University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

AUTUMN TRIMESTER EXAMINATION 2019/2020

MATH 20150

Graphs and Networks

Prof. B. Murphy

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Time Allowed: 2 hours

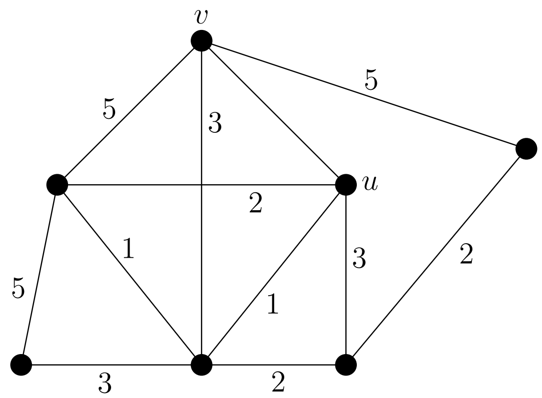
Instructions for Candidates

Full marks will be awarded for complete answers to all **five** questions.

Instructions for Invigilators

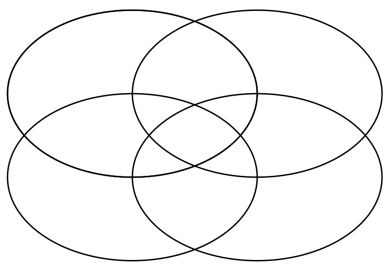
Candidates are allowed to use non-programmable calculators during this examination.

1. (a) What weight should be given to the edge uv in the folowing weighted graph so that it has a minimum weight spanning tree of weight 11? Produce such a minimal weight spanning tree.



(b) Is the graph in question 1(a) planar? Justify your answer.

2. (a) Can you draw the following figure without lifting your pen, without drawing the same arc twice, and with the pen finishing where it started? Justify your answer (you do not have to produce an explicit way to draw the figure).



(b) Let $G = (V, E)$ be a graph with adjacency matrix A. Show that the sum of the diagonal entries of the matrix A^2 is equal to $2|E|$.

3. Let $G = (V, E)$ be a connected graph.

(a) Show that the following two statements are equivalent:

i. G has a single cycle;

ii. There is an edge e in G such that $G \setminus \{e\}$ is a tree.

(b) Let S be a set of edges of G such that $G \setminus S$ is disconnected, and let T be a spanning tree of G. Show that S and T must contain a common edge.

4. Let $G = (V, E)$ be a graph. Recall that the line graph of G, denoted $L(G)$ is the graph whose set of vertices is E, and where two vertices of $L(G)$ are adjacent if and only if the corresponding edges of G are adjacent.

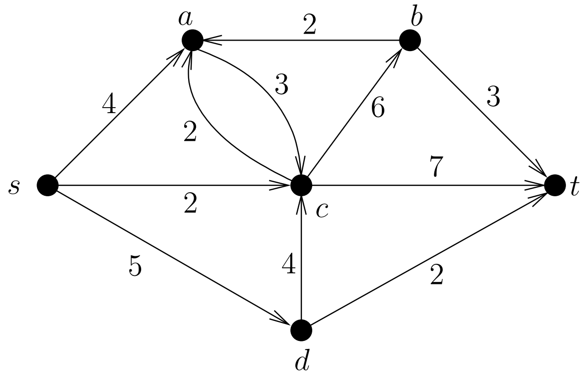
   (a) Draw the line graph of K_4 (use a dot for each vertex of the line graph).

   (b) Show that the line graph of an Eulerian graph is Eulerian.

   (c) If the line graph of G is Eulerian, must G be Eulerian?

5. (a) Let G be a connected planar graph with 6 vertices and 12 edges. Show that each face of G is bounded by 3 edges.

   (b) Find a maximum flow and a minimum cut in this network:



   (Justify your answer; the source of the network is the vertex s and the sink is the vertex t.)

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