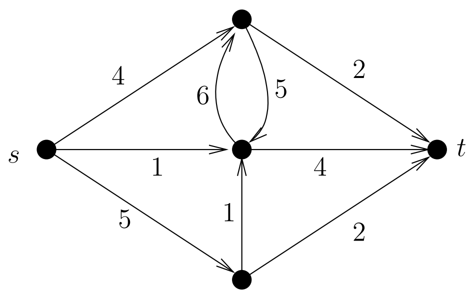
GRAPHS AND NETWORKS (MATH20150)

Problem sheet 11

1. We consider the following network, where the source is s, the sink is t, and the capacities are indicated next to the arcs.



   Find a maximmum flow and a minimum cut in this network. Justify why your cut is indeed minimum.

2. A company has to complete 3 projects, P_1, P_2 and P_3, in the next 4 months, subject to the following conditions:

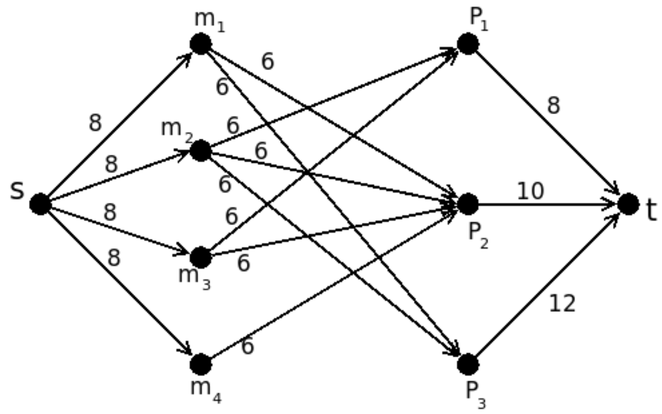
	P_1	P_2	P_3
can start from month	2	1	1
must be finished at the end of month	3	4	2
man-months required	8	10	12

   In any given month, only 8 workers can be employed, but at most 6 can work at the same time on a given project.

   Is it possible to to complete all projects on time, and how can it be organised?

   In order to solve this we build a network. The first step is to put vertices m_1, m_2, m_3, m_4 representing the 4 available months, and P_1, P_2, P_3 the three projects. We put an arc from m_i to P_j if it is possible to assign workers to project P_j during month m_i (for instance, only m_2 and m_3 are linked to P_1 since it cannot start before month 2 and must be finished by the end of month 3).

Explain why the following network models the problem, and why finding a maximal flow would give the answer.



Determine if there is a solution and, if yes, give it. Warning: Applying the Ford-Fulkerson algorithm in this case is a bit longer than usual (it took me 7 iterations).

3. (a) Let G be a graph with n vertices and less than $n-1$ edges. Show that G is not connected.

   (b) Let $G = (V, E)$ be a connected graph. Show that if $|E| \leq |V| - 1$ then G contains no cycle, and that if $|E| > |V| - 1$ then G contains a cycle.

   (c) Let $G = (V, E)$ be a connected graph such that $|E| \geq |V| - 1$. Show that G contains at least $|E| - |V| + 1$ cycles. Hint: Proceed by induction on $|E|$, starting with $|E| = |V| - 1$.