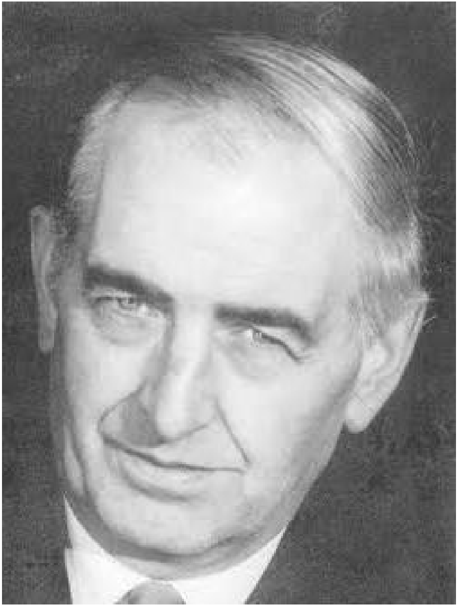
A Sample of Richard's contributions to Infinite Dimensional Analysis

Seán Dineen

AGA

9 May, 2019



Dick Timoney 1909-1985

Dear All,

I am saddened to hear of Richard's passing. I cannot come to Dublin for the fest, but I have a story I wish to share.

In Spring 1978 I was a Postdoc at MIT and I gave a job talk at Illinois. There was a party for me, at which a senior faculty member was pontificating about investments.

I wasn't interested so instead I chatted at length with the only graduate student there, Richard Timoney. He told me a bit about his work on Bloch functions in several complex variables.

He asked me a few questions about strongly pseudoconvex domains; I was impressed with his questions and his interests.. I took the job, and I have been at Illinois ever since.

He gave me a very positive impression of the quality of the graduate program, which was a decisive reason for me to accept the offer. (In fact, the program was nowhere near as good as he was!)

Over the years we had several interactions; I invited him back here for something in honor of his advisor which he attended and spoke, and I refereed papers for him a few times.
I owe him a huge debt for being such a great graduate student; he was a crucial reason for my choosing where I spent my mathematical career. I once told him so, and his response was quite modest.
I think the world of him.
Best regards,
John P. D'Angelo.

PERSONAL ITEMS

Professor J.R. Timoney retired as Professor of Mathematical Analysis at University College Dublin on November 30, 1979. Professor Timoney joined the staff of University College Dublin in 1932. Dr. Seán Dineen was appointed to the position vacated by Professor Timoney.

Professor Michael Hayes (U.C.D.) has been elected to membership of the Royal Irish Academy.

Dr. Michael Mortell has been appointed Professor of Applied Mathematics and Registrar at University College, Cork.

Dr. Richard Ward joined the staff of Trinity College in October, 1979. Dr. Ward works in differential geometry and wrote his thesis under the guidance of Roger Penrose at Oxford.

Dr. Richard Timoney joined the staff of Trinity College in January of 1980. Dr. Timoney works in Complex Analysis and obtained his Ph.D. from the University of Illinois at Champagne under the direction of Lee Rubel. Prior to joining Trinity College he held a position in the Mathematics Department of the University of Indiana at Bloomington.

Definition 1

A sequence of finite dimensional Banach subspaces $(X_n)_{n=1}^{\infty}$ of a Banach space X is a 1-unconditional finite dimensional decomposition of X if for each $x \in X$ there is a unique sequence $(x_n)_{n=1}^{\infty}, x_n \in X_n$ all n, such that $x = \sum_{n=1}^{\infty} x_n =: \lim_{m\to\infty} \sum_{n=1}^{m} x_n$ and

$$\|x\| = \sup\{\|\sum_{n=1}^{\infty} \lambda_n x_n\| : |\lambda_n| \leq 1, n = 1, 2, \ldots\}.$$

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Definition 2

A bounded symmetric domain is the open unit ball $\mathcal{B}$ of a Banach space X such that for each $x \in \mathcal{B}$ there exists a biholomorphic automorphism of $\mathcal{B}$, φ_x, such that $\varphi_x^2 = \mathbf{1}_{\mathcal{B}}$ and x is the unique fixed point of φ_x.

A deep result of W. Kaup says that bounded symmetric domains can also be described as the unit balls of JB^* triples, where these triples are Banach spaces endowed with a triple product $\{x, y, z\}$ obeying a number of axioms including the Jordan triple identity. A typical example of a JB^* triple is a $\mathcal{C}^*$ algebra endowed with the product

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In finite dimensions there are 4 infinite collections of irreducible bounded symmetric domains, these are the unit balls of the $n \times m$ matrices, the symmetric $n \times n$ matrices, the skew-symmetric $n \times n$ matrices, the spin factors and 2 exceptional domains of dimension 16 and 27. Every finite dimensional bounded symmetric domain can be written as a finite product, unique up to permutation, of irreducible bounded symmetric domains.

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Theorem 3

(Barton, D, T) If $\{X_n\}_{n\in\mathbb{N}}$ is a 1-unconditional finite dimensional decomposition of the Banach space X and the open unit ball of X is a bounded symmetric domain then X is isometrically isomorphic to $\left(\sum_m F_m\right)_{c_0}$ where the unit ball of each F_m is either an irreducible finite dimensional bounded symmetric domain or is isometrically isomorphic to $\mathcal{L}(H_1, H_2)$ where each H_i is a separable Hilbert space at least one of which is finite dimensional.

We followed this by answering a question posed by W. Kaup. Kaup proved the following: if B_H is the closed unit ball of a Hilbert space H and the domain $\mathcal{D}$ satisfies $B_H \subset \mathcal{D} \subset \sqrt{2}B_H$ then $\mathcal{D}$ is irreducible, that is it is not biholomorphically equivalent to a non-trivial product of domains, and he also showed that every bounded domain in a Hilbert space is biholomorphically equivalent to a finite product of irreducible domains.

He wished to know in which Banach spaces there are similar results. Let $\phi_X(r)$ denote the maximum number of irreducible factors for a domain $\mathcal{D}$, $B_X \subset \mathcal{D} \subset rB_X$.

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2. $\phi_X(r) < \infty$ *for all* $r \geq 1$ *if and only if X has finite cotype.*
3. *If* $q(X) := \inf\{q : X \text{ has cotype } q\} < \infty$ *then for every* $\varepsilon > 0$ *there is* $A(\varepsilon, X) > 0$ *such that*

$$\left(\frac{r-1}{2}\right)^{q(X)} \leq \phi_X(r) \leq A(\varepsilon, X) r^{q(X)+\varepsilon}.$$

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4. *If* $2 \leq p < \infty$ *and* $X := \mathcal{L}^p(\Omega, \mu)$ *then* $\phi_X(r) = [r^p]$ *if* X *is infinite dimensional and* $[\cdot]$ *denotes integer part.*

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5. **[Conjecture]** $\phi_X(r) \geq [r^2]$ *for any infinite dimensional X.*

If X is a Banach space then a continuous linear operator $T : X \longrightarrow X$ is a multiplier if each extreme point ϕ of $B_{X'}$ is an eigenvalue of ${}^tT$ with eigenvalue $\lambda_T(\phi)$. The centralizer $Z(X)$ of X consists of all multipliers for which there is a another multiplier S such that $\lambda_S(\phi) = \overline{\lambda_T(\phi)}$. The space X is irreducible if $Z(X)$ is one dimensional.

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The centroid $Z(X)$ of a JB^* triple $(X, \{\cdot,\cdot,\cdot\})$ consists of all continuous linear operators $T : X \longrightarrow X$ such that $T\{x, y, z\} = \{Tx, y, z\}$ for all $x, y, z \in X$.

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When domains cannot be written as a product of a finite number of irreducibles one has to look at infinite products. These come in various forms and following the above Richard, Maciej Klimek and myself investigated that situation.

Definition 6

A maximal function module representation of the Banach space X is a quadruple $(K, (X_k)_{k\in K}, \widetilde{X}, \rho)$ where K is a compact Hausdorff space, $(X_k)_k$ is a collection of Banach spaces, $\widetilde{X}$ is a Banach subspace of $\ell^\infty(\{X_k\}_{k\in K}) := \{(x(k))_{k\in K} : \sup_{k\in K} \|x(k)\| < \infty\}$ and ρ is an isometric isomorphism of X onto $\widetilde{X}$ such that

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5. $Z(X) = \{\varrho^{-1} \circ m_h \circ \varrho : h \in \mathcal{C}(K)\}$ where $[m_h(x)](k) = h(k)x(k)$ for $x \in \widetilde{X}$ and $k \in K$.

Every Banach space has a maximal function module representation and this is unique in a canonical way. X is irreducible if and only if K is connected. If X is a dual Banach space then K is extremelt disconnected and the mappings in (3) are continuous. Moreover, a dual Banach space is irreducible if and only if K is a singleton.

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$$\rho(\psi(x))(k) = \theta_k(\rho(x)(k)).$$

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$$\rho(\psi(x))(k) = \theta_k(\rho(x)(k)).$$

3. *If X is a Banach space then X may be isometrically embedded as a weak*-dense subspace of $Y := \ell^\infty(\{Y_i\}_{i\in I})$ such that each $\psi \in Aut(B_x)$ extends to an element of $Aut(B_Y)$.*

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Let X denote a JB^* triple which is also a dual Banach space. Then X has a unique predual X_* and the triple product on X is separately $\sigma(X, X_*)$ continuous.

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Because of time considerations I will skip any other references to bounded symmetric domains and just mention a very different type of result which had been extensively developed by several people in the audience here today.

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2. For each continuous semi-norm p on X there exists a continuous semi-norm q such that for all $x = \sum_{n=1}^{\infty} x_n e_n \in X$

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It had been known for some time that the monomials form an absolute basis for the space of holomorphic functions with the compact open topology on any open polydisc in a fully nuclear space with basis. Richard and I were interested in the converse: does the presence of a monomial absolute basis imply nuclearity?

If every compact set in X is contained in a compact polydisc then the existence of an absolute basis implies that for every compact polydisc $\{\sum_{n=1}^{\infty} z_n e_n : |z_n| \leq \alpha_n\}$ there exists another compact polydisc $\{\sum_{n=1}^{\infty} z_n e_n : |z_n| \leq \beta_n\}$ and $C > 0$ such that for any collection of scalars $(a_m)_{m\in\mathbb{N}^{(\mathbb{N})}}$

$$\sum_{m\in\mathbb{N}^{(\mathbb{N})}} \|a_m z^m\|_{\{|z_n|\leq\alpha_n\}} = \sum_{m\in\mathbb{N}^{(\mathbb{N})}} |a_m|\alpha^m \leq C \sup\Big\{\Big|\sum_{m\in\mathbb{N}^k} a_m z^m\Big| : |z_n| \leq \beta_n\Big\}$$

where $m = (m_1, \ldots, m_n)$ and $z^m = z_1^{m_1} \cdots z_n^{m_n}$. If $r_n = \alpha_n/\beta_n$ this is equivalent to

$$\sum_{m\in\mathbb{N}^{(\mathbb{N})}} |a_m| r^m \quad \leq \quad C \sup\Big\{\Big|\sum_{m\in\mathbb{N}^k} a_m z^m\Big| : |z_n| \leq 1\Big\}. \tag{2}$$

This reminded us of the following H. Bohr inequality from 1914: if $0 \leq r \leq 1/3$ then

$$\sum_{n=0}^{\infty} |a_n| r^n \quad \leq \quad \sup\Big\{\Big|\sum_{n=0}^{\infty} a_n z^n\Big| : |z| \leq 1\Big\}$$

and $1/3$ is best possible in this case. Initially we hoped that extending Bohr's inequality would lead us to solving the basis problem. However, an inequality of Hardy-Littlewood and (2) for scalars a_m with $|m| = 2$, that is the existence of an absolute basis for the 2-homogeneous polynomials, allowed us prove nuclearity and solve the basis problem. Moreover, the same approach and an estimate of Mantero-Tonge in place of Hardy-Littlewood for a_m with $|m| = k$, $k = 1, 2, \ldots,$ allowed us to prove the following several variables version of Bohr's Inequality:-

Theorem 10

For any $\varepsilon > 0$ *there exists* $C(\varepsilon) > 0$ *such that if* $(r_j)_{j=1}^{\infty}$ *satisfies (2) then for all* n

$$\sum_{j=1}^{n} r_j \leq C(\varepsilon) n^{\frac{1}{2}+\varepsilon}. \tag{3}$$

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