

# Algebraic Structures

## MST20010

Contents of the course

2020-2021

# Content of the course

First part of the course: Rather computational

Second part of the course: More abstract

## **First part of the course:**

You can already compute with  $\mathbb{N}$ ,  $\mathbb{Z}$ ,  $\mathbb{Q}$ ,  $\mathbb{R}$ ,  $\mathbb{C}$ ...

We will see 2 different situations where it is possible to compute with something else than the usual numbers with their usual rules.

Why?

- Because both are useful, for maths and for applications.
- Because it serves as an introduction to the second part.

## **Second part of the course:**

We will introduce the notion of group. There are at least two (similar) ways to present this.

I present one in each of the following two pages.

As already mentioned, you can easily compute with “numbers” in different forms:  $\mathbb{Z}$ ,  $\mathbb{R}$ , etc. And also with some functions, for instance functions from  $\mathbb{R}$  to  $\mathbb{R}$ .

# Content of the course

## Question

Are there other (useful) situations where it is also possible to compute with two operation ( $+$  and  $\cdot$ ) that behave in a “nice” way?

Answer: Yes.

(and it is the starting point of a lot of modern mathematics, including applications)

## First step toward an answer:

Are there situations where we can compute “nicely” with just one operation?

The notion of “group” is a possible way to describe situations where you have one operation with properties that are “nice” and allow us to compute well with it.

# Change of point of view for computations

Until now, the most common setup for you to do computations was

## Common point of view

You know exactly what you are computing with (integers, real numbers, functions from  $\mathbb{R}$  to  $\mathbb{R}$ , etc) and when you compute with them you use the properties of these elements.

The notion of group will be an illustration of a reversed approach:

## New point of view for groups

We will have a list of properties of the operation (at first just one), but we will not know what elements we are computing with. We will only be able to use the properties of the operation in a “blind” way (not really knowing what we are computing with).

We will then try to see how far we can go with this, and also what kind of elements could be used with an operation having these properties.