

ALGEBRAIC STRUCTURES (MST20010)

Problem sheet 2

1. (a) Write the multiplication table of $\mathbb{Z}/3\mathbb{Z}$. Is $(\mathbb{Z}/3\mathbb{Z} \setminus \{0\}, \cdot)$ a group (where \cdot denotes the product in $\mathbb{Z}/3\mathbb{Z}$)?
(b) Is $(\mathbb{Z}/4\mathbb{Z} \setminus \{0\}, \cdot)$ a group?
2. Let G be a group, and let $a \in G$. Show that $(a^{-1})^2 = (a^2)^{-1}$.
Notation: If $b \in G$, by b^k we mean $b \cdot b \cdot \dots \cdot b$ (k times).
Observe that the same reasoning will give $(a^{-1})^k = (a^k)^{-1}$.
3. Let G be a group.
 - (a) Let $a, b, x \in G$. Solve $axb = c$ for x (in terms of a , b and c).
 - (b) Let $x, y \in G$ and assume that $xy = yx^2$. Write $xyxy$ in the form $y^i x^j$ for some integers i and j .
4. (This exercise uses Definition 2.12 from the course notes, so you may need to read a little bit more of them.) Let G be a group and let $g \in G$. Let $k = |g|$, the order of g . Show that $g^{mk} = e$ for every $m \in \mathbb{N}$.
So: If you raise an element to the power of a multiple of its order, you get e . The converse is true (if $g^k = e$ then k is a multiple of the order of g) and will be done in a future exercise sheet.