IMS September Meeting 2007 Seminar

Title: Unfinished business: some open questions in classical analysis

Speaker: D. Armitage (QUB)

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Location: ENG226

Abstract:

Some easily stated open problems will be discussed. (i) It is known (Zalcman, 1982) that the Radon transform is not injective: there exist non-trivial continuous functions $f : \mathbb{R}^2 \to \mathbb{R}$ with zero (proper) integral on every (doubly infinite, straight) line. All known examples of such functions have extremely rapid overall growth. Can such a function have slow growth, or even be bounded? Is it true that a continuous function on $\mathbb{R}^3$ with zero integral on every line must be identically zero? (ii) Every polygonal domain D in $\ntrianglerighteq\mathbb{R}^2$ has the Pompeiu property (PP): if $f : \mathbb{R}^2 \to \mathbb{R}$ is continuous and $\int_{\sigma(D)} f(x)dx = 0$ for every rigid motion x03C3;, then f x2261; 0. The corresponding assertion for functions on the sphere S2 is false: there are infinitely many (non-congruent) regular spherical polygons that lack PP, and they can be characterised. But it still seems unclear whether, for example, all non-trivial regular spherical triangles have PP, and whether (up to congruence) the known example of a spherical square lacking PP is unique.

(iii) One formulation of the maximum principle asserts that if h is a non-constant

harmonic function on a ball centred at the origin O in $\mathbb{R}^n$ and $h(O) = 0$, then h takes positive values and negative values on every neighbourhood of O. In the case $n = 2$, this can be quantified: it is easy to show that, with h as above, the subset of $\{x : \|x\| < r\}$ where $h > 0$ and the subset where $h < 0$ have roughly the same area. (The ratio of the areas tends to 1 as $r \to 0^+$.) What can be said in the case $n \geq 3$?