

ALGEBRAIC STRUCTURES (MST20010)

Problem sheet 1

1. Write the addition and multiplication tables of $\mathbb{Z}/4\mathbb{Z}$ (they are in the notes, so do them without looking, and check afterwards).
2. Find the remainder in the division by 5 of each of the following numbers (i.e. compute them in $\mathbb{Z}/5\mathbb{Z}$):

$$8^{92}, \quad (13^{15} \cdot 5^{26}) + (4 \cdot 26^{32}).$$

Hint: When computing the power of a number in $\mathbb{Z}/n\mathbb{Z}$, look at the sequence of successive powers of this number, there will be a repeating pattern.

3. (a) If today is a Monday, what day will it be in 47×642 days? Hint: Division by 7.
(b) You have 7 pieces of paper, and you apply the following procedure as many times as you want: Pick any one of your pieces of paper and cut it in 7.
Show that you can never get 1997 pieces of paper. Hint: Think modulo 6.
4. (a) Show that $2 \cdot 2 = 1$ in $\mathbb{Z}/3\mathbb{Z}$.
(b) Let $a, b \in \mathbb{Z}$ be such that $3a + 5b = 8$. Show that we must have $b = 1 \pmod{3}$ (i.e., $b = 1$ in $\mathbb{Z}/3\mathbb{Z}$).
(Depending on how you do this, it can be useful to realise that $2 \cdot 2 = 1$ in $\mathbb{Z}/3\mathbb{Z}$.)