# Proof by "rewriting"

It is often convenient to prove a statement by rewriting it (usually in several steps) until you get a true statement. Something like

$$\begin{aligned} & \text{Statement to be proved} \\ \Rightarrow\quad & \text{Statement 1} \\ & \quad\vdots \\ \Rightarrow\quad & \text{Statement n} \\ \Rightarrow\quad & \text{Something true.} \end{aligned}$$

**You need to be careful when doing this because if you do not do it properly it may not work. For instance, what I wrote above may not work.**

Why? A proof is an argument that uses the hypotheses and what you know to be true, and finishes with the statement that you want to prove.

In the above, in order to finish with the "Statement to be proved" you need to be able to start from the "Something true" and got back up your sequence to finish with "Statement to be proved". In other words, you need to have:

$$\begin{aligned} & \text{Statement to be proved} \\ \Leftrightarrow\quad & \text{Statement 1} \\ & \quad\vdots \\ \Leftrightarrow\quad & \text{Statement n} \\ \Leftrightarrow\quad & \text{Something true.} \end{aligned}$$

It must be possible to go back at each step, and it is a good idea to formally indicate that it is possible (for instance by writing "$\Leftrightarrow$", or by saying something like "the following statements are equivalent") to show that you are aware of it and paid attention to it.

Here is an example of what can go wrong if you do not do this (this example is a bit extreme, but it should illustrate the problem):

We show that $1 = 2$ in $\mathbb{R}$:

$$\begin{aligned} & 1 = 2 \\ \Rightarrow\quad & 0 \cdot 1 = 0 \cdot 2 \\ \Rightarrow\quad & 0 = 0, \text{ which is true.} \end{aligned}$$

Of course, we don't have $1 = 2$ in $\mathbb{R}$. What does not work is that you cannot go "backwards" in the sequence from $0 \cdot 1 = 0 \cdot 2$ to $1 = 2$.