UCD School of Mathematics and Statistics

University College Dublin
Belfield, Dublin 4, Ireland

Tel +353 1 716 2580
Fax +353 1 716 1196

Scoil na Matamaitice agus na Staitisticí UCD

An Coláiste Ollscoile, Baile Átha Cliath
Belfield, Baile Átha Cliath 4, Éire

Email seminars@maths.ucd.ie
Web maths.ucd.ie/seminars

Analysis Seminar

L. Harris (Kentucky)

will speak on

Derivatives of bivariate polynomials, Markov's theorem and Geronimus nodes

Tue 14th February 2012 at 3:00PM

Location: Mathematical Sciences Seminar Room (Ag 1.01)

An outstanding problem that has been recently solved is to prove V. A. Markov's theorem for derivatives of polynomials on any real normed linear space. An elementary argument leads to a reduction of the problem to a certain directional derivative on two dimensional spaces. To state this, let $\mathcal{P}_m(R^2)$ denote the space of all polynomials $p(s,t)$ of degree at most m and let

$$N_k = \{(\cos(n\pi/m), \cos(q\pi/m)) : \; n - q = k2, \; 0 \leq n, q \leq m\}.$$

Then to prove the Markov theorem it suffices to show that the maximum of the values $|\hat{D}^k p(1,1)(1,-1)|$ over polynomials p in $\mathcal{P}_m(R^2)$ satisfying $|p(x)| \leq 1$ for all x in the set N_k of nodes is attained when $p(s,t) = T_m(s)$, where T_m is the Chebyshev polynomial of degree m. We consider more general sets of nodes, called Geronimus nodes, where the extremal polynomials sought are orthogonal polynomials satisfying a three-term recurrence relation with constant coefficients. For example, this includes the Chebyshev polynomials of kinds 1-4.

In the course of our discussion we obtain an explicit formula for Lagrange polynomials and a Lagrange interpolation theorem for the Geronimus nodes. We also deduce a bivariate cubature formula analogous to Gaussian quadrature.

This talk is part of the **Analysis** series. For more, see
`https://maths.ucd.ie/seminars`