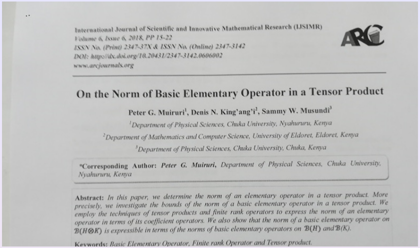

International Journal of Scientific and Innovative Mathematical Research (IJSIMR)
Volume 6, Issue 6, 2018, PP 15-22
ISSN No. (Print) 2347-37X & ISSN No. (Online) 2347-3142
DOI: http://dx.doi.org/10.20431/2347-3142.0606002
www.arcjournals.org

On the Norm of Basic Elementary Operator in a Tensor Product

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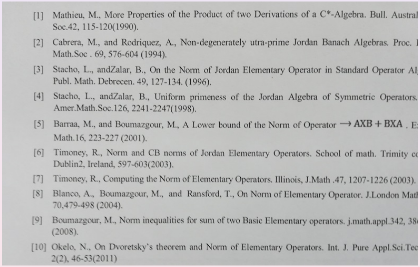
Abstract: *In this paper, we determine the norm of an elementary operator in a tensor product. More precisely, we investigate the bounds of the norm of a basic elementary operator in a tensor product. We employ the techniques of tensor products and finite rank operators to express the norm of an elementary operator in terms of its coefficient operators. We also show that the norm of a basic elementary operator on $\mathcal{B}(H \otimes K)$ is expressible in terms of the norms of basic elementary operators on $\mathcal{B}(H)$ and $\mathcal{B}(K)$.*

Keywords: *Basic Elementary Operator, Finite rank Operator and Tensor product.*



[1] Mathieu, M., More Properties of the Product of two Derivations of a C*-Algebra. Bull. Austral
Soc.42, 115-120(1990).
[2] Cabrera, M., and Rodriquez, A., Non-degenerately utra-prime Jordan Banach Algebras. Proc. I
Math.Soc . 69, 576-604 (1994).
[3] Stacho, L., andZalar, B., On the Norm of Jordan Elementary Operator in Standard Operator Al
Publ. Math. Debrecen. 49, 127-134. (1996).
[4] Stacho, L., andZalar, B., Uniform primeness of the Jordan Algebra of Symmetric Operators.
Amer.Math.Soc.126, 2241-2247(1998).
[5] Barraa, M., and Boumazgour, M., A Lower bound of the Norm of Operator $\longrightarrow$ AXB + BXA . E
Math.16, 223-227 (2001).
[6] Timoney, R., Norm and CB norms of Jordan Elementary Operators. School of math. Trimity co
Dublin2, Ireland, 597-603(2003).
[7] Timoney, R., Computing the Norm of Elementary Operators. Illinois, J.Math .47, 1207-1226 (2003).
[8] Blanco, A., Boumazgour, M., and Ransford, T., On Norm of Elementary Operator. J.London Math
70,479-498 (2004).
[9] Boumazgour, M., Norm inequalities for sum of two Basic Elementary operators. j.math.appl.342, 38
(2008).
[10] Okelo, N., On Dvoretsky's theorem and Norm of Elementary Operators. Int. J. Pure Appl.Sci.Tec
2(2), 46-53(2011)

$$\min_{\|A\|=\|B\|=1} \max_{\|X\|=1} \|AXB+BXA\| \color{red}{= 1}$$

C_0-semigroups of holomorphic Carathéodory spin-isometries

L.L. STACHÓ

10/05/2019, Dublin

SPIN FACTORS

$\big(\mathbf{H},\langle\cdot|\cdot\rangle\big)$ Hilbert space, $x \mapsto \overline{x}$ conjugation, $\langle x|y\rangle^- = \langle\overline{x}|\overline{y}\rangle$
$\mathcal{S} := \mathcal{S}(\mathbf{H},\bar{\cdot})$ JB*-triple
$\{xay\} = \langle x|a\rangle y + \langle y|a\rangle x - \underbrace{\langle x|\overline{y}\rangle}_{\langle y|\overline{x}\rangle}\,\overline{a}$

$e = \{eee\}$ TRIPOTENT:
(1) $e = \lambda v,\quad \lambda \in \mathbb{T},\quad v \in \mathrm{Re}(\mathbf{H}),\quad \|v\| = 1;$
(2) $e = \lambda u + iv,\quad \lambda \in \mathbb{T},\quad u, v \in \mathrm{Re}(\mathbf{H}), u \perp v,\quad \|u\| = \|v\| = 1/2.$

$\mathcal{S}$-unitary op.s: $U_t = \kappa_t V_t$: $\quad V_t : \mathrm{Re}(\mathcal{S}) \to \mathrm{Re}(\mathcal{S})$ $\langle\cdot|\cdot\rangle$-unitary, $\kappa_t \in \mathbb{T}$.

Norm formula: for $a = x + iy$ with $x = \overline{x}, y = \overline{y}$,
$$\|a\| = \big\|x + iy\big\| = \Big[\big[\langle x\rangle^2 + \langle y\rangle^2\big] + 2\big[\langle x\rangle^2\langle y\rangle^2 - \langle x|y\rangle^2\big]^{1/2}\Big]^{1/2}$$

HOLOMORPHIC ISOMETRIES OF THE UNIT BALL

E Banach space, **B** open unit ball, **D** $\subset$ **E** bded. dom.

Infinitesimal Carathéodory metric of **D** at a point $a \in$ **D**

$$\delta_{\mathbf{D}}(a,v) := \sup \big\{ \big|\varphi'(a+\zeta v)\big| : \varphi \in \mathrm{Hol}(\mathbf{D},\mathbb{D}),\ \varphi(a)=0 \big\}.$$

Metric: $\underline{d_{\mathbf{D}} = \int \text{ form of } \delta_{\mathbf{D}}}$; $\mathrm{Iso}(\mathbf{D}) := \big\{\text{holomorphic } d_{\mathbf{D}}\text{-isometries}\big\}$.

$$d_{\mathbf{B}}(0,x) = \mathrm{arth}\,\|x\| \ \ (x \in \mathbf{B}), \quad \delta_{\mathbf{B}}(v) = \|v\| \ \ (v \in \mathbf{E}).$$

Problem. Is $\Phi \in \mathrm{Iso}(d_{\mathbf{B}})$ LINEAR if $\underline{0 = \Phi(0)}$?

Remark. Ctrex: $\Phi : (\zeta_1, \zeta_2, \dots) \mapsto (\zeta_1^2, \zeta_1, \zeta_2, \dots)$ in c_0.
Cartan's Linearity Thm. if Φ surjective.

Problem. Ctrex with $\mathcal{C}_0$-sgr. $[\Phi^t : t \in \mathbb{R}_+]$?

Def. $\boldsymbol{\Phi} := \left[\Phi^t : t \in \mathbb{R}_+\right]$ $\mathcal{C}_0$-sgr. in $\mathrm{Iso}(d_{\mathbf{D}})$ if
$\Phi^{t+h} = \Phi^t \circ \Phi^h \quad (t, h \in \mathbb{R}_+), \quad \Phi^0 = \mathrm{Id}_{\mathbf{D}}$
$t \mapsto \Phi^t(x)$ cont. on $\mathbb{R}_+ \ \forall\, x \in \mathbf{D}$

Inf. gen. $\Phi'(x) := \frac{d}{dt}\Big|_{t=0+} \Phi^t(x)$ if exists.

Problem. Is $\mathrm{dom}(\Phi') = 0$ possible? (YES in real setting!)

Cauchy estimates $\longrightarrow$ basic hol. Hille-Yosida
(i) $\mathrm{dom}(\Phi') = \left\{x \in \mathbf{D} : t \mapsto \Phi^t(x) \text{ is continuously diff.}\right\}$.
(ii) $\frac{d}{dt}\Phi^t(x) = \Phi'\big(\Phi^t(x)\big) = \left[D_x\Phi^t\right]\Phi'(x) \qquad \big(x \in \mathrm{dom}(\Phi')\big)$.
(iii) The graph of Φ' is closed.
$\Phi' = \Psi'$, $\Rightarrow \Phi^t|\mathcal{D} \equiv \Psi^t|\mathcal{D}$ on $\mathcal{D} = \mathrm{dom}(\Phi')\big(= \mathrm{dom}(\Psi')\big)$

EARLY RESULTS ON HOL. $\mathcal{C}_0$-SGR IN JB*-TRIPLES

Kaup 1976-83. Banach-Lie theory of $\mathrm{Aut}(\mathbf{B})$ in gen. Banach setting
Möbius trf. in gen. JB*-triples
Uniformly cont. one-prg $(\mathrm{dom}(\Phi') = \mathbf{B})$ instead of $\mathcal{C}_0$-sgr.

$$(*) \qquad \Phi'(x) = a - \{xax\} + i\big[\mathbf{E}\text{-hermitian}\big]x, \quad a := \Phi'(0)$$

Def. *Kaup's type vector field*: $(*)$ with $0 \in \mathrm{dom}(\Phi')$ dense lin. in $\mathbf{B}$

Vesentini 1987-92. Hilbert ball, TRO ball, Spin ball:
Linearity of non-surjective 0-preserv. in $\mathrm{Iso}(d_{\mathbf{B}})$
$\rightarrow \Phi^t$ extends hol. to some nbh. of $\partial\mathbf{B}$; $\rightarrow \overline{\mathbf{\Phi}}$ on $\overline{\mathbf{B}}$, $\mathrm{Fix}(\overline{\mathbf{\Phi}}) \neq \emptyset$
Linear representations + Hille-Yosida theory
Tacitly used continuity properties of (ambiguous!) representation terms
No closed formulas for $\mathbf{\Phi}$, $\sim$ sketch for num. meth.

Stachó 2014-18: Jordan-approach in ∞-dim. refl. TRO,
recently: Spin factors

Thm. **E** gen. JB*-triple, $\mathrm{dom}(\Phi'), \mathrm{Fix}(\overline{\boldsymbol{\Phi}}) \neq \emptyset \Longrightarrow \mathrm{dom}(\Phi')$ dense in **B**

Thm. Let **E** be reflexive. Then
1) 0-preserving $d_{\mathbf{B}}$-isometries are lin. $\{\ldots\}$-hom.
2) Φ^t factor preserving
3) Up to Möbius-equivalence: $e \in \mathrm{Fix}(\overline{\boldsymbol{\Phi}})$ trip., Kaup's type Φ',
$\mathrm{dom}(\Phi') = \mathbf{B} \cap \big[$dense Jordan subtriple$\big]$

Application. Closed formulas for $\boldsymbol{\Phi}$ in case of refl. TRO
$\big(= \bigoplus_{\text{finite}} \mathcal{L}(\mathbf{H}_j, \mathbf{K}_j)$ with $\dim(\mathbf{K}_j) < \infty\big)$.

HOLOMORPHIC SPIN BALL ISOMETRIES

History. Pauli matrices $\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$, $\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$\longrightarrow$ $\mathbb{S}$ cl. selfadj. subs. $\subset \mathcal{L}(\mathbf{H})$, $s^2 \in \mathbb{C}\mathrm{Id}$ $(s \in \mathbb{S})$.

Fractional lin. form for some $\mathcal{C}_0$-groups of inner automorphisms.

Vesentini 1989. No fr.lin. form for gen. $\boldsymbol{\Phi}$ in $\mathbb{S}$-setting

$\mathcal{S} \equiv (\mathbf{H}, \bar{\cdot})$ with spin norm

Hierzbruch 1965. finite dim. $\rightarrow$ **Vesentini 1992.**

$$\Phi^t = R\big(G^t\big), \quad G^t = \begin{bmatrix} M_t & B_t \\ C_t^{\mathrm{T}} & E^t \end{bmatrix} \in \mathcal{L}(\mathbf{H} \oplus \mathbb{C}^2)$$

with $[G^t : t \in \mathbb{R}_+]$ $\mathcal{C}_0$ sgr. in $\mathcal{L}(\mathbf{H} \oplus \mathbb{C}^2)$.

MATRIX REPRESENTATION

$$G^t = \begin{bmatrix} M_t & B_t \\ C_t^{\mathrm{T}} & E^t \end{bmatrix} \qquad B_t = [b_1^t, b_2^t] \in \mathbf{H}^2,\ C_t^{\mathrm{T}} = \begin{bmatrix} \overline{c_1^t}^* \\ \overline{c_2^t}^* \end{bmatrix},\ E = \left[E_{k\ell}\right]_{k,\ell=1}^2$$

$$\Phi^t(x) = R(G^t)(x) = F^t(x)/\varphi^t(x)$$

$$F^t(x) = (b_1^t - ib_2^t) + 2M_t x + (x^{\mathrm{T}}x)(b_1^t + ib_2^t)$$

$$\varphi^t(x) = (E_{11}^t + E_{22}^t - iE_{12}^t + iE_{21}^t) + 2(c_1^t + ic_2^t)^{\mathrm{T}} x + (E_{11}^t - E_{22}^t + iE_{12}^t + iE_{21}^t)x^{\mathrm{T}}x$$

Alg. constrains:

$$[G^t]^* \mathrm{diag}(I, -I_2) G^t = \mathrm{diag}(I, -I_2),\ \ \det(E^t) > 0 \qquad (t \in \mathbb{R}_+),$$

$$C_t E^t = M_t^{\mathrm{T}} B_t,\ \ M_t^{\mathrm{T}} = I + C_t C_t^{\mathrm{T}},\ \ [E^t]^{\mathrm{T}} E^t = I_2 + B_t^{\mathrm{T}} B_t.$$

THE INFINITESIMAL GENERATOR

$$\Phi' = \frac{d}{dt}\Big|_{t=0+} \frac{F^t}{\varphi^t} = -\frac{\varphi'}{(\varphi^0)^2} F^0 + \frac{1}{\varphi^0} F'$$

where, for $x \in \mathrm{dom}(G')$,

$$\Phi'(x) = \left[\tfrac{1}{2}(b_1' - ib_2')\right] + \left[M' - iE_{21}'\right]x - \left[x(b_1' + ib_2')^{\mathrm{T}} x - \tfrac{1}{2}(b_1' - ib_2')x^{\mathrm{T}} x\right].$$

Proposition. If $\Phi'(x) = a + iAx - \{xa^*x\}$ is of Kaup's type then

$$G' = \begin{bmatrix} iA - i\varepsilon I & 2\,\mathrm{Re}(a) & -2\,\mathrm{Im}(a) \\ 2\,\mathrm{Re}(a)^{\mathrm{T}} & 0 & -\varepsilon \\ -2\,\mathrm{Im}(a)^{\mathrm{T}} & \varepsilon & 0 \end{bmatrix} \quad \text{where} \quad \varepsilon := E_{21}'$$

and $iA = M' + i\varepsilon I$ with $M' = -[M']^{\mathrm{T}}$, $\mathrm{dom}(M'), \mathrm{ran}(M') \subset \mathrm{Re}(\mathbf{H})$.

TRIANGULARIZATION WITH FIXED POINTS

Assume (up to Möbius equiv): $0 \neq e \in \partial \mathbf{B} \in \mathrm{Fix}(\overline{\mathbf{\Phi}})$ tripotent,
$\Phi^t = R(G^t)$, $[G^t : t \in \mathbb{R}_+]$ $\mathcal{C}_0$-sgr in $(\mathbf{H} \oplus \mathbb{C}^2)$

$\Phi'(x) = a + iAx - \{xa^*x\} =$
$\left(\frac{1}{2}b_1 - \frac{i}{2}b_2\right) + M'x + i\varepsilon x - \langle x|b_1 - ib_2\rangle x + \langle x|\overline{x}\rangle\left(\frac{1}{2}b_1 + \frac{i}{2}b_2\right)$,

$$G' = \begin{bmatrix} M' & b_1 & b_2 \\ b_1^{\mathrm{T}} & 0 & -\varepsilon \\ b_2^{\mathrm{T}} & \varepsilon & 0 \end{bmatrix}$$ where $b_1 := 2\mathrm{Re}(a), b_2 := -2\mathrm{Im}(a)$,
$M' = \overline{M'} = -[M']^{\mathrm{T}}$, $\varepsilon \in \mathbb{R}$.

Cases up to lin. equiv.

1) $e = \overline{e}$, $\langle e|e\rangle = 1$ (real extreme point),

2) $e \perp \overline{e}$, $\langle e|e\rangle = \frac{1}{2}$ (face middle point).

In any case, $\Phi'(e) = 0$ $\quad$ ($\longleftarrow$ extension to $\overline{\mathbf{B}}$)

CASE (1) REAL TRIP. $e = \overline{e}$, $\langle e|e\rangle = 1$

$$0 = \Phi'(e) = a + iAe - \{ea^*e\} =$$
$$= \left(\tfrac{1}{2}b_1 - \tfrac{i}{2}b_2\right) + M'e + i\varepsilon e - \langle e|b_1 - ib_2\rangle e + \langle e|e\rangle\left(\tfrac{1}{2}b_1 + \tfrac{i}{2}b_2\right).$$

$b_j := \rho_j e + x_j \quad (\rho_j \in \mathbb{R},\ x_j \perp e)$ orth. decomp. $\underbrace{\mathbb{C}e \oplus e^\perp}_{\mathbf{H}} \oplus \mathbb{C} \oplus \mathbb{C}$

$M'_0 := \mathrm{P}_{e^\perp} M'|e^\perp$ restricted operator

$$M' = \begin{bmatrix} 0 & -(M'e)^{\mathrm{T}} \\ M'e & M'_0 \end{bmatrix}, \quad b_1 = \begin{bmatrix} \rho_1 \\ x_1 \end{bmatrix}, \quad b_2 = \begin{bmatrix} -\varepsilon \\ x_2 \end{bmatrix}$$

$$G' = \begin{bmatrix} M' & b_1 & b_2 \\ b_1^{\mathrm{T}} & 0 & -\varepsilon \\ b_2^{\mathrm{T}} & \varepsilon & 0 \end{bmatrix} = \begin{bmatrix} 0 & x_1^{\mathrm{T}} & \rho_1 & -\varepsilon \\ -x_1 & M'_0 & x_1 & x_2 \\ \rho_1 & x_1^{\mathrm{T}} & 0 & -\varepsilon \\ -\varepsilon & x_2^{\mathrm{T}} & \varepsilon & 0 \end{bmatrix}.$$

QUASI-TRIANGULARIZATION, CASE (1)

$$T := \begin{bmatrix} 1/2 & 0 & 0 & 1 \\ 0 & I_0 & 0 & 0 \\ -1/2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad T^{-1} G' T = \begin{bmatrix} -\rho_1 & 0 & 0 & 0 \\ -x_1 & M_0' & x_2 & 0 \\ -\varepsilon & x_2^{\mathrm{T}} & 0 & 0 \\ 0 & x_1^{\mathrm{T}} & -\varepsilon & \rho_1 \end{bmatrix}.$$

Proposition. G' is triangularizable if $y = \langle z|y\rangle z - M_0' z$ has a solution $z \in e^{\perp}$. This happens whenever $y \in \mathrm{ran}(M_0')$.

Remark. M_0' is a possibly unbounded skew symmetric closed real-linear operator on a dense lin. submanifold of $e^{\perp}$, $\longrightarrow$ LIN ISOM. $\mathcal{C}_0$-SGR

CASE (2) FACE CENTER $0 = \Phi'(e)$, $e \perp \overline{e}$, $\langle e\rangle^2 = 1/2$

$$0 = \Phi'(e) = \left(\tfrac{1}{2}b_1 - \tfrac{i}{2}b_2\right) + M'e + i\varepsilon e - \langle e|b_1 - ib_2\rangle e.$$

$e = \frac{1}{2}u + \frac{i}{2}v$, $\quad u \perp v,\ u, v \in \mathrm{Re}(\mathbf{H})$ and $\langle u\rangle^2 = \langle v\rangle^2 = 1$.

Orthogonal decomposition: $\underbrace{\mathbb{C}u \oplus \mathbb{C}v \oplus \{u,v\}^\perp}_{\mathbf{H}} \oplus \mathbb{C} \oplus \mathbb{C}$

$b_j = \rho_j u + \sigma_j v + x_j$, (where $x_1, x_2 \perp \{u, v\}$)
$P := \mathrm{P}_{\{u,v\}^\perp}$, $M'_0 := PM'|\{u,v\}^\perp$,

$$G' = \begin{bmatrix} M' & b_1 & b_2 \\ b_1^{\mathrm{T}} & 0 & -\varepsilon \\ b_2^{\mathrm{T}} & \varepsilon & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2\rho_2 - \varepsilon & x_1^{\mathrm{T}} & \rho_1 & \rho_2 \\ \varepsilon - 2\rho_2 & 0 & -x_2^{\mathrm{T}} & \rho_2 & -\rho_1 \\ -x_1 & x_2 & M'_0 & x_1 & x_2 \\ \rho_1 & \rho_2 & x_1^{\mathrm{T}} & 0 & -\varepsilon \\ \rho_2 & -\rho_1 & x_2^{\mathrm{T}} & \varepsilon & 0 \end{bmatrix}$$

TRIANG. FORM OF GEN. CASE (2)

$$T^{-1}G'T = \begin{bmatrix} -\rho_1 & \rho_2-\varepsilon & 0 & 0 & 2\varepsilon \\ \varepsilon-\rho_2 & -\rho_1 & 0 & 0 & 0 \\ -x_1 & x_2 & M_0 & 0 & 0 \\ \rho_1 & \rho_2 & x_1^{\mathrm{T}} & \rho_1 & \varepsilon-\rho_2 \\ \rho_2 & -\rho_1 & x_1^{\mathrm{T}} & \rho_2-\varepsilon & \rho_1 \end{bmatrix}$$

with

$$T := \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & I_0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

INTEGRATION OF THE ALMOST TRIANG. SYSTEMS

Up to linear equiv.: $G' \sim A + B$,
A lower triang. + integrable diag., B rank 1

Notation.
$[U^t : t \in \mathbb{R}]$ lin. $\mathcal{C}_0$-sgr, inf. gen. $U' = A + B$ $\longrightarrow \Phi^t = R(U^t)$,
$[T^t : t \in \mathbb{R}]$ lin. lower triang. $\mathcal{C}_0$-sgr with inf. gen. A

[Stachó 2016] $\Rightarrow$ Closed (complicated) formulas for T^t

Convolution equation of Volterra type

(V) $$U^t = \int_{s=0}^{t} T^{t-s} B U^s ds + T^t$$

VOLTERRA GENERATORS

Case (1) $A := \begin{bmatrix} -\rho & 0 & 0 & 0 \\ -x & M_0' & 0 & 0 \\ -\varepsilon & y^{\mathrm{T}} & 0 & 0 \\ 0 & x^{\mathrm{T}} & -\varepsilon & \rho \end{bmatrix}, \quad B := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & y & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$

Case (2)

$$A = \begin{bmatrix} -\rho_1 & \rho_2 - \varepsilon & 0 & 0 & 0 \\ \varepsilon - \rho_2 & -\rho_1 & 0 & 0 & 0 \\ -x_1 & x_2 & M_0' & 0 & 0 \\ \rho_1 & \rho_2 & x_1^{\mathrm{T}} & \rho_1 & \varepsilon - \rho_2 \\ \rho_2 & -\rho_1 & x_1^{\mathrm{T}} & \rho_2 - \varepsilon & \rho_1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 & 0 & 2\varepsilon \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

DYSON-PHILLIPS SERIES FOR (V) Case (1)

$U^t = \sum_{n=0}^{\infty} S_n(t)$ $\qquad S_0(t) := T^t,\ S_{n+1}(t) = \int_0^t T^{t-s} B S_n(s)\, ds$

$\sum_{n=0}^{\infty} S_n(t)$ converges locally uniformly in norm.

$$T^{t-s} B S_n(s) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \left[T_{k,2}^{t-s} y [S_n(s)]_{3,\ell} \right]_{\substack{2 \le k \le 4 \\ 1 \le \ell \le 4}} & & & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \end{bmatrix}.$$

column $\times$ row

$$U^t = \begin{bmatrix} e^{-\rho t} & 0 & 0 & 0 \\ \left[U_{k,\ell}^t \right]_{k>1, \ell<4} & & & \begin{matrix} 0 \\ 0 \\ e^{\rho t} \end{matrix} \end{bmatrix}, \qquad U_{k,\ell}^t = \int_{r=0}^t \left[T_{k,2}^{t-r} y \right] \underbrace{U_{3,\ell}^r}_{\text{govern}} dr + T_{k,\ell}^t$$

CONTROLLING SCALAR V-EQU.

Control functions $w(t) := T_{32}^t y, \quad V_\ell(t) := T_{3,\ell}^t \quad (\ell = 1, 2, 3),$

$$\big[T^{t-s} B S_n(s)\big]_{3,\ell} = T_{32}^{t-s} y [S_n(s)]_{3,\ell}, \Longrightarrow$$

with locally uniform convergence in t,

$$U_{3,\ell}^t = T_{3,\ell}^t + \sum_{n=1}^{\infty} S_n(t)_{3,\ell} =$$

$$V_\ell(t) + \big\{w * V_\ell\big\}(t) + \sum_{n=2}^{\infty} \big\{\underbrace{w * \cdots * w}_{n \text{ terms}} * V_\ell\big\}(t) = \big\{W * V_\ell\big\}(t),$$

$$U_{3,\ell}^t = w * U_{3,\ell}^t + V_\ell.$$

LAPLACE TRF.

$$\mathcal{L}v = \mathcal{L}_t\{v(t)\} : s \mapsto \int_{t=0}^{\infty} e^{-st} V(t)\, dt,$$

$$\mathrm{dom}(\mathcal{L}v) = \left\{ s \in \mathbb{C} : \int_{t=0}^{\infty} \left| e^{-st} v(t) \right| dt < \infty \right\};$$

$[U_0^t : t \in \mathbb{R}_+]$ isometries with $\mathrm{Re}(\mathbf{H}_0) \to \mathrm{Re}(\mathbf{H}_0)$, generator M_0',

$$U_0^t z = \int_{\lambda \in \mathbb{R}} e^{i\lambda t} P(d\lambda)\, z \qquad \big(z \in \mathrm{Re}(\mathbf{H}_0)\big) \qquad \longleftarrow \text{Deddens}$$

$$w(t) = \int_{\lambda \in \mathbb{R}_+} \frac{\sin(\lambda t)}{\lambda}\, dp(\lambda)$$

$$p(\Lambda) = 2\left\langle y \middle| P(\Lambda) y \right\rangle \ (\Lambda > 0) \ \ p(\{0\}) = \left\langle y \middle| P(\{0\}) y \right\rangle, \ \ p(\mathbb{R}_+) = \|y\|^2 < 1.$$

LAPLACE SOLUTION

$|w(t)| \leq \mu e^{\Omega t} \quad (t \in \mathbb{R}_{++})$ for some $\mu, \Omega > 1$,

$$\mathcal{L}_t\big\{e^{-\Omega t} U^t_{3,\ell}\big\} = \frac{\mathcal{L}_t\big\{e^{-\Omega t} T^t_{3,\ell}\big\}}{1 - \mathcal{L}_t\big\{e^{-\Omega t} T^t_{32} y\big\}} \qquad (\ell = 1, 2, 3)$$

$\longrightarrow$ Closed formulas in ters of inverse Laplace trf.
CASE (2) analogous

CONCLUSIONS

Theorem. Let $\mathbf{\Phi} = [\Phi^t : t \in \mathbb{R}_+]$ be $\mathcal{C}_0$-sgr of hol. Carathéodory isometries of the unit ball $\mathbf{B}$ of a spin factor $\mathcal{S} := \mathrm{SPIN}(\mathbf{H}, \overline{\cdot})$.

Alternatives up to Möbius equivanece:

(0) $\mathbf{\Phi}$ linear,
(1) Φ' Kaup's type, $\exists\, e \in \mathrm{Fix}(\overline{\mathbf{\Phi}})$ tripotent, $\langle e \rangle^2 = 1$ extreme point of $\overline{\mathbf{B}}$,
(2) Φ' Kaup's type, $\exists\, e \in \mathrm{Fix}(\overline{\mathbf{\Phi}})$ tripotent, $\langle e \rangle^2 = 1/2$ face center of $\overline{\mathbf{B}}$.

Cases (1,2): the $\Phi^t = R(G^t)$ with $[G^t : t \in \mathbb{R}_+] \subset \mathcal{L}(\mathbf{H})$,

$G^t = TU^t T^{-1}$ in a linar chart T, $U' = A + B$ quasi triang, op. matrix

A lower triang., diag. entries: max. closed symm. op $\oplus$ finte dim. op.,
B is rank 1 with a unique non-vanishing sup-diag. entry.

ALGEBRAIC STUCTURE

The entries of U^t can be expressed with finite formulas in fractional lin. terms of a lin. isometry $\mathcal{C}_0$-sgr., rank 1 ops., classical spec. functions and the solution of a Volterra type scalar convolution equation which admits a closed form by means of Laplace and inverse Laplace transformations.

Remark. Using a Deddens type $\mathcal{C}_0$-group dilation (with enlarged Hilbert space), we can construct a $\mathcal{C}_0$-group dilation for $\mathbf{\Phi}$ on the unit ball of a suitable covering spin factor of $\mathcal{S}$.

Open problem. Simplify the procedure with Laplace transform.

ACKNOWLEDGEMENT

This research was supported by the Ministry of Human Capacities, Hungary grant 20391-3/2018/FEKUSTRAT.

THANKS FOR YOUR ATTENTION