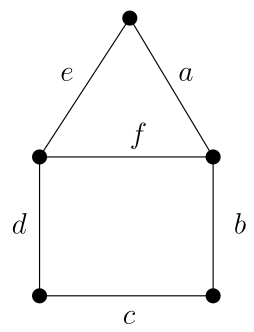
Graphs and Networks (MATH20150)

Problem sheet 1

1. Let $G = (V, E)$ be a graph. The line graph of G, denoted $L(G)$ is the graph that has E as set of vertices (i.e., the edges of G become the vertices of $L(G)$), and such that for every $e_1, e_2 \in E$, there is an edge in $L(G)$ between e_1 and e_2 if and only if the edges e_1 and e_2 have a common vertex in G.

   Draw the line graph of this graph G (keep the labels a, b, c, d, e, f for the edges of G, so that a, b, c, d, e, f are the vertices of $L(G)$).

2. The completement G^c of a graph $G = (V, E)$ is the graph with vertex set V and such that two vertices are adjacent in G^c if and only if they are not adjacent in G.

   (a) Draw an example of G^c, for some graph G of your choice.

   (b) Describe the graph $(K_n)^c$.

3. Are there graphs with the following degree sequences (produce such a graph, or explain why there is no such graph):

   (a) 2,2,2,3 (b) 1,2,2,3,4 (c) 2,2,4,4,4 (d) 1,2,3,4

   Hint for the final two: Consider what these numbers mean in term of how the vertices would be connected.

4. Let $G = (V, E)$ be a pseudograph with set of vertices $V = \{v_1, \ldots, v_n\}$. The adjacency matrix of G is the matrix $(a_{i,j})$ with $1 \leq i, j \leq n$ where $a_{i,j}$ is the number of edges in G from v_i to v_j.

   (a) Determine the adjacency matrix of two pseudographs of your choice.

   (b) Draw a picture of a pseudograph with adjacency matrix

$$\begin{pmatrix} 2 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 2 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$