Graphs and Networks (MATH20150)

Problem sheet 5

1. Apply Kruskal's algorithm to find a minimal weigth spanning tree on two weighted graphs of your choice (don't take them too simple).

2. Let G be a connected graph and let e be an edge in G. Show that e is a cut edge in G if and only if e is in every spanning tree of G.

3. The purpose of this exercise is to show that if T is a tree with at least 2 vertices, then T has at least 2 leaves.

   (a) Show it using the degree sum formula.

   (b) Show it by showing that if P is a path of maximal length in the tree, then the first and last vertices in P both have degree 1.

4. Let T be a tree and let v be a vertex of T such that $d(v) = k$.

   (a) Show that $T \setminus \{v\}$ has exactly k components.

   (b) Show that T has at least k leaves.