Algebraic Structures (MST20010)

Problem sheet 7

1. (a) Show that $\{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\}$ is a subgroup of $(\mathbb{R}, +)$.

   (b) Show that $\{\mathrm{id}, (1\ 2\ 3), (1\ 3\ 2)\}$ is a subgroup of S_3.

2. Let G be a group. Show that each row in the Cayley table of G contains each element of G exactly once (we do not consider the first column and first row as part of the table, they are rather just the lists of the elements of the group). The same is true for the columns, with a very similar justification.

3. Let $G = \{a, b, c, d\}$ be a group with 4 elements. Complete the Cayley table of G:

$\cdot$	a	b	c	d
a			a	
b		d		a
c		b		
d			d	

   Which element is the identity element? What is the inverse of a? Is the group Abelian (how can you see it on the table?)?

4. Let G be a group and let H be a nonempty finite subset of G such that for every $a, b \in H$ we have $ab \in H$.

   (a) Let $a \in H$. Show that $a^{-1} \in H$. Hint: We did something similar twice in the course.

   (b) Show that H is a subgroup of G.