Analysis Seminar

Title: Lipschitz-free spaces and descriptive set theory II

Speaker: Richard Smith

Date: Tue 14th April 2026 at 3:00PM

Location: E0.32 (beside Pi restaurant)

Abstract: This is a continuation of my talk 'Lipschitz-free spaces and descriptive set theory' that I gave here last September. The study of Lipschitz-free (hereafter free) spaces is a lively area of contemporary functional analysis which interacts strongly with metric geometry and optimal transport theory. Two enduring questions are (a) when are two free spaces isomorphic, and (b) when does a free space have or fail one or more of Grothendieck's classical approximation properties? In the September talk I showed how descriptive set theory can be used to yield insights into the isomorphic theory of free spaces. In this talk I will show how it can be used to prove results about approximation properties.

Firstly, I will introduce again Vershik's Polish space of complete separable metric spaces, which serves as a natural descriptive set-theoretic framework for separable free spaces. Secondly, I will present the main result, which is that there exists a countable complete discrete metric space whose free space has the approximation property (AP) but not the bounded approximation property (BAP). To the best of my knowledge, this is the first example of a free space which (a) has the AP but not the BAP, and (b) has the Radon-Nikodým property but is not isomorphic to a dual space. Interestingly, the existence of such a space is obtained indirectly via a 'complexity

argument': I have no concrete description of the space.

Finally, I will show how Lipschitz-free spaces can be used to show that the class of separable Banach spaces having the AP is a complete coanalytic set with respect to the standard descriptive set-theoretic framework (the corresponding class of spaces having the BAP is known to be Borel).

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