

## GRAPHS AND NETWORKS (MATH20150)

### Problem sheet 6

1. Let  $e$  be an edge in a connected graph  $G$ . Turn  $G$  into a weighted graph by giving the weight 1 to  $e$  and the weight 2 to all the other edges. Then Kruskal's algorithm will produce a spanning tree containing  $e$  ( $e$  will be the first edge put in the spanning tree when you apply the algorithm).
2. That it is hamiltonian.
3. The closure of the first graph is the first graph itself (no changes). The closure of the second graph is  $K_7$ . So the second graph is hamiltonian. The first graph is also hamiltonian (despite its closure not being  $K_7$ ): A hamiltonian cycle can be found if you start at the lower left vertex, go to the top left one, then to the lower right one, then go back following the "zigzags".
4. Consider the graph where the people are the vertices, and where there is an edge between 2 people if they know each other. You want to find a Hamiltonian cycle in this graph. But it is possible since the degree of each vertex is at least 6 (so the closure of the graph will be  $K_{12}$ , which is Hamiltonian).
5. Consider the graph with vertices 1, 2, 3, 4, 5, 6 and an edge between vertices  $i$  and  $j$  iff there is a domino  $(i, j)$  in the list. This graph is Eulerian, so it is possible to obtain such a circle.