Analysis Seminar

Title: Derivatives of bivariate polynomials, Markov's theorem and Geronimus nodes

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Location: Mathematical Sciences Seminar Room (Ag 1.01)

Abstract: An outstanding problem that has been recently solved is to prove V. A. Markov's theorem for derivatives of polynomials on any real normed linear space. An elementary argument leads to a reduction of the problem to a certain directional derivative on two dimensional spaces. To state this, let $\mathcal{P}_m(\mathbb{R}^2)$ denote the space of all polynomials $p(s,t)$ of degree at most m and let

$$\mathbb{N}_k = \{(\cos(n\pi/m), \cos(q\pi/m)) : \ n - q = k2, \ 0 \le n, q \le m\}.$$

Then to prove the Markov theorem it suffices to show that the maximum of the values $|\hat{D}^k p(1,1)(1,-1)|$ over polynomials p in $\mathcal{P}_m(\mathbb{R}^2)$ satisfying $|p(x)| \le 1$ for all x in the set $\mathbb{N}_k$ of nodes is attained when $p(s,t) = T_m(s)$, where T_m is the Chebyshev polynomial of degree m. We consider more general sets of nodes, called Geronimus nodes, where the extremal polynomials sought are orthogonal polynomials satisfying a three-term recurrence relation with constant coefficients. For example, this includes the Chebyshev polynomials of kinds 1-4.

In the course of our discussion we obtain an explicit formula for Lagrange polynomials and a Lagrange interpolation theorem for the Geronimus nodes. We also deduce a bivariate cubature formula analogous to Gaussian quadrature.