

## Algebra and Number Theory Seminar

Title:	Central Simple Algebras, the Procesi-Schacher Conjecture, and Positive Polynomials
Speaker:	Igor Klep (Universities of Maribor and Ljubljana)
Date:	Mon 14th June 2010 at 4:00PM
Location:	Mathematical Sciences Seminar Room

Abstract: Consider a central simple algebra A with involution \*. The involution is called emphpositive if the involution trace form  $xmapstor(x^*x)$  is positive semidefinite (w.r.t. a fixed ordering of the center F of A). A symmetric element b is defined to be emphpositive if the scaled involution trace form  $xmapstor(x^*bx)$  is positive semidefinite, giving rise to an emphordering of the central simple algebra A. We discuss how these can be used to give a Positivstellensatz characterizing polynomials in noncommuting variables that are positive semidefinite or trace-positive on *dimesd* matrices. Along the way we give a counterexample to a conjecture of Procesi and Schacher. Here is a sample result:

egintheorem For a real polynomial f in n free noncommuting variables, the following are equivalent: eginenumerate[m (i)] item  $r(f(A_1, ldots, A_n))geq0$  for all  $A_iinM_2(R)$ ; item there exist a nonvanishing central polynomial c, and a polynomial identity h of 2imes2 matrices, such that [ c f c\*inh + os.]endenumerateHereosdenotesthesetofallpolynomialsthatcanbewrittenassumsofhermitiansquaresg\*g and commutators pq - qp. endtheorem We shall also explain how this statement fails for d > 2, and how this fact pertains to the Procesi-Schacher conjecture.

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