



Algebra and Number Theory Seminar

Title: Exact certification in global polynomial optimization via sums-of-squares of rational functions with rational coeffs.

Speaker: Erich Kaltofen (North Carolina State University)

Date: Mon 31st May 2010 at 4:00PM

Location: Mathematical Sciences Seminar Room

Abstract: Any positive semidefinite polynomial f with real (rational) coefficients, i.e., a polynomial that does not evaluate to a negative value, can be written as a finite sum

$$f(x_1, \dots, x_n) = (g_1(x_1, \dots, x_n)^2 + \dots + g_k(x_1, \dots, x_n)^2) / g_0(x_1, \dots, x_n)^2$$

where g_i are polynomials with real (rational) coefficients. If there exist g_i with $g_0 = 1$, f is said to be SOS. This is equivalent to $f - h$ being positive semidefinite; h is a global optimization is the real infimum (or rational $h = 1/g_0^2(g_1^2 + \dots + g_k^2)$ constitute a proof (exact certificate) for the inequality/optimum. Optimization called *Positivstellensätze*.

Two recent developments have made it possible to compute such certificates. The first are the numerical optimization algorithms for semidefinite programming. The second is a symbolic technique for converting an imprecise SOS with floating point coefficients to an exact identity over the rational numbers. Among our recent successes are a proof

of the Monotone Column Permanent Conjecture for $n = 4$, which was completed shortly before the general conjecture could be established, proofs for sharp factor coefficient bounds of degree ≤ 17 ("Rump's model problem"), new SOS proofs for many known inequalities, and a deformation analysis approach to Seidenberg's problem.

In my talk I will explain how our certificates have been found and what difficulties remain. This is joint work with Sharon Hutton, Bin Li, Zhengfeng Yang, and Lihong Zhi.

<http://www4.ncsu.edu/~kaltfen/>