# Analysis Seminar

**Title:** Isolated singularities for elliptic equations with convolution terms in a punctured ball

**Speaker:** Zhe Yu

**Date:** Tue 7th October 2025 at 3:00PM

**Location:** E0.32 (beside Pi restaurant)


**Abstract:** The purpose of this research is two-fold. First, we investigate the inequality

$$-\Delta u + V(x)u \geq f \quad \text{in } B_1 \setminus \{0\} \subset R^N, N \geq 2,$$

where $f \in L^1_{loc}(B_1)$. If $V \geq 0$ is radially symmetric, we provide optimal conditions for which any solution $0 \leq u \in \mathcal{C}^2(B_1 \setminus \{0\})$ of the above inequality satisfies $u, \Delta u, V(x)u \in L^1_{loc}(B_1)$. This extends a result of H. Brezis and P.-L. Lions (1982), originally established for constant potentials $V$. Second, we investigate the equation

$$-\Delta u + \lambda V(x)u = (K_{\alpha,\beta} * u^p)u^q \quad \text{in } B_1 \setminus \{0\},$$

where $0 \leq V \in \mathcal{C}^{0,\nu}(\overline{B}_1 \setminus \{0\})$, $0 < \nu < 1$, $\lambda, p, q > 0$ and

$$K_{\alpha,\beta}(x) = |x|^{-\alpha} \log^{\beta} \frac{2e}{|x|}, \quad \text{where } 0 \leq \alpha < N, \beta \in R.$$

For $N \geq 3$, we establish sharp conditions on the exponents $\alpha, \beta, p, q$ under which singular solutions exist and exhibit the asymptotic behavior $u(x) \simeq |x|^{2-N}$ near the origin. For $N = 2$, we provide a classification of the existence and boundedness of solutions based on the local behavior of the potential $V(x)$ near the origin.

(Based onjoint work with Marius Ghergu)

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