

ALGEBRAIC STRUCTURES (MST20010)

Problem sheet 5

1. For each of the following permutations of S_7 , indicate if it is odd or even.

(a) $(1\ 7\ 4\ 5\ 2)(2\ 4\ 7\ 6)$.

(b) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 6 & 7 & 4 & 5 & 2 & 1 \end{pmatrix}$.

2. Show that the inverse of an even permutation is even, and the product of an odd and an even permutation is odd.
3. Show that there is no solution in S_n to the equation

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \sigma = \tau$$

with σ cycle of length 4 and τ even (think of the parity of the permutations).

4. (a) Let S be the set of all UCD students and B the relation “having the same birthday”, i.e. if x and y are students, xBy means that x and y have the same birthday. Show that B is an equivalence relation on S .
(b) Let $f : A \rightarrow B$ be a function. We define a relation R on A by

$$xRy \Leftrightarrow f(x) = f(y).$$

Show that R is an equivalence relation on A .

5. We define a relation on \mathbb{Z} by: xRy if and only if x and y have a common divisor greater than 1. Show that R is not an equivalence relation.

The following exercise is for you to practice computing with permutations (if you have not done so already, do practice it, it is important. Just compute a few random products –also of more than 2 permutations–, inverses, how to write a permutation as a product of disjoint cycles, how to determine the order, the parity). It will not be corrected in the tutorials, the solution is upside-down just after it. Ask me in class if you want more explanations or had difficulties. (Do ask! It is very important!)

1. (a) Compute $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 1 & 7 & 5 & 2 & 6 & 3 \end{pmatrix}^{-1}$, write it as a product of disjoint cycles, and compute its order (hint: having it written as a product of disjoint cycles is useful for this).
- (b) Compute $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 3 & 2 \end{pmatrix}$, and its order.
- (c) Determine the following permutation of S_6 : $(1\ 2)(2\ 3)(1\ 4)$ (i.e., write it in the form of a table with two lines that we use to represent permutations)..

<p style="text-align: right;">Answer 1 (a) $= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 5 & 7 & 1 & 4 & 6 & 3 \end{pmatrix} = (1\ 2\ 5\ 4)(3\ 7). \text{ Its order is } lcm(4, 2) = 4.$</p> <p style="text-align: right;">(b) $= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} = (1\ 4)(2\ 3)$ so its order is 2.</p> <p style="text-align: right;">(c) $= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 1 & 2 & 5 & 6 \end{pmatrix}.$</p>
