



Analysis Seminar

Title: Spirals in Spaces of Holomorphic Functions

Speaker: Andrew D. Smith

Date: Tue 17th September 2024 at 3:00PM

Location: E0.32 (beside Pi restaurant)

Abstract: Functions $W(t, z)$ of real time $t \geq 0$ and $z \in \mathbb{C}$ satisfy the spiral relation:

$$W(2t, z) = (1 + e^z)W(t, z)$$

For fixed t , these are holomorphic functions of z in the region:

$$|\Im z| < \cos^{-1} [-12e^{-|\Re z|}]$$

Viewed as functions of t , for fixed z , the functions $W(t, z)$ are Hölder continuous and nowhere differentiable. They have a time-homogeneity property if $\Re z = 0$, while for $\Im z = \pm \frac{1}{2}\pi$ the paths have finite quadratic variation; a property also associated with semi-martingale paths in the theory of stochastic processes.

The W functions can produce beautiful images. Familiar fractal sets: Lévy's C-curve, Heighway's dragon curve and van Roy's unicorn curve arise as the loci of $W(t, z)$ when $0 \leq t \leq 1$ and $z = \pm \frac{1}{2}i\pi$, that is, functions of t that satisfy both the time-homogeneity and quadratic variation criteria.

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