

## Midterm exam

- Put your student number on top of each sheet.
  - Start your paper by writing the following statement:  
**The following is my own work, and I have not received any help during this exam.**
  - This midterm is graded out of 120, but grades are capped at 100.
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1. (25%) Let  $f : R \rightarrow S$  be a morphism of rings. Let  $I$  be a 2-sided ideal of  $S$ . Show that  $f^{-1}(I)$  is a 2-sided ideal of  $R$ .

Recall that  $f^{-1}(I) = \{x \in R \mid f(x) \in I\}$ .

2. Let  $R$  be a ring without zero divisors. Suppose that  $R$  has a minimal non-zero left ideal  $I$  (we may have  $I = R$ ).

(a) (5%) Show that, for every  $x \in R$ , if  $x \neq 0$  then  $x^2 \neq 0$ .

(b) (20%) Let  $i_0 \in I \setminus \{0\}$ . Show that  $Ri_0^2 = I$ .

(Where  $Ri_0^2 = \{ri_0^2 \mid r \in R\}$ .)

(c) (20%) Show that  $I = R$ . Hint:  $i_0$  belongs to  $I$ .

3. Let  $R$  be a ring and let  $M$  be an  $R$ -module.

(a) (25%) Show that  $M$  is simple if and only if  $M \neq \{0\}$  and  $M$  is generated by any non-zero element of  $M$ .

(b) (25%) Let  $N$  be a submodule of  $M$ . Show that if  $M$  is Artinian then  $M/N$  is Artinian.

Recall that Artinian means satisfying the descending chain condition on submodules.