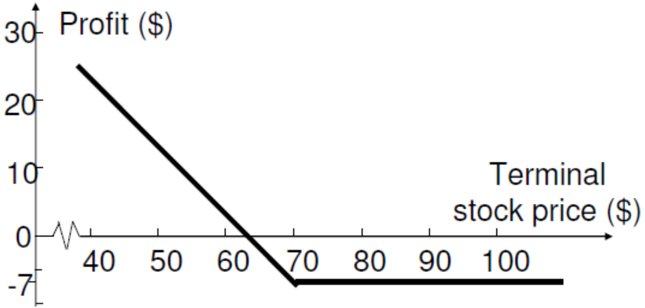
Profit from writing one European call option: option price = $5, strike price = $100



Option Payoffs

Long Put

Profit from buying a European put option: option price = $7, strike price = $70



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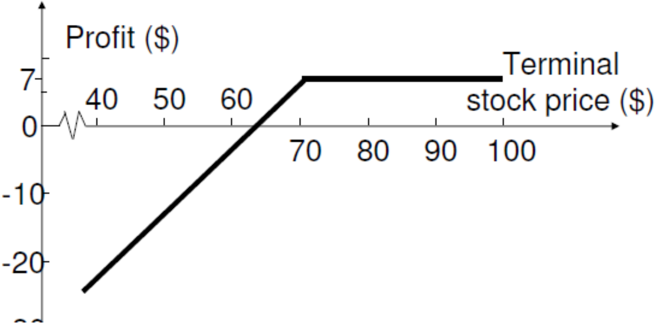


Valuing Options

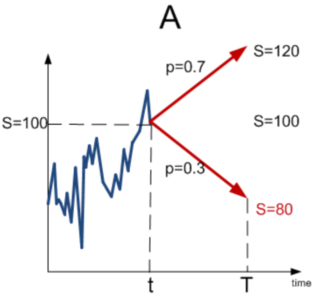
What determines the value of an option?

- what is the asset price today S_t?
- how long there is until expiry $T - t$?
- how volatile is the asset S_t?

General principles:

- The longer the time to expiry, the more time there is for the asset to rise or fall;
- The more the asset is volatile the higher the chance that it will rise or fall;

Valuing Options



For $K = 100$ which call option is more expensive $C_A < C_B, C_A > C_B, C_A = C_B$? To find the answer we follow the reasoning of replicating an option.

Binomial Pricing Model

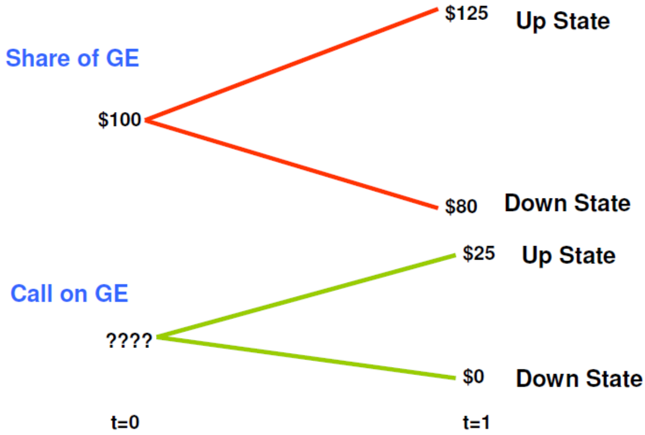
Consider the following example:

- The stock of GE today ($t = 0$) is \$100.
- You analyse the firm and conclude that one year from now ($t = 1$) the stock will sell for either
- \$125 (a rise of 25%) or \$80 (a drop of 20%).
- The risk free rate is 8% compounded continuously.

Binomial Pricing Model

- Consider a call option on GE
- Let us say that the call's exercise price is \$100 and that the expiration date is one year from now.
- One year from now, the call will have a value of either \$25 (if GE sells at \$125) or \$0 (if GE sells at \$80).

Binomial Pricing Model



Binomial Pricing Model

Three investments are of interest for us:

- Stock
- Option
- Risk-free bond

Payoffs and prices of different instruments

Security	Payoff: *Up state*	Payoff: *Down state*	Current Price
Stock	\$125.00	\$ 80.00	\$100.00
Bond	108.33	108.33	\$100.00
Call	25.00	0.00	???

Replicating Portfolio

- The Call Option on GE can be valued by finding a portfolio that replicates the payoff of the Call in all states of nature.
- The replicating portfolio will look as follows:
  - A position in GE stock
  - A position in a risk free bond
- The replicating portfolio will generate the same cash flows as a call and hence they must have the same value.
- The portfolio's cost will be the fair value of the option.

Replicating Portfolio

- The two portfolios (Portfolio A: Holding a Call and Portfolio B: Stock and risk free bond) must have the same value otherwise there will be an arbitrage opportunity.
- This will happen if the investor could buy the cheaper of the two alternatives and sell the more expensive one.

Replicating Portfolio

- Consider a portfolio with shares of GE and risk free bonds. What are the payoffs of such a portfolio?
- In the Up State: $\$125 N_s + \$108.33 N_b$
- In the Down State: $\$80 N_s + \$108.33 N_b$

Replicating Portfolio

- Composition of the Replicating Portfolio: Consider a portfolio with N_s shares of GE and N_b risk-free bonds
- In the Up State: $\$125N_s + \$108.33N_b = \$25$
- In the Down State: $\$80N_s + \$108.33N_b = 0$
- Solving the two equations simultaneously

$$(125 - 80)N_s = 25 \Rightarrow N_s = 0.5556$$

- Substituting in either equation yields

$$N_b = -0.4103$$

Replicating Portfolio

What is the meaning of the numbers we have just obtained?

- The investor can replicate the payoffs from the call by short selling \$41.03 of the risk-free bond
- and buying 0.5556 shares of GE stock.
- The payoffs will confirm this...

Replicating Portfolio

Portfolio Part	Up State	Down State
GE Stock	0.5556 x $125 =$69.45	0.5556 x $80 =$44.45
Risk-free Bond	-$41.03 x 1.083 =-$44.45	-$41.03 x 1.083 =-$44.45
Net Payoff	$25	$0

Replicating Portfolio

Cost of building the replicating portfolio?

- \$55.56 must be spent to purchase .5556 shares of GE at \$100 per share
- \$41.03 income is provided by the bonds (i.e. this money is borrowed)
- Total cost is:

$$\$55.56 - \$41.03 = \$14.53$$

Value of the Option

The main conclusion is the value of the option is given by

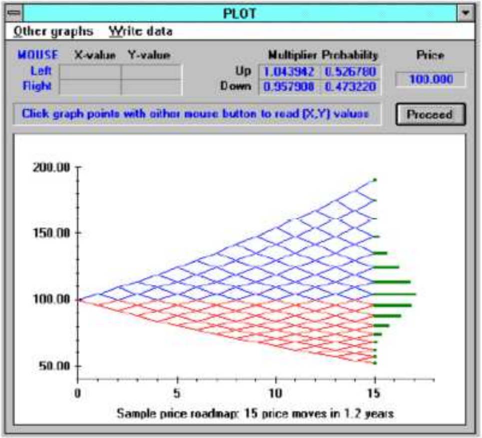
$$C = S_0 N_s + K N_b$$

where

- S_0 is the share price
- K is the value of the bond purchased (in this example $K = \$100$)
- and N_s and N_b are the number of units of the stock and the bond purchased to replicate the option.

Value of the Option

In reality option prices are valued using multiperiod binomial models or continuous time stochastic processes



Summary

- The first main topic was the modeling of stock prices in a probabilistic fashion.
- The second main topic was derivatives pricing using the very simple binomoial option pricing model.
- Those interested should consider the MSc Quantitative Finance at Smurfit Business School.
- Email me conall.osullivan@ucd.ie for more information.