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Scoil na Matamaitice agus na Staitisticí UCD

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Algebra and Number Theory Seminar

Igor Klep (Universities of Maribor and Ljubljana)

will speak on

Central Simple Algebras, the Procesi-Schacher Conjecture, and Positive Polynomials

Mon 14th June 2010 at 4:00PM

Location: Mathematical Sciences Seminar Room

Consider a central simple algebra A with involution $*$. The involution is called emphpositive if the involution trace form $xmapstor(x^*x)$ is positive semidefinite (w.r.t. a fixed ordering of the center F of A). A symmetric element b is defined to be emphpositive if the scaled involution trace form $xmapstor(x^*bx)$ is positive semidefinite, giving rise to an emphordering of the central simple algebra A. We discuss how these can be used to give a Positivstellensatz characterizing polynomials in noncommuting variables that are positive semidefinite or trace-positive on $dimesd$ matrices. Along the way we give a counterexample to a conjecture of Procesi and Schacher. Here is a sample result:

egintheorem For a real polynomial f in n free noncommuting variables, the following are equivalent: eginenumerate[m (i)] item $r(f(A_1, ldots, A_n))geq 0$ for all $A_i in M_2(R)$; item there exist a nonvanishing central polynomial c, and a polynomial identity h of $2imes 2$ matrices, such that [c f c$^*inh+os.]endenumerateHere$ os$denotes the set of all polynomials that can be written as sums of hermitian squares$ g$^*g$ and commutators $pq - qp$. endtheorem

We shall also explain how this statement fails for $d > 2$, and how this fact pertains to the Procesi-Schacher conjecture.

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igskip The talk is partially based on joint work with Thomas Unger.

This talk is part of the **Algebra and Number Theory** series. For more, see
`https://maths.ucd.ie/seminars`