Markushevich bases and projectional skeletons for JBW*-triple preduals

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AGA (Analysis, Geometry, Algebra)
Hamilton Mathematics Institute,
A Tribute to Professor Richard Timoney
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MATH. SCAND. 59 (1986), 177–191

WEAK*-CONTINUITY OF JORDAN TRIPLE PRODUCTS AND ITS APPLICATIONS

T. BARTON and RICHARD M. TIMONEY

S. Dineen [3] has shown that if E is a JB*-triple, then so is its bidual E^{**}. We observe here that the triple product on E^{**} is in fact separately w^*-continuous (Theorem 1.4). This result is used to show that if E is a JB*-triple and a dual Banach space, then E has a unique predual and the triple product an E is separately w^*-continuous (Theorem 2.1). From this it will follow that the closed ideals of any JB*-triple are precisely its M-ideals,

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This is one of the first papers I studied as a Ph.D. student, around 1999, in Granada.

Later on.... I discovered the persons behind the names, I shared with Richard several moments here, in Dublin, in London, Hong-Kong ...

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Some of the preceding speakers knew Richard much closely than me; all I can add is that he was a positive scientific and personal influence.

At this stage I should confess that I have changed the title and subject of this talk. The reason will be revealed very soon. Before presenting the new one, I apologize for the inconveniences and I ask for certain patience. Let me continue from the previously commented theorem.

At the same time that I discovered the mentioned Barton-Timoney theorem, I was also exposed to the influence of Grothendieck's contribution to Functional Analysis.

Let me place you on the exact historic background....

[A. Grothendieck, *Résumé de la théorie métrique des produits tensoriels topologiques*'1956]

There exists a universal constant $G > 0$ satisfying that for every couple (Ω_1, Ω_2) of compact Hausdorff spaces and every bounded bilinear form V on $C(\Omega_1) \times C(\Omega_2)$ there exist two probability measures μ_1 and μ_2 on Ω_1 and Ω_2, respectively, such that

$$|V(f,g)| \leq G\|V\| \left(\int_{\Omega_1} |f(t)|^2 d\mu_1(t)\right)^{\frac{1}{2}} \left(\int_{\Omega_2} |g(s)|^2 d\mu_2(s)\right)^{\frac{1}{2}}$$

for all $f \in C(\Omega_1)$ and $g \in C(\Omega_2)$.

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for all $f \in C(\Omega_1)$ and $g \in C(\Omega_2)$.

By identifying, via Riesz's representation theorem, the probability measures with norm-one positive functionals in $C(\Omega_i)^*$ we have.....

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$$|V(f,g)|^2 \leq G^2 \|V\|^2 \varphi_1\left(|f|^2\right) \varphi_2\left(|g|^2\right),$$

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Grothendieck already conjectured in 1956 that a similar conclusion should hold for general C*-algebras......

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(*Non-commutative Grothendieck inequality*)

[G. Pisier, *J. Funct. Anal.*'1978, U. Haagerup, *Adv. Math.*'1985]

For every bounded bilinear form V on the cartesian product of two C*-algebras A and B, there exist two states ϕ in A^* and ψ in B^* satisfying

$$|V(x,y)| \leq 4\,\|V\|\,\phi\left(\frac{xx^*+x^*x}{2}\right)^{\frac{1}{2}}\psi\left(\frac{yy^*+y^*y}{2}\right)^{\frac{1}{2}},$$

for all $(x,y)\in A\times B$.

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moduli of continuous functions $\rightsquigarrow$ $|x|^2 = \dfrac{xx^* + x^*x}{2}$

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for all $(x,y) \in A \times B$.

In the non-commutative setting, the pre-Hilbertian semi-norms of the form

$$\|x\|_\phi^2 := \phi\left(\frac{xx^* + x^*x}{2}\right),$$

with ϕ running through the set of all states on a C*-algebra, are valid to factor all bounded bilinear forms.

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for all $x \in A$.

That is, every C*-algebra encodes enough information to control every bounded linear operator from itself into an arbitrary complex Hilbert space. Its algebraic structure hides all the Euclidean information.

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First, we need to identify "appropriate" preHilbertian semi-norms.

Every C*-algebra A is a JB*-triple with respect to the triple product

$$\{x, y, z\} = \frac{1}{2}(xy^*z + zy^*x), \ (x, y, z \in A).$$

Actually the same triple product remains valid to produce an structure of JB*-triple on the space $B(H, K)$ of all bounded linear operators between complex Hilbert spaces.

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If φ is a state on A ($\varphi(1) = \|\varphi\| = 1$), then the preHilbert semi-norm

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[Barton and Y. Friedman, *J. London Math. Soc.*'1987]

Let φ be a functional in the dual space E^* of a JB*-triple E, and let z be a norm-one element in E^{**} such that $\varphi(z) = \|\varphi\|$. Then the mapping

$$(x, y) \mapsto \varphi\{x, y, z\}, (x, y \in E)$$

is a semi-positive sesquilinear form on E which does not depend on the choice of z. The corresponding semi-norm is denoted by $\|x\|_{\varphi}^2 = \varphi\{x, x, z\}$.

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For every bounded linear operator T from a complex JB*-triple E into a complex Hilbert space H there is a norm-one functional $\varphi \in E^*$ satisfying

$$\|T(x)\| \leq \sqrt{2}\|T\|\|x\|_\varphi \text{ for every } x \in E.$$

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(*Grothendieck inequality*) [Barton-Friedman, *J. London Math. Soc.*'1987] [Chu-Iochum-Loupias, *Math. Ann.*'1989]

There exists a universal constant $K \in [2, 3 + 2\sqrt{2}]$ satisfying the following property: for every bounded bilinear form V on the cartesian product of two JB*-triples E and F there exist norm-one functionals $\varphi \in E^*$ and $\psi \in F^*$ satisfying

$$|V(x, y)| \leq K \, \|V\| \, \|x\|_\varphi \, \|y\|_\psi, \tag{1}$$

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LGT $\rightsquigarrow$ GT !!

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Unfortunately, the original proof of the LGT contains a gap!!!

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for all $(x,y) \in E \times F$.

[Borut Zalar, MathSciNet, MR1851084]

"...its main importance is the discovery that some technical result from the Banach space geometry on weak*-continuous bilinear forms is not true. A counterexample is provided. Therefore, previously published results cannot be considered fully proved. The present authors do not provide a counter-example to the version of Grothendieck's inequality for complex JB^*-triples, which was given by Barton and Friedman."

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"*The problem of extending the non-commutative Grothendieck theorem from C^*-algebras to JB^*-triples was considered notably by Barton and Friedman around 1987, but seems to be still incomplete.*"

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"*The problem of extending the non-commutative Grothendieck theorem from C^*-algebras to JB^*-triples was considered notably by Barton and Friedman around 1987, but seems to be still incomplete.*"

Let me know introduce the real title of this talk.

Finally a proof for the Barton-Friedman conjecture

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[Pe., Rodríguez, *Proc. London Math. Soc.*'2001]

Let $K > \sqrt{2}$ and $\varepsilon > 0$. Then, for every complex JBW*-triple M, every complex Hilbert space H, and every weak*-continuous linear operator $T : M \to H$, there exist norm-one functionals $\varphi_1, \varphi_2 \in M_*$ such that the inequality

$$\|T(x)\|^2 \leq K^2 \|T\|^2 \left(\|x\|^2_{\varphi_2} + \varepsilon^2 \|x\|^2_{\varphi_1}\right) \tag{2}$$

holds for all $x \in M$.

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holds for all $x \in M$.

This was enough to fix the problems for all consequences.... let me borrow some words...

[Bunce, *Quart. J. Math.*'2001]

"The remarkable recent article [PeRodriguez'2001] (see also [Pe2001]) provides antidotes to some subtle difficulties in [BarFri87] and subsequent work, including certain results on the important strong* topology of a JBW*-triple N."

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holds for all $x \in M$.

Problem:

Does the inequality in (2) hold for some universal constant and $\varepsilon = 0$?

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[Pe., *Math. Inequal. Appl.*'2005]

Let A be an atomic JBW*-triple. Then for every weak*-continuous linear operator T from A into a complex Hilbert space there exists a norm-one functional $\varphi \in A_*$ satisfying

$$\|T(x)\| \leq 32\sqrt{2}\ \|T\|\ \|x\|_\varphi,$$

for all $x \in A$.

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[Pe., *Math. Inequal. Appl.*'2005]

Let V and W be atomic JBW*-triples. Then for every separately weak*-continuous bilinear form U on $V \times W$, there exist norm-one functionals $\varphi \in V_*$, and $\psi \in W_*$ satisfying

$$|U(x,y)| \leq 2^{11}\ (1+2\sqrt{3})\ \|U\|\ \|x\|_\varphi \|y\|_\psi$$

for all $(x,y) \in V \times W$.

Finally, after almost twenty years pursuing the Barton-Friedman conjecture, today we can recover the status-quo valid from 1987 to 2001.

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[Hamhalter, Kalenda, Pe., Pfitzner, *arXiv:1903.08931*]

Let M be a JBW*-triple. Then given any two functionals φ_1, φ_2 in M_*, there exists a norm-one functional $\psi \in M_*$ such that

$$\|x\|_{\varphi_1,\varphi_2} = \sqrt{\|x\|^2_{\varphi_1} + \|x\|^2_{\varphi_2}} \leq \sqrt{2} \cdot \sqrt{\|\varphi_1\| + \|\varphi_2\|} \cdot \|x\|_\psi ,$$

for all $x \in M$. Furthermore, given $K > 2$, for every complex Hilbert space H, and every weak*-to-weak continuous linear operator $T : M \to H$, there exists a norm-one functional $\psi \in M_*$ satisfying

$$\|T(x)\| \leq K \|T\| \|x\|_\psi$$

for all $x \in M$.

We can now conclude that the Grothendieck's inequality in the case of JB^*-triples is valid for semi-norms given a single functional in the corresponding dual spaces.

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Suppose $G > 8(1+2\sqrt{3})$. Let E and B be JB*-triples. Then for every bounded bilinear form $V : E \times B \to \mathbb{C}$ there exist norm-one functionals $\varphi \in E^*$ and $\psi \in B^*$ satisfying

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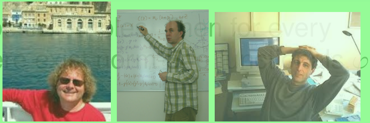

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$$|V(x,y)| \leq G \, \|V\| \, \|x\|_{\varphi} \, \|y\|_{\psi}$$

for all $(x,y) \in E \times B$.

That was all I had in mind for today.
Thanks for your time!

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